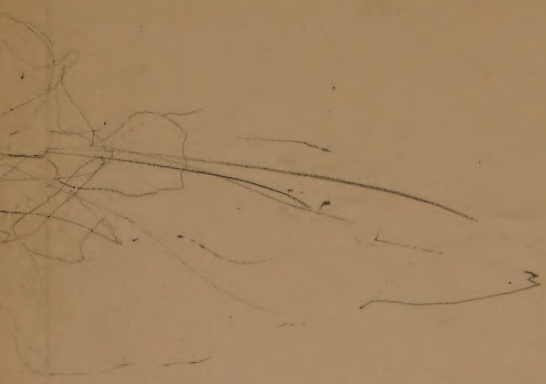
SMITH'S ARITHMETIC OF BUSINESS

LYONS AND CARNAHAN



Harvey Smith

1.00

When finding time, principal on It.
carry at least 6 places

SMITH'S ARITHMETIC OF BUSINESS

By

OLIVER S. SMITH, LL.B.

SOUTHEASTERN HIGH SCHOOL
DETROIT, MICH.

LYONS AND CARNAHAN

CHICAGO NEW YORK

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PREFACE

The course in arithmetic should do two things, at least, for the student. It should give him the ability to handle figures rapidly and accurately, and it should give him a knowledge of the problems of business and of life. The aim of the author has been to provide a course which would effect both of these results.

Accuracy and speed are developed in the first chapters through the use of material for drill which covers the field in the most economical fashion possible. The drill charts in addition, subtraction, and multiplication present all necessary combinations and give the maximum of drill in the minimum of space.

Smith's Arithmetic of Business is not devised to compel the pupil to labor through long chapters of unencouraging abstract work before interesting problems are presented. Work of a very practical and attractive character is introduced in the first chapter, and from the first chapter to the last practical problems are made the basis of the course.

These practical problems are not incidental — not merely fortunate selections for the illustration of arithmetical processes. They are the heart of the course. No process or method is taught that does not lead directly to the solution of some practical and important problem which the student knows that he will some time meet. It is unnecessary to state that this motivates the student's work at every stage.

The problems touch both the accounting side and the practical side of virtually every important line of trading, manufacture, or other occupation. There are many important problems involving the relations of individuals to each other and to the government that are treated. Problems of the home, of the investment of savings, of taxes, of municipal expenditure, of farm management, of corporation financing — what citizen but comes into contact with many such problems outside of the daily routine of his vocational employment?

Many of these important problems can be introduced when the student has had addition and subtraction only, and it is a matter

of some astonishment that no text heretofore published has done this to any extent. The average student is tremendously bored with the first few weeks of his work in arithmetic. The first few pages of many well-known texts are little used. The teacher knows that the student needs practice in addition, subtraction, multiplication, and division at the outset, but the student seldom feels this necessity. Too often, therefore, the teacher passes lightly over this part of the work, trusting to incidental practice throughout the year to bring facility. *Smith's Arithmetic of Business,* through its early use of practical, interesting material, obviates this difficulty.

This book presents business conditions as they exist. The material for the problems was obtained from first hand investigation in the various lines of business presented. The problems are modern and new. The conditions surrounding the problems are accurately portrayed. Textbook traditions have not been the guide, but every process, every rule, every fact has been subjected to the scrutiny of experts in the line of business being developed in each case.

The strongest statements that can be made on behalf of *Smith's Arithmetic of Business* are: (1) That it introduces practical work at the very start, making the work in the processes interesting and effective; (2) that the central idea of the course is an effective presentation of the problems of business and of life, nothing being included that does not relate thereto; (3) that practically every important type of problem has been included; and (4) that the problems are new, and modern, and faithfully present modern business conditions, as will be seen by referring to Chapter 39, which was prepared by M. E. Studebaker, Director of Commerce Dept., Ball State Teachers College, Muncie, Indiana.

Attention is also directed to the plan of standardization which has been followed throughout. In connection with each class of drill problems a time standard has been established wherever practicable. Experience has shown that best results can be secured when students are given, in this definite way, an understanding of what is expected of them—a reasonable standard of attainment.

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COMMERCIAL ARITHMETIC

CHAPTER I

ADDITION PRACTICE

First Step. The first step in learning to add is to master all possible combinations in the addition of two numbers of one digit each. Without reversing the order, or using zero, there are 45 of these, as shown in the following table:

1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1
	2	3	4	5	6	7	8	9
	2	2	2	2	2	2	2	2
		3	4	5	6	7	8	9
		3	3	3	3	3	3	3
			4	5	6	7	8	9
			4	4	4	4	4	4
				5	6	7	8	9
				5	5	5	5	5
					6	7	8	9
					6	6	6	6
						7	8	9
						7	7	7
							8	9
							8	8
								9
								9

There are 100 combinations in simple addition. These are the 45 given on the previous page, 45 formed by using the same numbers in reverse order, and 10 found by combining zero with the 9 digits and with itself.

These 100 combinations are given below arranged in the order of difficulty as shown by recent extended investigations, the most difficult being placed first.

Oral Drill Chart No. 1

6	8	9	7	9	5	9	3	3	2	5	4	8	7	7	9	7	2	7	9
9	5	5	9	6	8	7	9	2	3	9	8	7	5	8	4	3	2	1	8
6	5	8	3	9	8	3	6	0	2	8	6	4	0	0	4	5	0	0	0
8	7	6	4	3	3	3	2	6	6	0	7	9	9	4	6	3	3	8	7
0	1	5	3	6	6	6	4	9	3	8	0	7	1	8	7	2	3	3	2
1	9	6	8	0	3	6	7	0	7	9	5	0	7	4	4	8	6	0	5
0	6	4	2	2	1	6	8	7	5	4	5	2	5	4	9	2	7	6	5
2	4	0	0	9	0	1	8	2	2	3	1	4	0	1	1	1	6	5	5
4	3	9	1	8	2	1	1	0	9	4	4	1	1	3	1	1	8	5	7
4	5	2	6	2	7	5	4	0	9	5	2	3	8	1	2	1	1	4	7

Beginning with 0, count by 2's to 94; by 3's to 84; by 5's to 125; by 6's to 96; by 7's to 112; by 8's to 128.

Beginning with 16, count by 7's to 149.

Beginning with 19, count by 4's to 91.

Beginning with 0, count by 12's to 156; by 9's to 117.

Beginning with 21, count by 10's to 111.

Beginning with 56, count by 11's to 287.

Beginning with 0, count by 15's to 225, by 14's to 196.

Second Step. The second step in learning to add is to master all possible new combinations arising from adding another one-digit number to each mental result of the 100 combinations. One hundred of these are shown in Oral Drill Chart No. 2.

ORAL. Drill on the following until the total of each group can be given without hesitation.

Oral Drill Chart No. 2

6	8	9	7	9	5	9	3	3	2	5	4	8	7	7	9	7	2	7	9
9	5	5	9	6	8	7	9	2	3	9	8	7	5	8	4	3	2	1	8
1	0	6	5	3	7	8	2	4	9	8	3	2	9	0	5	4	1	6	7
6	5	8	3	9	8	3	6	0	2	8	6	4	0	0	4	5	0	0	0
8	7	6	4	3	3	3	2	6	6	0	7	9	9	4	6	3	3	8	7
5	0	3	9	1	2	6	7	8	4	5	0	7	6	9	8	2	1	9	3
0	1	5	3	6	6	6	4	9	3	8	0	7	1	8	7	2	3	3	2
1	9	6	8	0	3	6	7	0	7	9	5	0	7	4	4	8	6	0	5
6	7	0	2	9	8	5	3	1	4	9	0	6	5	3	1	7	2	4	8
0	6	4	2	2	1	6	8	7	5	4	5	2	5	4	9	2	7	6	5
2	4	0	0	9	0	1	8	2	2	3	1	4	0	1	1	1	6	5	5
1	3	2	5	8	7	9	6	0	4	7	0	9	6	2	5	4	3	1	8
4	3	9	1	8	2	1	1	0	9	4	4	1	1	3	1	1	8	5	7
4	5	2	6	2	7	5	4	0	9	5	2	3	8	1	2	1	1	4	7
4	6	8	2	9	5	4	3	0	1	5	1	0	7	2	4	3	9	8	6

Each of the foregoing rows of figures should be added in from 20 to 30 seconds. Do better if you can.

TEACHER'S NOTE. It is not sufficient for the student to be able to compute accurately with figures that are written or printed. He should be taught to concentrate upon what he hears as well as upon what he sees. Read the foregoing numbers aloud, calling upon the students in turn for the sums. Do not permit the use of pencils nor allow books to be open.

Third Step. The third step in learning to add is to master all possible new combinations arising from adding to the mental total of a column at any point a single digit which will change the figure in the tens column of the mental total. Seventy-three of these are shown in Oral Drill Chart No. 3.

Oral Drill Chart No. 3

32	68	59	39	76	21	88	43	87	57	69	35
8	6	5	8	4	9	9	7	3	9	6	7
65	54	74	38	92	69	45	86	79	29	98	59
5	9	7	3	9	9	8	7	3	2	2	9
49	87	26	79	47	75	87	36	85	68	96	47
7	8	6	9	4	8	9	7	9	9	8	7
97	49	36	65	58	88	27	59	78	39	97	79
5	9	8	6	7	4	6	8	7	9	6	7
25	68	53	77	36	79	66	28	89	98	48	27
9	8	9	7	9	8	6	7	9	8	9	9
83	99	28	47	96	56	38	19	64	44	99	54	29
8	9	9	8	9	5	8	9	9	8	8	6	9

It is suggested that the class be drilled upon this chart frequently.

Add, reading upward.

1.

8	4	6	8	3	6	4	2	9	3	9	6	7	6	9
7	6	3	6	9	5	9	7	1	7	3	8	6	3	1
3	3	5	4	6	8	7	3	7	8	8	9	9	7	8
2	2	5	2	5	4	8	8	4	2	4	7	5	4	2

2.

4	7	6	7	9	6	4	8	3	8	4	4	3	8	7
9	2	8	6	8	3	6	2	7	6	7	4	9	6	2
6	8	9	9	2	7	7	4	9	3	6	8	4	3	6
3	7	1	4	7	4	3	6	4	5	2	6	4	3	4

3.

9	7	3	4	9	6	9	6	7	9	3	9	6	7	9
6	6	8	8	6	6	4	8	6	8	8	8	6	7	8
3	4	5	4	8	8	6	7	5	6	4	8	7	3	6
2	6	4	4	4	8	3	3	2	4	5	6	4	8	4

4.

7	8	3	6	7	2	5	9	4	6	4	7	5	4	7
4	7	6	6	7	2	5	9	9	7	5	4	8	9	3
8	6	4	9	8	4	4	6	5	9	6	3	7	6	4
9	2	8	9	8	4	4	6	8	3	8	7	2	1	6

Add, reading downward.

5.

9	8	7	6	9	9	4	8	3	8	9	4	8	8	7
3	6	9	9	5	8	8	7	5	6	5	7	8	7	8
6	9	7	3	4	5	6	9	9	8	8	6	9	6	8
2	5	7	5	2	8	9	5	7	4	7	3	4	4	9

6.

6	3	7	8	6	4	5	4	6	4	7	6	9	4	8
7	5	6	9	4	8	6	9	8	6	5	9	5	7	3
8	9	7	3	9	6	6	3	3	4	7	6	9	4	8
9	6	9	8	8	2	3	8	9	8	3	7	8	9	9

7.	5	3	4	3	6	4	5	9	4	7	3	6	5	3	6
	3	8	9	8	9	9	9	9	8	9	7	4	9	9	6
	8	6	7	9	8	8	8	6	8	8	8	7	6	4	4
	9	9	8	4	5	6	3	6	7	7	9	8	8	4	8

8.	4	7	9	7	4	3	6	4	7	3	4	5	7	9	8
	9	8	4	6	3	8	3	7	8	8	7	8	7	9	9
	6	6	8	8	8	7	8	8	6	7	6	3	8	6	9
	3	4	7	7	9	4	7	9	5	9	3	4	8	6	8

WRITTEN. An important aid in addition is the ability to see instantly groups or combinations of numbers the sum of which is 10. Study the following before adding. Then add, writing totals only. Add the twelve totals of each problem for a grand total. Try to add the 1st and 2d problems in $1\frac{1}{2}$ minutes each, the 3d in 2 minutes, the 4th in 3 minutes and the 5th in 4 minutes.

	a	b	c	d	e	f	g	h	i	j	k	l
1.	87	39	47	43	29	36	42	45	54	39	57	26
	53	74	62	82	32	43	34	69	36	71	62	35
	56	86	41	25	56	21	66	38	73	63	38	65
	42	36	47	26	82	86	18	62	57	47	72	47
2.	47	63	54	42	37	39	63	67	32	65	59	69
	53	49	38	63	21	72	54	49	78	36	86	45
	78	21	96	41	80	68	49	38	94	84	34	38
	22	67	72	24	92	21	74	73	78	32	73	62
3.	37	45	52	26	29	67	92	65	46	49	59	58
	46	27	63	32	24	43	26	50	38	37	62	72
	59	32	78	57	66	55	82	75	95	81	38	64
	38	61	27	74	83	98	55	81	64	45	71	97
	72	74	56	88	57	68	87	76	79	63	46	82
	63	26	24	73	25	74	98	63	81	74	92	37

4.	23	48	75	34	15	83	97	27	52	29	25	36
	84	68	59	96	36	42	45	46	91	50	36	38
	33	84	76	82	64	68	63	83	78	49	58	29
	57	86	82	38	51	77	38	27	35	83	17	41
	29	44	18	75	78	81	74	62	61	71	93	58
	58	55	95	58	83	63	51	74	74	59	58	63
	64	67	16	64	16	92	63	39	53	63	27	45
	56	48	49	93	62	74	66	47	89	75	39	84

5.	45	64	53	29	74	62	59	39	54	37	94	36
	62	37	89	64	36	91	36	29	79	95	16	59
	37	83	76	32	92	37	39	86	93	61	39	74
	95	61	44	71	57	58	74	36	85	73	61	78
	61	39	59	56	81	29	58	57	62	54	74	35
	74	45	31	39	45	64	37	27	23	81	26	62
	37	61	78	58	63	59	89	92	94	39	49	70
	28	27	95	74	78	27	57	81	76	64	27	81
	64	39	27	96	56	32	65	36	53	78	67	59
	75	25	36	15	39	78	39	35	85	64	32	27
	39	18	58	17	78	56	87	42	64	27	57	68
	14	94	97	53	12	37	49	28	53	35	28	75

ORAL. The following figures may be read horizontally and vertically. The students should add, silently, each number as it is read by the teacher. When the last number of any row or any column is called the students should give the total in unison.

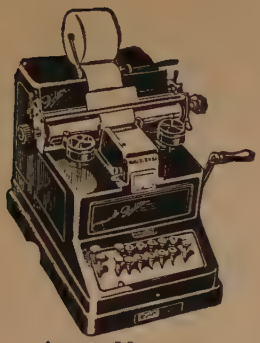
	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s
1.	6	7	8	9	4	3	7	6	9	4	3	4	7	8	4	9	3	8	7
2.	9	6	3	8	7	4	2	9	3	8	7	4	6	3	9	7	4	3	8
3.	4	3	0	4	7	3	6	8	9	4	7	7	4	3	8	5	0	1	6
4.	9	0	6	5	3	4	5	5	6	8	2	9	0	3	7	4	3	8	7
5.	3	8	6	4	7	3	9	0	6	8	4	2	3	7	9	8	6	5	9
6.	4	6	8	3	2	9	0	7	4	2	6	9	3	8	1	4	8	3	7

7.	3	9	4	6	4	8	6	9	0	8	8	9	2	5	5	5	8	7	6
8.	7	6	3	8	9	4	2	6	7	8	3	9	6	7	4	7	8	4	8
9.	3	4	9	8	3	6	4	9	2	7	3	0	9	6	7	6	3	8	5
10.	4	4	9	8	3	7	6	2	0	9	5	5	4	6	3	8	5	6	4
11.	6	8	5	7	4	7	8	9	6	4	3	8	3	6	4	0	8	9	0
12.	2	4	6	4	9	4	6	3	9	2	4	7	8	2	3	8	9	5	4
13.	9	6	9	3	7	8	7	4	4	7	8	0	6	7	3	3	8	8	9
14.	7	9	1	2	3	9	4	6	9	2	4	5	4	3	8	4	3	6	8
15.	2	2	3	6	8	4	8	3	2	3	5	9	2	6	4	6	7	8	3
16.	8	6	3	9	0	4	3	7	9	5	8	6	4	9	3	4	2	7	8
17.	9	3	8	5	3	7	6	5	4	7	5	9	6	3	8	7	5	9	8
18.	5	4	3	7	8	9	6	4	5	9	4	3	8	9	5	4	7	3	8

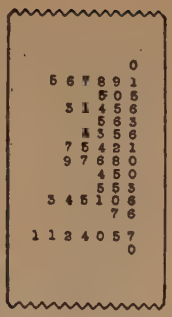
WRITTEN. Try to add each in 18 or 20 seconds.

1.	2.	3.	4.	5.	6.	7.
$43.50	$63.75	$25.09	$67.50	$ 7.47	$ .70	$ 65.43
60.00	47.00	33.00	245.46	236.00	22.40	350.40
46.00	85.00	40.50	74.40	420.00	10.20	7.60
35.00	35.20	25.38	754.79	158.47	750.00	18.32
22.50	22.00	51.99	346.00	42.50	900.00	128.31
14.00	417.06	25.00	24.46	64.88	242.67	8.00
62.07	204.66	36.35	54.79	36.06	31.87	14.06
352.67	635.45	46.24	36.68	15.55	35.44	25.29
406.00	344.00	135.00	43.69	457.46	241.70	103.64
204.00	200.35	200.00	27.47	123.00	403.06	1205.29
213.20	35.30	300.00	53.69	324.35	55.00	1250.00
214.60	24.06	600.23	25.40	45.90	70.40	2.63

WRITTEN. The best results can be secured if the following columns are added under competitive conditions. The teacher, noting the time of starting, should give the signal to the students to start. The students should all start at the same instant. As each student indicates that he is through, the teacher can tell him



ADDING MACHINE



SPECIMEN OF WORK DONE ON ADDING MACHINE

his exact time in minutes and seconds. The first student to hand in correct results is the winner in the contest. Friendly rivalry, such as these contests inspire, adds zest to the work and gives increased speed from day to day such as can be secured in no other way. Students should in time become able to complete columns such as these in forty seconds or less for each column.

1.	2.	3.	4.	5.	6.	7.
7627	4381	4578	4258	3264	4639	7824
4432	6297	6536	6479	7946	3066	4636
3396	3811	2568	3624	8309	4947	3899
4623	7292	4479	2457	4093	3073	6473
9287	4623	2529	2413	6377	6048	3868
4630	3269	7342	9850	4708	4724	7487
3841	7426	3456	4620	3824	3966	4966
4683	3611	5667	3449	7418	4747	4389
4792	1724	6634	7357	1929	4383	6474
4638	3127	1457	3624	6346	7014	4967
7426	3674	4325	6348	9633	4029	7438
7152	2119	5779	3521	8748	3723	8693
4675	4642	2435	5155	4997	8648	4622
3874	3874	3524	6369	7634	2964	3874
3946	4960	2357	1936	3875	3085	3297
4781	3811	3260	1779	4636	3706	4646
4354	4672	5745	7256	9387	4697	3897
3926	7821	3943	5926	7463	3873	6439

8.	9.	10.	11.	12.	13.	14.
31466	23253	62438	2429	398	35644	2636
1035	236	3960	673	11000	2431	11324
167	11100	983	11238	21369	23056	40072
657	368	2469	3087	198	4011	22693
855	44578	11240	7494	91245	75087	1445
44	173	800	646	2537	913	337
45089	36111	1100	1073	62636	15578	8088
753	589	2109	14737	31111	604	11057
22036	945	382	3088	942	384	4178
391	246	96	963	821	41000	9064
41815	10367	11246	47209	177	21245	38646
11356	751	3284	38041	379	6500	24825
71234	782	963	74	24065	61489	2737
571	57	1482	6036	891	3013	6188
12401	232	7328	808	21001	398	324

15.	16.	17.	18.	19.	20.	21.
$25.61	$ 7.84	$23.62	$29.86	$48.54	$45.91	$241.30
38.92	5.38	19.24	2.58	2.95	27.48	13.00
45.09	2.19	6.70	63.71	56.23	52.63	45.00
6.73	63.42	23.51	4.23	32.41	5.67	1.24
24.20	100.40	3.00	9.45	20.03	82.95	69.35
.68	202.00	6.00	16.72	35.20	52.84	25.79
31.40	31.01	8.00	3.49	6.00	13.41	33.17
4.25	10.40	54.07	8.95	7.00	5.64	100.00
.70	37.35	24.26	26.78	17.00	28.32	36.35
9.14	13.00	547.00	31.06	14.00	9.65	325.46
146.00	7.00	350.60	45.71	10.00	14.58	11.36
326.20	8.00	255.40	8.24	8.00	9.24	12.63
3.24	9.00	325.00	65.39	40.47	18.72	21.46
5.27	22.00	4.00	7.89	210.75	27.95	43.79
6.48	60.00	6.00	24.64	63.20	16.43	36.13
8.43	3.00	78.00	35.92	27.58	5.67	22.60
2.69	7.58	56.00	4.81	9.07	28.34	127.46
35.36	5.24	45.00	15.82	35.57	56.27	102.79
217.61	33.70	60.00	19.28	476.90	25.46	24.48

WRITTEN. Errors made in addition are largely the results of disorderly columns and illegible figures. The student should make plain figures and the figures should be written in orderly columns.

1.	2.	3.	4.	5.	6.
$7.46	$13.74	$7.46	$6.34	$2.84	$98.63
3.89	6.38	.48	12.63	13.43	137.37
16.04	.97	112.60	138.48	6.68	158.39
3.74	112.63	38.92	6.90	8.73	230.50
112.00	39.86	6.47	14.89	114.37	100.20
6.89	4.73	146.04	396.21	23.89	53.00
4.63	45.00	29.78	78.36	174.60	60.88
4.67	68.00	6.39	6.74	11.74	27.09
387.09	1.48	14.74	14.85	2.26	260.36
11.00	3.86	196.00	9.43	9.38	361.47
4.00	.97	38.64	138.05	7.93	102.60
6.27	146.00	19.36	60.36	26.67	4.60
.79	11.80	7.38	174.69	4.08	5.50
3.86	9.63	.47	63.77	13.46	7.89
12.72	.79	1.25	118.44	8.74	303.00
14.56	1.04	12.00	90.32	.37	3.67
3.86	.38	6.00	17.85	112.08	11.30
17.94	14.50	3.21	3.07	10.73	22.00
7.86	6.72	17.40	4.73	28.49	6.33
4.32	386.00	8.06	14.99	9.36	750.00
7.24	112.60	10.14	126.67	17.64	450.00
.86	39.40	386.00	38.34	2.07	789.00
32.49	7.86	74.14	9.85	3.73	5.43
72.83	3.87	48.96	178.93	281.88	3.68
136.00	116.00	52.47	38.12	74.39	5.20
4.78	49.00	3.60	7.48	6.70	7.80
6.97	7.82	17.46	38.60	.40	24.00
3.87	6.47	6.38	9.09	397.27	52.50
138.40	3.86	126.14	16.36	11.92	67.30
10.10	.74	38.69	3.84	40.01	24.00
3.89	14.36	3.84	217.73	16.34	41.46

NOTE. Additional work may be provided for the class by having the students add the above columns omitting the 2 bottom numbers, 4 numbers, etc.

Horizontal Addition

It is sometimes necessary to add columns horizontally in preparing statistical reports, comparative statements, etc. Add the following numbers horizontally:

1.	2.	3.
624+312+386=	327+268+321=	741+638+841=
327+874+472=	726+432+729=	783+392+396=
436+692+386=	232+455+622=	417+467+782=
532+461+347=	343+529+628=	413+524+635=
214+543+471=	613+421+336=	131+228+769=

4.	5.	6.
475+496+312=	462+387+474=	397+461+378=
474+386+474=	369+246+382=	264+216+387=
382+675+464=	746+412+370=	464+327+496=
763+126+698=	542+255+868=	973+413+798=
987+552+839=	174+787+999=	285+654+546=

In this form of addition care must be taken to see that units are added to units, tens to tens, etc., the same as in vertical addition. The numbers should not be rearranged in vertical columns, but added as they stand.

Time yourself upon Problems 7 to 10.

7. The following sales were made in the different departments of a store:

Department	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Totals
Dry Goods.....	$540.20	$638.92	$684.12	$700.92	$562.50	$982.60	.....
Groceries.....	1178.90	973.47	853.61	912.60	873.26	1239.47	.....
Clothing.....	358.50	470.25	532.46	510.72	465.38	549.38	.....
Furniture	780.20	715.92	683.70	749.38	510.20	732.60	.....
Boots and Shoes	257.12	284.32	213.46	312.30	262.49	385.72	.....
Totals.....	.....	.....	.....	.....	.....	.....	.....

Find the total sales of each department, the total sales for each day and the total sales for the week. The total of the daily totals should agree with the total of the departmental totals; this gives the student an absolute check upon the accuracy of his work.

8. The following represents the expenditures by an electric railroad company on its different lines during one fiscal period.

	City Line	Rapid Ry.	D. M. & T.	D. J. & C.	Total
Engineering and Superintendence	$ 1,861.00	$17,679.45	$ 1,729.45	$ 3,780.50	
Right of Way	41,819.27	15,300.77	67,809.37	28,571.10	
Other Land used in Electric Operations	100.00	75,558.19	2,440.00	2,500.00	
Grading	26,535.92	38,529.69	3,357.19	11,485.49	
Ballast	3,812.06	35,750.58	1,237.96	5,662.12	
Ties	1,629.39	17,338.94	1,020.42	1,759.23	
Rails, Rail Fastenings, and Joints	4,358.95	52,426.07	1,214.86	3,473.27	
Special Work	67.90	27,461.42	1,013.44	105.98	
Paving	1,707.07	36,017.09	1,755.58	2,855.03	
Track Laying and Surfacing	5,182.21	36,478.70	1,487.40	2,347.80	
Bridges, Trestles, and Culverts	11,353.24	761.54	7,146.24	10,036.56	
Crossings, Fences, Cattle Guards, and Signs	2,591.31	487.40	2,035.58	94.20	
Interlocking and Other Signal Apparatus	7,719.72	6,156.09	190.33	499.20	
Telegraph and Telephone Lines	207.60	105.59	13.50	18.40	
Poles and Fixtures	1,030.47	3,427.84	5,858.23	1,314.72	
Distribution System	8,459.08	76,661.26	1,290.10	2,843.89	
Shop Equipment	2,752.51	21,294.69	395.21	35.18	
TOTALS					

Find the total expenditures for each account, and for each line; and find the total for all accounts on all the lines. What absolute check upon the accuracy of your addition have you?

9. The following is the trackage table of the C. C. C. & St. L. Railway Co.

MAIN LINE		Miles Main Track	Miles Second Track	Miles Yards and Sidings	Total Miles
FROM	TO				
Cleveland, Ohio	Columbus, Ohio	137.58	68.40	143.78	
Columbus, Ohio	Springfield, Ohio	45.13	.74	29.10	
Springfield, Ohio	Ludlow Grove, Ohio	72.82	28.25	66.46	
Delaware, Ohio	Springfield, Ohio	49.89	2.06	41.09	
Galion, Ohio	Indianapolis, Ind	203.11	6.58	124.45	
Indianapolis, Ind	E. St. Louis, Ill	261.62	102.92	169.31	
Hillsboro, Ill	Lenox, Ill	43.64	43.59	11.09	
Cincinnati, Ohio	Lafayette, Ind	170.71	79.02	144.05	
Templeton, Ind	Kankakee, Ill	56.72		34.61	
Cairo, Ill	Danville, Ill	261.64	5.93	164.22	
Sandusky, Ohio	Springfield, Ohio	129.92		38.79	
Benton Harbor, Mich	North Vernon, Ind	248.17	1.14	82.81	
TOTALS					

Find the total trackage between each city and the next; the total mileage of main track, second track, and yards and sidings; and the total of all trackage.

10. A traveling salesman hands in the following expense account for October:

Wilson & Gibbons—Travelers' Monthly Expense Sheet

Salesman _____ Month of _____ 19___

Date	Carfare	Hotel	Baggage	Incidentals	Total
1	2 78	2	1 20	2 15	
2	1 70	2 50	90	3 10	
3	1 10	3	75	2 80	
5	2 54	3	1 40	3 40	
6	3 25	2 50	1 80	1 75	
7	90	2	15	50	
8	2	2 50	80	75	
9	2	2	70	60	
10	40	2 50	10	25	
12	1 35	3	45	1	
13	1 12	2 50	65	90	
14	1 60	2 50	45	1 25	
15	85	2	60	40	
16	1 10	2	75	25	
17	3	2	1 20	1 10	
19	1 80	2	40	30	
20	65	2 50	20	80	
21	1 25	2 50	1 10	45	
22	30	2	20	20	
23	96	2	40	60	
24	50	2 50	20	10	
26	42	2 50	20	18	
27	70	2	25	1	
28	3 12	3	1 40	20	
29	2 40	2 50	1 80	2 35	
30	1 76	2	85	47	
31	2 20	2 50	1	1 15	

Find the total expense for each day, the total expense of each kind, and the total of all expense for the month.

Proofs of Addition

In business houses the additions made by a clerk cannot be checked for accuracy like schoolroom exercises. The clerk himself must prove his work correct. Following are several methods by which the accuracy of addition may be proved:

(1) **Perform the addition twice,** adding downwards the first time and upwards the second time. This is the most common method of verification and is probably as good as any under ordinary conditions. If the second addition is absolutely independent of the first, and the same result is secured both times, the result can be relied upon.

(2) **Cashier's method.**

	Proof
342	
697	
846	34
372	45
496	34
382	3884
749	
3884	

Add each column independently, placing the right hand figure of each column total under the second figure of the total above it, as shown. Then add the column totals as they stand. This method of proof is a good one for cashiers, bank clerks, and others whose work is subject to frequent interruption.

(3) **Using 9 as a check figure, or casting out the 9's.**

462 —	3
786 —	3
436 —	4
387 —	0
694 —	1
783 —	0
564 —	6
4112 —	17
8	8

First cast out the 9's from the first number in the column thus: From the sum of all three digits (4+6+2=12), take the highest multiple of nine that you can (in this case, 9.) The difference (3) is set to the right of the number, as in the illustration.*

* This is in effect finding the remainder after dividing by 9. Write any multiple of 9 (as 27, 126, or 648) and you will see that the sum of its digits is some multiple of 9. If in any number the sum of its digits is in excess of 9, that excess is the same as the excess found by casting the 9's out of the figure itself or the remainder after division by 9. Thus 651 exceeds 648, a multiple of 9, by 3; and 6+5+1=12, which exceeds 9 by 3. Find all check figures.

Now check the total, 4112, against the total of excess numbers, 17, in the same manner. The excess of 4112 after casting out the 9's is 8. (4+1+1+2=8). The excess of 17 after casting out the 9's is 8. Both results being the same, the addition is probably correct.

This is not an absolute check upon accuracy, as it will not detect a transposition, an omission of nine, nor a "slide" in figures, *i.e.*, the writing of a figure in the wrong column.

NOTE. To determine upon the check figure of an amount it is not always necessary to add all its digits and then cast out the 9's from the total. A 9 can be rejected entirely — that is, neither added into nor cast out of the total. Thus in finding the check figure for the number 93645275 the first figure (9) is rejected entirely (because it is a 9 — why add it and then cast it out? Just ignore it); next, reject the 3 and the 6 because their sum is 9; for the same reason reject the 4 and 5, and the 2 and 7. This leaves only the 5. Use 5 as the check number.

It must be understood that no checking or proving system can reveal an error which results from writing a wrong figure or an illegible one. Write your figures carefully and plainly.

For a drill in the use of the check figure 9 prove the addition of the problems on page 9.

(4) Using 11 as a check figure, or casting out the 11's.

9.60 —	3
11.40 —	7
3.15 —	7
3.00 —	3
27.15	20

First cast out the 11's from the first number in the column, thus: From the total of the figures in the odd places (0 and 9), take the total of the figures in the even places (6). The difference (3) is set to the right of the number, as in the illustration.*

If the total of the figures in the odd places is less than the total of the figures in the even places, add 11 or any multiple of 11 to the total of the figures in the odd places, and subtract the total of the figures in the even places, as in the second number of the illustration. $(0+1+11=12;\ 4+1=5;\ 12-5=7.)$

Proceed in the same manner with the remaining numbers and the results will be as follows: 7, 3.

Then take the totals 27.15 and 20 and check in the same manner. The result for 27.15 will be 9. $(5+7=12;\ 1+2=3;\ 12-3=9.)$ The result for 20 will be 9. $(0+11=11$, and $11-2=9.)$ Both results being the same (9), the addition is assumed to be correct.

The 11 check is not infallible, as it will not reveal certain errors, *e.g.*, a "slide" of two places, as when \$8 is written for 8 cents, or 3 cents for \$3.

These "slides" frequently occur, as any bookkeeper well knows. Even the adding machine is not proof against such mistakes, and when a list of figures has been added on an adding machine, the entire list should be carefully

* This is in effect finding the remainder after dividing by 11. Write any multiple of 11 (for instance, 77, 132, or 1254), and you will see that the sum of figures in the even columns agrees with the sum of figures in the odd columns. If in any number the sum of the figures in the odd columns exceeds the sum of the figures in the even columns (as in 136, in which $6+1$ is in excess of 3), this excess (4) invariably indicates the excess of the whole number (136) over the next lower multiple of 11 (132). $.136-132=4$.

checked back to see that the amounts have been properly listed. This serves to show that no device, mechanical or otherwise, will take the place of a clerk who is at all times absolutely accurate in addition.

For a drill in the use of 11 as a check figure, prove the addition of the problems on page 10.

It is not always necessary to perform fully the additions suggested above. Often time and effort can be saved by "pairing off" figures in the even columns against the same figures in the odd columns. This process will be described in detail later.

Business Computations

1. John Blackwell, a salesman for the Grand Rapids Furniture Co., made sales as follows on Saturday, May 16, 1927: 1 dresser, $35; 1 brass bed, $32.48; 1 set chairs, $19.50; 1 rocker, $11.75; 1 stand, $6.25; 1 ice box, $14.56; 1 pedestal, $4.98; 1 electric lamp, $7.78; 1 small rug, $3.84; 1 lawn swing, $9.87; 1 mirror, $1.39; 1 stove, $27.82. What were his total sales for the day?

2. Following is E. W. Carson's purchase book for June. Find the total purchases for the month.

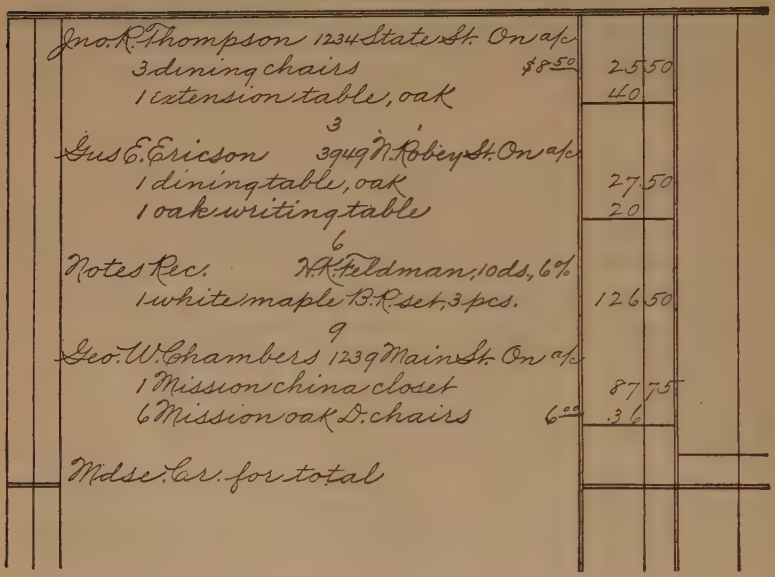
Purchases, June, 19—

19—					
June	2	Groves & Rood	2/10 1/30	199	60
	7	Walker & Co.	2/10 1/30	356	25
	8	Carpenter Bros.	3/10 1/30	582	43
	10	Walker & Co.	2/10 1/30	462	51
	13	Carpenter Bros.	2/10 1/30	322	46
	16	Groves & Rood	2/10 1/30	59	84
	25	Groves & Rood	2/10 1/30	161	91
	30	Mdse. Dr. for total			

From what one house did he purchase the most?

3. The purchase book of Miller Brothers showed the following purchases for the month of April, 1928: $137.40, $286.90, $374.29, $11.67, $58.71, $120.17, $378.56, $1163.40, $796.80, $211.64, $14.51, $2.39, $470.42. What were the total purchases made by them during this month?

4. Herewith is an extract from McGuire & Co.'s sales book. Find the total of each sale, and the total of sales for the period covered.


October 2, 19—
Jno. R. Thompson 1234 State St. On a/c
3 dining chairs $85.00 25.50
1 extension table, oak 40
3
Gus E. Ericson 3949 N. Robey St. On a/c
1 dining table, oak 27.50
1 oak writing table 20
6
Notes Rec'd. H. K. Feldman, 10 ds, 6%
1 white maple B. R. set, 3 pcs. 126.50
9
Geo. W. Chambers 1239 Main St. On a/c
1 Mission china closet 87.75
6 Mission oak D. chairs 6.00 36
Mdse. Acct. for total

5. William Douglas, on January 1, 1927, had a building and lot worth $4500; cash in bank, $1200; notes due him, $754.12; debts due him, $1387.46; First National Bank stock, $1471.85. What were his total assets on that date?

6. The Union Gas & Electric Company has 5 cashiers. Their collections by the day for one week follow in the order of the days of the week: Miss Hoyt, $473.92, $511.60, $527.83, $431.57, $397.46, $629.31; Miss Wells, $276.47, $385.91, $302.78, $297.46, $356.92, $407.38; Miss Stuart, $189.46, $275.41, $237.94, $185.16, $212.86, $295.85; Miss Carrol, $285.70, $163.74, $396.75, $297.60, $218.79, $314.82; Miss Barker, $116.93, $257.83, $274.12, $256.90, $212.74, $297.87. Using imaginary dates, arrange the above in a form similar to the form used for problem 7, showing the total collections for each cashier for one week, and the total collections

for the office for each day. Then find the total collections for the week.

7. Henry Collister made the following weekly collections from six vending machines located in the city park:

Date 1917	Machine No. 248	Machine No. 679	Machine No. 387	Machine No. 475	Machine No. 638	Machine No. 846	Totals
Mar. 7	11.63	4.78	9.24	4.62	3.29	8.74	
14	9.70	11.20	3.87	5.12	6.37	9.60	
21	10.32	9.87	5.32	7.24	7.63	4.71	
28	9.29	8.78	3.29	6.52	7.12	8.22	
Apr. 4	11.63	7.46	5.87	7.90	6.84	9.27	
11	8.54	4.60	4.74	4.18	5.30	7.38	
18	9.60	4.44	3.86	7.32	6.45	6.43	
25	10.39	6.29	9.74	8.78	7.81	9.81	
May 2	7.42	5.58	3.87	4.29	3.92	7.12	
9	6.27	6.15	8.42	7.83	6.87	6.39	
16	4.78	8.36	11.54	9.27	8.54	4.63	
23	3.70	5.48	6.28	7.43	6.39	3.86	
30	7.12	9.84	5.36	6.39	5.52	7.19	
June 6	11.84	10.39	7.82	7.26	6.97	6.87	
13	6.82	7.63	8.12	8.90	7.86	8.26	
20	3.97	4.74	5.65	6.38	5.62	7.42	
27	6.43	8.65	7.40	7.43	6.87	6.38	
July 4	19.81	11.49	3.82	9.85	8.90	4.42	
11	12.25	3.82	6.97	6.05	4.60	6.97	
18	7.43	4.62	4.38	4.27	3.71	7.29	
25	6.29	3.96	7.94	8.15	4.63	6.84	
Aug. 1	8.41	7.47	6.83	6.28	3.84	7.12	
TOTALS							

Find the total receipts for each week, the receipts of each machine for the entire period, and the total receipts of all machines for the period. Why are you certain that your result is correct?

The thirty minutes allowed for the written test (see next page) provides for a proof of the 9th and 10th problems according to the plan suggested in their outline, and allows time for copying if the teacher prefers not to require any horizontal addition. In marking, allow 5 points each for the first 8 problems, and 30 points each for problems 9 and 10. After deducting for inaccuracy, allow 1% for every minute saved, and deduct 1% for every minute taken more than 30.

Written Review Test

(Time, 30 minutes)

1.	2.	3.	4.
32196	43207	54318	65429
45374	56485	67596	78605
80539	91640	82532	43643
61428	72539	45907	56018
27854	34896	91253	23364
50612	80142	24637	75419
5.	**6.**	**7.**	**8.**
76531	21643	32754	43865
89716	42065	53176	64287
54754	47109	58210	69321
67129	68243	79354	81463
34765	57891	68725	75836

9. The report of sales for a certain wholesale house, by salesmen, for the first four months of business, was as follows. Find the total sales of each salesman, the total sales for each month, and the total sales for the four months. The total of totals by months should agree with the total of totals by salesmen.

Salesman	January	February	March	April	Totals
Johnson	$3297.20	$3065.82	$4356.12	$3759.42	
Townsend	3043.56	3174.50	3870.42	3512.96	
Harrison	2471.32	2892.73	3583.60	3463.87	
Richards	2238.90	2574.32	2471.18	2485.50	
Williams	2146.30	2368.41	2738.15	2811.96	
Totals					

10. The following table represents the liabilities of certain national banks of Chicago, as they stood at one time:

	Capital Stock	Surplus	Undiv. Profits	Notes Outstand'g	Deposits	Totals
Bowman National	$ 50,000	$ 10,000	$ 1,174	$ 35,000	$ 158,215	
Calumet National	100,000	50,000	12,635	99,500	1,187,958	
Commercial National	21,500,000	8,500,000	2,040,146	8,579,100	172,445,785	
Exchange National	3,000,000	5,000,000	1,554,344	1,199,997	57,258,483	
Drovers National	750,000	250,000	143,962	300,000	9,683,575	
First National	10,000,000	10,000,000	2,126,761	2,434,000	116,410,921	
Totals						

CHAPTER II

SUBTRACTION PRACTICE

There are only 45 cases in which a single figure is subtracted from another single figure. There are 45 additional cases in which the unit figure in the minuend is smaller than the unit figure in the subtrahend. All 90 cases are shown in the following table:

Subtraction Table

1	2	3	4	5	6	7	8	9	10									
1	1	1	1	1	1	1	1	1	1									
	2	3	4	5	6	7	8	9	10	11								
	2	2	2	2	2	2	2	2	2	2								
		3	4	5	6	7	8	9	10	11	12							
		3	3	3	3	3	3	3	3	3	3							
			4	5	6	7	8	9	10	11	12	13						
			4	4	4	4	4	4	4	4	4	4						
				5	6	7	8	9	10	11	12	13	14					
				5	5	5	5	5	5	5	5	5	5					
					6	7	8	9	10	11	12	13	14	15				
					6	6	6	6	6	6	6	6	6	6				
						7	8	9	10	11	12	13	14	15	16			
						7	7	7	7	7	7	7	7	7	7			
							8	9	10	11	12	13	14	15	16	17		
							8	8	8	8	8	8	8	8	8	8		
								9	10	11	12	13	14	15	16	17	18	
								9	9	9	9	9	9	9	9	9	9	

This table gives all possible combinations in subtraction, arranged in consecutive order. The same combinations are presented on the next page in miscellaneous order for drill.

To be quick and accurate enough in subtraction the student must become so familiar with these combinations that their results are recognized instantly.

Following are the same 90 fundamental combinations in subtraction, arranged in miscellaneous order for drill.

Oral Drill Table

10	7	7	12	8	5	7	14	9	6	13	8	6	9	7
6	4	1	4	4	1	2	7	4	3	9	7	4	3	6
13	7	11	7	4	16	5	8	18	3	5	10	15	9	12
8	3	5	7	1	9	3	6	9	3	2	9	6	1	7
11	3	5	4	11	9	10	1	16	6	11	9	17	8	15
9	1	5	2	4	9	5	1	7	5	7	8	9	5	8
13	2	9	10	15	3	7	17	4	11	9	12	11	9	6
7	2	7	1	9	2	5	8	3	2	5	9	8	6	1
14	8	10	6	12	9	11	10	8	2	12	8	4	10	14
9	2	4	2	3	2	6	7	8	1	6	1	4	3	6
5	10	11	6	10	15	16	8	13	13	12	13	12	14	14
4	2	3	6	8	7	8	3	4	6	5	5	8	5	8

Return to the table for drill frequently.

ORAL. Count backwards

1. By 2's from 88, 100, and 124, until nothing remains.
2. By 3's from 75, 96, 102, 123, and 138, until nothing remains.
3. By 4's from 84, 96, 116, 128, and 136, until nothing remains.
4. By 5's from 95, 110, 115, 125, and 140, until nothing remains.
5. By 6's from 96, 114, 126, 144, and 156, until nothing remains.
6. By 7's from 91, 112, 133, 147, and 154, until nothing remains.
7. By 8's from 88, 96, 112, 120, and 136, until nothing remains.
8. By 9's from 86, 99, 112, 117, and 130, to less than 9.
9. By 10's from 89, 95, 100, 120, and 135, to less than 10.
10. By 11's from 91, 97, 99, 120, and 127, to less than 11.

Oral Drill

11	20	10	11	42	22	45	21	30	43	23	15	44	24	42
3	7	5	6	8	4	9	3	5	7	4	6	5	9	7
21	14	37	13	31	54	11	12	64	12	22	10	31	33	52
4	6	9	5	8	7	4	3	8	4	5	6	7	4	6
31	50	21	34	22	41	14	65	20	42	20	64	21	40	43
4	5	6	9	3	7	5	6	4	5	6	7	8	6	9
33	34	31	74	30	11	26	20	32	33	75	85	27	95	16
5	6	3	7	4	5	9	8	6	6	7	6	8	6	7
53	41	30	10	21	32	32	58	43	32	75	35	35	13	22
5	6	7	4	5	3	7	9	8	4	8	6	7	6	9
54	32	56	41	13	12	86	24	34	12	33	75	32	63	53
6	4	8	5	4	6	7	5	8	5	7	6	8	6	8

Written

The following problems contain all the combinations in subtraction shown in the Oral Drill Table on the opposite page:

1. 92780 44146	**2.** 87314 22349	**3.** 70031 16598	**4.** 40033 27127
5. 47823 17358	**6.** 64354 45858	**7.** 90320 82667	**8.** 52785 32859
9. 54121 51549	**10.** 58496 12743	**11.** 97442 38465	**12.** 25914 14136
13. 95977 58859	**14.** 39866 36939	**15.** 93697 79436	**16.** 71125 68237
	17. 95126 92717	**18.** 96162 12967	

Making Change

The mental processes of addition and subtraction are so closely related that it is difficult to determine which is employed in the process of making change. Certainly the oral process is one of addition. The cashier or salesman, returning change out of $1.00 offered in payment of a bill of 19¢:

Hands customer package		and says	"19¢"
Hands customer	1¢	and says	"20¢"
Hands customer	5¢	and says	"25¢"
Hands customer	25¢	and says	"50¢"
Hands customer	50¢	and says	"$1.00"

1. One dollar is tendered in payment of each of the following bills. State what change is returned to the customer: 14¢, 24¢, 32¢, 27¢, 42¢, 38¢, 51¢, 36¢, 47¢, 64¢, 72¢, 17¢, 21¢, 12¢, 26¢, 35¢, 81¢, 67¢, 71¢, 48¢, 54¢.

2. State what is returned in each case, if a two-dollar bill is offered in payment of each of the following amounts: $1.41, $1.22, $1.12, $1.16, $1.24, $1.38, $1.62, $1.46, $1.28, $1.76, $1.39, $1.73, $1.64, $1.83, $.97, $.86, $.37.

3. A $5.00 bill is offered in payment of each of the following amounts: $1.04, $2.09, $3.10, $1.76, $3.11, $4.49, $3.54, $2.97, $1.98, $3.58, $1.07, $4.02, $3.16, $4.51, $1.83, $2.19, $3.29. What change is returned in each case?

4. A $10.00 bill is offered in payment of each of the following bills: $3.27, $4.52, $5.79, $7.38, $5.95, $3.85, $8.46, $9.12, $7.25, $1.29, $1.76, $.84, $3.81, $4.73, $6.44, $2.28, $6.54, $3.70. What change is returned in each case?

Complement Method

A **complement** of a number is the amount which if added to it will make one unit of the next higher order. Thus the complement of 6 is 4, since added they make 10; the complement of 36 is 64; the complement of 475 is 525.

When a number to be subtracted is mixed in with a number of amounts which are to be added, a convenient method is to add the complement of the number and then deduct 1 from the next higher order in the result.

$398+686-475=398+686+525$ (the complement of 475) $-1000=609$

This method is valuable in connection with work listed on an adding machine which consists largely of addition, with an occasional subtraction. Subtraction is accomplished on the adding machine by adding the complement of the number to be subtracted, and deducting 1 from the next higher order.

Arrange the following for horizontal addition as in the illustration, but in place of the number to be subtracted write its complement, and change the sign from − to +. Add, and deduct 1000 from the total.

1. 397+783−425	**9.** 642+538−382	**17.** 1647+382−119
2. 627+354−274	**10.** 392+627−214	**18.** 1382+622−156
3. 438+472−187	**11.** 396+729−312	**19.** 1481+549−350
4. 629+108−267	**12.** 398+647−475	**20.** 1397+478−386
5. 529+216−483	**13.** 362+429−264	**21.** 1478+370−315
6. 718+956−112	**14.** 949+926−110	**22.** 1873+965−143
7. 927+832−126	**15.** 872+965−116	**23.** 1983+875−132
8. 854+963−285	**16.** 654+897−245	**24.** 1781+910−175

25. Here are four depositors' accounts taken from a bank ledger. Find the balance of each:

Wednesday, May 1, 19

DEPOSITORS	Balance	Checks in Detail		Total Checks	Deposits	Balance
Bauman & Co.	1297 20	60 140	120 40		892 75	
H. C. Spillman	2360 18	11 92 193 80	6 39		487 63	
Henry Clews	7452 86	47 83 238 56	89 42 296 12		1467 40	
Jas. R. Platt	3478 92	38 11 67 45	3 85 9 74		476 32	

First, find the total checks drawn against each separate account and extend the total to the column headed "total checks." Then find the balance of each account at the end of the day by adding to the previous day's balance (shown in the first column) the deposits (shown in the fourth column), and subtracting the total of the checks paid during the day.

Proof of Subtraction. Certainty of the accuracy of any result can be had by subjecting your work to a proper proof. Prove your work in subtraction by adding subtrahend and difference. If your result agrees with the minuend, your work is accurate.

How a Banker Proves His Depositors' Ledger. When the balance of each account has been obtained, the work is proved as follows: Add the total of the first *balance* column and the total of the *deposits* column. Subtract the total of the *total checks* column. The result should be the total of the final *balance* column.

26. Prove the depositor's ledger given in Problem 25.

27. Balance and prove the following accounts:

THURSDAY, OCTOBER 3, 1928

DEPOSITOR	Balance	Checks in Detail		Total Checks	Deposits	Balance
Barnes, Thos. F.	2798 12	129 60 83 57			375 96	
Brantley, Davis & Co.	3639 74	257 80 19 74	16 35 39 78		581 62	
Boston Paper Co.	10,853 11	176 45 93 82	15 39 16 87		937 46	
Berkley, F. M.	1297 80	27 38 125			385 07	
Burrows & Co.	3856 78	9 76 5 81 116 50	37 82 100		276 54	
Burlington R. R. Co.	27,391 86	*2283 97			3085 16	
Brooklyn R. T. Co.	35,811 17	475 19 389 11	1258 90 1000		4380 78	
Barker Drug Co.	676 59	578 20 64 00	179 56 87 11		163 47	
Burlage, Thos. & Co.	1879 50	278 50 392 76	113 50 39 71		250	
Baskin, Frank	830 57	253 12 349 14			82 12	

* This is a total of several checks, there being too many to enter separately.

Note that the Barker Drug Co. has overdrawn. Write the balance of this account in red ink, preceded by the abbreviation *O.D.* (overdraft). In finding the total of balances, omit this item. Then deduct the overdraft from the total of credit balances.

28. On Friday, Oct. 4, 1928, the following deposits were received by the bank a part of whose individual ledger was shown in problem 27: From Thos. F. Barnes, $427.36; Brantley, Davis & Co., $339.82; Burlington R. R. Co., $2546.84; Brooklyn R. T. Co., $3240.56; Barker Drug Co., $265.65; Boston Paper Co., $560.78; F. M. Berkley, $292.56; Burrows & Co., $157.62.

The following checks were paid the same day: Thos. F. Barnes, $124.20; Brantley, Davis & Co., $24.92; Boston Paper Co., $21.36; F. M. Berkley, $32.50; Brantley, Davis & Co., $124.36; Burrows & Co., $52.60; Frank Baskin, $12.75; Thos. Burlage & Co., $42.36; Brooklyn R. T. Co., $432.85; Brantley, Davis & Co., $129.75; Boston Paper Co., $85.46.

Rule for Oct. 4 a form similar to that shown in the preceding problem, entering the final Oct. 3 balances as the beginning balances for Oct. 4. Enter the deposits and checks listed above. Balance each account. Prove the ledger.

Accounts

Accounts with Cash

1. Richard Thompson received cash during September as follows: Sept. 1, $116.47; Sept. 2, $12.92; Sept. 3, $15.85; Sept. 4, $27.84; Sept. 6, $19.41; Sept. 7, $41.63; Sept. 10, $19.84; Sept. 11, $23.50; Sept. 13, $61.54; Sept. 16, $47.39; Sept. 20, $51.87; Sept. 24, $112.45; Sept. 26, $37.84; Sept. 30, $53.81. During September he paid cash as follows: Sept. 2, $15.98; Sept. 3, $42.63; Sept. 5, $27.84; Sept. 6, $18.42; Sept. 10, $43.86; Sept. 12, $53.72; Sept. 15, $61.45; Sept. 18, $81.46; Sept. 24, $112.68; Sept. 27, $161.45. How much cash did he have left?

2. On May 10, Wilson & Wingate opened business with $147.38 cash in the safe. During the day, their receipts and payments were as follows: Receipts, $12.47, $16.83, $9.78, $14.63, $12.75, $100, $111.47, $61.78, $193.56, $78.58. Payments, $75, $11.39, $71.53, $8.74, $13.37, $12.62, $51.80, $2.76, $13.11, $115.86, $150. How much cash did they have at the end of the day?

3. Brownlee's ten-cent store has ten cash registers. Saturday morning the following amounts were put in these registers for making change: In No. 1, \$12.85; No. 2, \$43.75; No. 3, \$25.54; No. 4, \$17.38; No. 5, \$31.62; No. 6, \$18.47; No. 7, \$31.79; No. 8, \$19.56; No. 9, \$23.80; No. 10, \$52.76. During the day the receipts registered by these were respectively: \$114.92; \$187.45; \$49.38; \$61.54; \$102.09; \$136.58; \$79.40; \$126.82; \$96.39; \$157.69. Payments were made from these registers during the day, as follows: From No. 1, \$27.56; No. 2, \$96.45; No. 3, \$10.87; No. 4, \$14.32; No. 5, \$57.86; No. 6, \$90.80; No. 7, \$23.59; No. 8, \$61.38; No. 9, \$17.74; No. 10, \$64.27. Prepare a statement showing in tabulated form all of the facts given above. Show at the right the balances in the different registers and at the bottom the total on hand at the beginning of the day, total receipts, total payments, and total balances at the end of the day.

4. During the month of October, Frank T. Davis received cash \$2754.85, and paid out cash \$2945.97. He had a balance of \$1358.12

Cash

19						19				
Mar.	1	Bal. on hand	523	69	Mar.	1	Rent	35		
	3	N. J. Harkness	125	60		2	Expense	5	25	
	11	M. W. Wilms	26	52		7	Postage	12	64	
	15	Mdse. Sales	354	27		15	Salaries	42	50	
	22	J. C. Rankin	195	43		24	Advertising	25	75	
	31	Mdse. Sales	586	29		31	Salaries	42	50	
						31	Expense	2	47	
						31	Balance (Red)	*******		
	1		*******					*******		
Apr.	1	Balance	*******							

at the end of the month. What was his cash balance at the beginning of the month?

5. On page 28 is an account showing items of cash received and cash paid out during March, 1928. The items of cash received are entered on the left side, and the payments on the right. What was the amount of cash on hand on March 31? Write the footings in pencil and the balancing entry in red ink. Rule in red ink and bring the balance down in black ink.

6. The Magruder Grocery Co. had a cash balance of $2751.18 on May 1. During the month the firm received $14,398.54 in cash and at the end of the month had a cash balance of $3267.45. What was the total of cash payments of The Magruder Grocery Co. for the month of May?

7. The cash book shown on pages 30 and 31 shows F. W. C. Holtkamp's receipts and payments from January 1 to January 16. The receipts are on the left hand page and the payments on the right. Copy the cash book, filling in the amounts of total receipts, total payments, balance on hand, and footings.

Accounts with Banks

8. James Cornwell opened an account with the Mercantile National Bank on October 1, 1928, by depositing $437.85. Thereafter he deposited as follows: Oct. 5, $276.87; Oct. 8, $194.37; Oct. 12, $226.92; Oct. 16, $150; Oct. 20, $217.24; Oct. 25, $100; Oct. 30, $453.40. His checks for October were: Oct. 3, $74.19; $142.57; Oct. 5, $13.74, $176.29, $712; Oct. 8, $35, $11.78, $15.83; Oct. 12, $9.47, $184.16, $50, $27.65; Oct. 16, $20, $17.39, $44.76, $110; Oct. 20, $3.18, $5.74, $34.75, $16.81; Oct. 24, $19.86, $58.43, $100; Oct. 29, $13.27, $16.74, $32.95. If all of the foregoing checks were paid by the bank, what was his balance in the bank on Oct. 31? Arrange the deposits and checks in separate columns, giving the date of each transaction.

9. Wm. Elliot had a balance in the bank on Mar. 31, 1927, of $247.38. He received on Apr. 1 a check for his salary, which was $250. This he deposited in the bank. During April he paid by check: Rent, $30; groceries, $17.34; meat, $15.18; milk, $3.85;

Dr. Cash

Date	L F	Credit Accounts	Explanations	Items	Total
Jan 1		F. W. C. Holthamp	Investment	1000 00	
3		Mdse.	Sold to Jas. Smith	175 50	
6		F. D. Naegele	On account	150 00	
9		Bills Rec.	Disc'td D. C. Meyers note	250 00	
16			Total Receipts		
Jan 18			Balance		

ice, $2.25; gas, $2.18; electric light, $2.61; coal, $16.70; insurance, $21.64; lodge dues, $5.20; church, $5.00; shoes, $10; clothing, $21.80. He also drew out cash $10. What was his bank balance at the end of April? Arrange your work in appropriate form.

10. John F. Gregory's check book on May 1, 1927, showed a balance of $1643.96. The German-American bank sent him a statement which gave his balance on May 1 as $1827.65. By comparing returned checks with the stubs of his check book he found that the bank had not yet received 3 checks that he had issued. What was the total of these checks?

11. Wm. Blackman & Co. received a statement from the First National Bank which showed a balance of $3290.84 on deposit. Blackman & Co.'s check book showed a balance of $3048.14. In checking over this account Blackman & Co.'s bookkeeper found a charge of 40¢ made by the bank for collecting an out-of-town draft, which was not entered by him in Blackman & Co.'s books. He also found that there was one outstanding check. What was the amount of the check?

12. On July 1, 1920, The Brown-Manly Co. had a bank balance of $8379.46, and on July 31 their bank balance amounted to

Cash

Cr.

1a Date	LF	Debit Accounts	Explanations	Items	Total
Jan. 2		Expense	Rent of store 1 mo.	75 00	
5		Expense	Paid clerk hire 1 wk.	12 50	
7		Mdse.	Paid for drayage	3 60	
9		Discount	On Meyer's note	2 75	
11		J. P. Shaw	On account	75 00	
14		Fixtures	Bot. of J. F. Sarley	125 00	
16			Total Payments		
16			Balance (Red Ink)		

$9741.16. If they deposited $19,746.83 during July, what was the total of checks drawn?

13. Henry Lane's check book showed a balance of $1157.34 but when he compared his books with a statement from the bank he found that he had omitted a deposit of $69.43, and had entered a check for $45 as $4.50. What should his balance have been?

14. The Midvale Trust Co. sent John Burnham & Co. a statement of their account which showed a balance of $9753.82. Burnham & Co.'s bookkeeper found that six checks issued had not been paid by the bank. The checks were as follows: $137.95, $32.50, $207.95, $75, $183.14, $379.53. What was Burnham & Co.'s balance as shown by their check book?

During the following month Burnham & Co. deposited $1524.63, $825.39, $627.54, $1256.92, and $694.87, and issued checks amounting to $5248.63. The bank paid the six checks that were not paid the preceding month and paid all checks issued this month except three, two of which were for $425.52 and $67.93 respectively. The bank's statement to Burnham & Co., showed a balance of $9258.46. What was the amount of the third unpaid check?

15. Here is a copy of one page of J. E. Baker's check book record. What balance should he forward to the top of the next page?

No.	Date			Amt. Drawn	Amt. Deposited	Balance
	'27					
211	Apr.	13	N. R. Denver	5		497 58
212	"	24	T. C. Van Dyke	2 65		
213	"	25	Burton & Holmes	15 50		
214	May	4	N. A. Hofer	40	250	
215	"	7	R. B. Updike	3 76		
216	"	12	N. L. Harmon	10 60		
217	"	13	Oscar Newman	4 83		
218	"	14	F. J. Smith	52 94		
219	"	17	N. N. Hill	5 65		
220	"	19	J. B. Owen	2 13		

Accounts with Persons

16. The Motor Sales Co. bought merchandise on account from Strong, Carlisle, & Hammond, and made payments to them, as follows: **Jan. 10, 1927,** bought mdse, $389.50; Jan. 18, bought mdse, $197.56; Jan. 20, paid cash, $389.50; Jan. 28, paid cash, $79.50; Jan. 29, bought mdse, $216.47; Feb. 5, bought mdse, $384.93; Feb. 8, paid cash, $216.47; Feb. 9, bought mdse, $156.23; Feb. 10, returned mdse, $75; Feb. 15, paid cash, $384.93; Feb. 17, paid cash, $43.06. What was the balance due Strong, Carlisle, & Hammond on Feb. 18?

17. On Sept. 1, 1927, Park & McKay's ledger showed that there was a balance of $562.75 due from James Dupont. On Sept. 3 they sold him mdse, $194.80. Sept. 7, sold him mdse, $49.35. Sept. 9, received from him cash, $243.29. Sept. 12, sold him mdse, $78.50. Sept. 13, received from him cash, $44.80, and his note for $150. Sept. 17, received from him cash, $49.35. Sept.

20, sold him mdse, $84.39. Sept. 22, received from him cash, $78.50. How much was due Park & McKay from James Dupont on Sept. 23?

18. Shelton & James owed the National Biscuit Company $160.40 on May 1, 1928. During May they made purchases and payments on account as follows: May 3, bought mdse, $52.69. May 7, paid cash, $28.50. May 10, bought mdse, $72.34. May 11, returned mdse, $12.75. May 12, bought mdse, $13.78. May 13, paid cash, $52.69. May 15, bought mdse, $125.83. May 20, paid cash, $22.34, and gave them a note for $50. May 21, bought mdse, $15.60. How much did Shelton & James owe the National Biscuit Co. on May 31?

19. On April 1, 1928, H. L. and B. W. Smith owed us $142.50. This balance, and their purchases and payments during April, are shown in the account herewith. Our charges against them are shown on the left side of the account. Their payments are shown on the right. How much did they owe us on April 30?

H. L. & B. W. Smith

Jacksonville, Ill.

19				19			
Apr. 1	Bal.	142	50	Apr. 12	Cash	41	65
3	Mdse.	52	69	20	Cash	50	85
14	Mdse.	75	32	20	Note	50	
23	Mdse.	47	83	23	Cash & Disc.	52	69
29	Mdse.	25	64	24	Cash	25	
				29	Cash & Disc.	50	32

During May, H. L. and B. W. Smith bought merchandise and made payments as follows: May 4, bought mdse., $92.74. May 5, bought mdse., $63.82. May 13, paid cash, $47.83. May 16, bought mdse., $62.75. May 19, returned mdse., $10, and paid cash, $15.64. May 23, bought mdse., $29.63, May 24, paid cash, $92.74. May 27, bought mdse., $39.62. How much did they owe us on May 31?

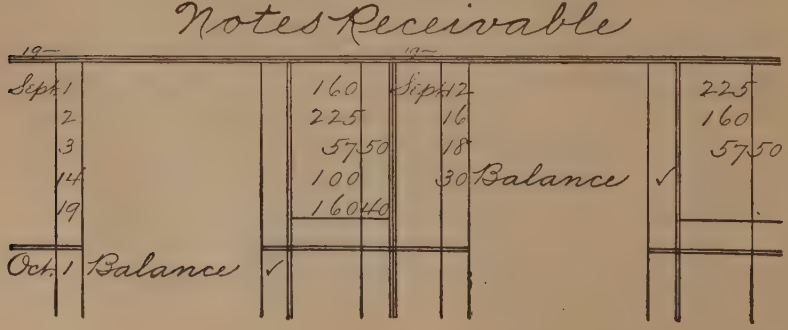
Accounts with Notes

20. On Oct. 1, 1927, J. W. Statler held notes for amounts receivable from others as follows: $250, $165.25, $246.90. During the month he received notes for amounts receivable from others as follows: $100, $230.65, $75.75, $50.72. He received payment and returned to the makers the notes for $250 and $100; and he sold to the bank the notes for $246.90 and $230.65. What was the face value of the notes in his hands on Oct. 31?

21. A. G. Robyn owed $267.58 on notes when he began business on August 1, 1928. On Aug. 10 he issued a note for $135.40; on Aug. 17, a note for $85.21; on Aug. 26, a note for $110.94. He made payments on these notes as follows: Aug. 5, $175; Aug. 15, $125; Aug. 31, $102.98. What was the balance of his notes payable account on Aug. 31? Can you tell what notes had not been paid?

22. The notes receivable account of John H. French showed a balance of $3297.84 on Mar. 1, 1927. During the month he received from his customers notes receivable amounting to $2732.18 and discounted at his bank (sold to his bank at a discount) notes receivable to the value of $2113.90; and his customers paid notes that fell due amounting to $3017.85. What was the balance of his notes on hand on March 31?

23. Our account with notes receivable showed that on Sept. 1, 1926, we had on hand other person's notes amounting to $160.00. During the month we received notes as shown on the left side of the following account and disposed of notes as shown on the right. What was the balance due on notes at the end of the month?



Accounts with Capital

24. William Harris invested in his business on June 1, 1927, $6397.40. During June, July, and August he lost $275.81. What was the balance of his capital account on Sept. 1?

25. On January 1, 1928, Thomas Adams invested $4500 in his business. During the first three months his gains were $983.78, and during the next three months he lost $471.95. What was the balance of his capital account at the end of the six months?

26. On January 1, 1927, R. C. Grant's capital account showed a balance of $11,627.90. During the first half year he gained $3462.04. During the second half year he lost $879.26. What was the balance of his capital account on July 1? On Jan. 1, 1928?

27. The balance of Harper Bros.' capital account on Jan. 1, 1928, was $16,859.30. Their profits and losses for the year follow: Jan. profits, $387.94; Feb. profits, $470.13; Mar. profits, $415.83; Apr. profits, $632.90; May profits, $761.48; June profits, $843.64; July losses, $256.44; Aug. profits, $112.80; Sept. losses, $369.78; Oct. profits, $284.39; Nov. profits, $576.19; Dec. profits, $311.61. Find the balance of Harper Bros.' capital account on December 31. What was the average balance of this account for the 12 months?

28. Kelsey Bros.' investment at the close of 1928 (their second year in business) amounted to $37,593.12. During 1928 their net loss was $4732.94, while in 1927 their profit had been $6597.65. How much did they invest at the beginning of business?

Investment

29. Hollis & Sprague began business with the following property: Cash, $1450.80; store and lot, $10,000; stock of goods, $15,785.46; horses and wagons, $935; trade fixtures, $500; notes receivable, $1847.80. They owed on notes $1167.93, and on accounts $1319.27. What was their net investment?

30. J. C. Armstrong owns: 5 horses valued at $1175; 2 wagons valued at $247.50; house and lot valued at $3200; bank stock valued at $10,453.92; a life insurance policy valued at $2687.46; a farm valued at $6478.40; and other property worth $1450.85. He owes: On a note, $695.37; on a mortgage on his farm, $1500.56; and on book accounts, $179.58. What is he worth?

31. At the close of business on Dec. 31, 1928, Kirkman & Co. had the following property: Land and buildings, $25,300; machinery for manufacturing goods, $15,387.90; lumber and other material not manufactured, $7384.75; manufactured goods ready for sale, $12,329.85; cash in the bank, $9,537.67. Their customers owed them $21,329.85 on notes and on accounts. They owed $10,385.74 on a mortgage. They owed $4856.79 to other business houses from whom they had purchased goods. They owed their employees $763.58, not payable until pay day. What was Kirkman & Co.'s net investment?

Profits from Trading and Manufacture

32. Waring & Co. bought cotton costing $3475.38. They then made the following sales: $1250.60, $937.65, $1127.32, $846.93. If all the cotton was sold, what was Waring & Co.'s gross profit on this lot?

33. Eastman and Company began business on Jan. 1, 1928. During January they bought merchandise which cost them $1569.25. On Jan. 31 they had on hand goods which had cost them $926.75. What was the cost of goods sold during the month? If their sales for January amounted to $837.62, what was their gross profit on merchandise for January?

34. On Sept. 1, 1927, Smith & Hobart had on hand merchandise which had cost $2573.60. Their purchases and sales of merchandise during the month were as follows: Sept. 5, sale, $950.64; Sept. 7, sale, $262.72; Sept. 11, purchase, $1006.50; Sept. 15, sale, $827.63; Sept. 18, purchase, $215.40; Sept. 20, sale, $765.92; Sept. 30, sale, $1927.65. On Oct. 1 no merchandise remained on hand. What was the cost of the goods sold? What was their gross profit on merchandise for the month?

35. Lynn & Haven had merchandise on hand Oct. 1, 1928, which had cost $5627.39. During the month of October their sales amounted to $3762.93 and their purchases amounted to $2573.86. They had on hand on Oct. 31 merchandise costing $6120.70. What was the cost of goods sold during October? What was the gross profit on merchandise for October?

36. Vinson & Co. had on hand on Nov. 1, 1928, merchandise which had cost $6257.84. Sales for November, $2479.50. Pur-

chases for November, $1596.27. Inventory of merchandise Nov. 30, $6329.30. In-Freight for November, $35.26. What was the cost of goods sold, freight paid, during November? What was the gross profit on merchandise, In-Freight being considered a legitimate addition to cost?

37. Mdse on hand Dec. 1, 1928, at cost, freight paid, $6729.34. Dec. 2, sale, $345.62. Dec. 3, sale, $472.63. Dec. 5, purchase, $629.34. Dec. 7, sale, $150. Dec. 9, sale, $75.93. Dec. 10, sale, $376.27. Dec. 11, purchase, $125.25. Dec. 13, sale, $262.93. Dec. 15, sale, $172.95. Dec. 19, purchase, $50.60. Dec. 21, sale, $127.60. Dec. 26, sale, $439.48. Dec. 29, sale, $278.64. In-Freight for December, $42.50. Goods returned by us during December, $75.60. Goods returned to us and claims allowed by us during December, $40.50. Other expenses of the business, $429.50. Mdse. on hand Dec. 31, $6327.50. Find the cost of goods sold, the gross profit on merchandise, and the net profit of the business for the month.

38. Ebeling & Reuss are importers of glass and china ware and do a general wholesale business. During three months their purchases amounted to $53,869.18. They paid freight on the goods purchased amounting to $432.64; tariff on importations was paid to the amount of $3179.45. The sales for the period were $71,316.32; the inventory at beginning was $11,512.76; and at closing the unsold goods amounted to $12,593.55. What were the profits from trading?

39. The Eldridge Mfg. Co. purchased during February material (lumber, cloth, glue, etc.) costing $3264.37. At the beginning of the month they had on hand material valued at $843.95 and at the end of the month the material on hand was valued at $927.80. They paid $4756.94 for labor during the month, and had miscellaneous manufacturing expenses amounting to $1279.81. If the articles manufactured during the month were sold for $11,394.27, what was the profit or loss on manufacture?

Profit and Loss Accounts

40. In a certain business the profits during the year were from the following sources: Profits on merchandise, $5264.83; rent from sub-let premises, $400; received from customers for interest, $87.35; discount on bills paid before due, $187.64. The expenses

were: Rent, $1000; hiring one clerk, $780; advertising of all kinds, $438.25; and general expenses, $1394.82. What was the net profit or loss for the year?

41. Smith's profits and losses for a certain year were as follows: Profit on merchandise, $15,645.92; loss on interest, $265.29; cash discounts allowed to his debtors, $562.70; cash discounts allowed by his creditors, $840.60; losses on bad accounts, $350; general expenses, $6245.93. What was his net profit or loss for the year?

42. From the following showing of the yearly footings of the profit and loss accounts in Jones & Co.'s ledger, and other figures given, compute the profit or loss on each account and the net profit or loss of the business for the year. *Merchandise*: Invty. Jan. 1, $16,729.65; net sales for year, $39,625.62; net purchases for year, $27,638.54; Invty. Dec. 31, $14,524.32. *Interest and Discount*: Losses for year, $462.50; gains for year, $273.61. *Rent*: Costs for year, $1565.75; returns from sub-tenants, $300.00. *General Expense*: Invty. Jan. 1, 4 days' pay roll to be paid this year but which was charged as loss against preceding year, $87.50. Losses paid this year, $2564.70. Invty. Dec. 31, 5 days' pay roll not yet paid, but to be charged against this year's business, $112.50.

43. Find the profit on each of the following contracts and the total profit on all:

SUMMARY OF CONTRACTS

Job No.	FOR	Contract Price	Cost	Profits
1	Advanced Bldg. Co.....	$37588.00	$33594.27	.....
2	Broadway Parkway Co.	8346.85	7589.25	.....
3	John B. Berry.....	14500.00	12047.01	.....
4	Chas. Beckman.....	4717.47	4328.73	.....
5	Chas. Meyer.....	491.90	373.08	.....
6	Geo. Schuck.....	16224.60	14695.01	.....
7	Manderkin Bldg. Co.....	12061.70	10696.87	.....
8	Ittner Realty Co.....	6455.00	5887.63	.....
9	Beck Realty Co.....	14806.17	12661.11	.....
10	Home Realty Co.....	1003.83	865.56	.....

44. Fill the blank spaces in the following:

COMPARATIVE BALANCE SHEET

Assets	Dec. 31 1927	Dec. 31 1928	Increase	Decrease
Cash	565 29	627 58		
Accounts Receivable	5267 56	6348 92		
Notes Receivable	3560 50	4258 00		
Real Estate	8675 00	8675 00		
Chattels	1522 50	1493 75		
Total Assets				
Liabilities				
Accounts payable	2250 60	2341 25		
Mortgage payable	5000	4000		
Notes payable	1250	1175 50		
Capital	10000	10000		
Surplus	1090 25	3886 50		
Total Liabilities				

45. Following is a form used in bookkeeping, called the trial balance. The footings of both columns should be the same. One item, representing the amount of the proprietor's investment, is omitted from the right hand column. Find out what it should be, write it in the right hand column, and add both columns.

C. V. Radcliffe's Trial Balance, June 1, 1928

C.V.Radcliffe, Prop.			????	??
Real Estate	5000	00		
Merchandise	3256	35		
Expense	272	65		
Interest			13	27
Cash	1462	74		
Notes Receivable	525	50		
H. A. Watson	242	37		
Notes Payable			675	50
J. R. Hadley			325	
	?????	??	?????	??

Use of a Check System in Bookkeeping

Many bookkeepers check the accuracy of their footings by casting out 9's, 11's, or 13's. The following ledger account shows how these check numbers (found by casting out 11's) are kept in a separate column, and how they are used to test the accuracy of the balance of the account.

H. G. Brown
3629 Aldine Place

19			C.R.No.			19			C.R.No.		
Jan.	1	Balance	5	331	15	Jan.	2	Cash & Dis.	1	243	11
	5	Mdse.	7	424	23		9	Cash & Dis.	4	212	12
	6	Mdse.	3	300			13	Returned	4	88	04
	16	Mdse.	1	286	45		22	Cash & Dis.	0	176	
	19	Mdse.	6	50			23	Cash & Dis.	3	212	11
	25	Mdse.	3	321	67		26	Cash & Dis.	7	150	
	28	Mdse.	1, 26	100, 1813	50		27	Cash & Dis.	1, 20	100, 1813	50
							31	Balance (Red)	6	632	12
				1813	50					1813	50
Feb.	1	Balance	6	632	12						

EXPLANATION. **The 11's are cast out of each amount on each side, and the** remainder entered in the check figure column. This may be done at the time of making the entry or at a later time.

At the end of the month the account is balanced. The balance of the check figure columns (6) is then found and must agree with the check figure for the balance of the account. (Casting out the 11's from 63212 gives 6.) In case of failure to agree, test the accuracy of the two footings. In each case this is done by casting out the 11's and comparing the result with the check figure of the footing of the check figure column. By this plan the accuracy of the balance is tested before the account is ruled and closed.

Some bookkeepers do this work as a part of their regular routine. Others only use the device after failure to get a trial balance has shown that there is an error or errors.

NOTE. The student will observe that in every case the check figure is the remainder that would result were the division by 11 actually performed.

SHORTENING THE WORK. It is not always necessary to find the sum of all numbers in the odd places and the sum of all the numbers in the even places, and then to take the excess of the former over the latter for the check figure. Often much or all of this work can be eliminated by "pairing off" or by some other instantaneous device. For instance, in the first amount on the left side it is apparent at a glance that the 3 in the fifth column offsets the 3 in the fourth column and that the 1 in the third column offsets the 1 in the second column. This leaves only the 5 in the first column, and that is written as the check figure. In the case of the second amount on the left side, it is immediately apparent that the figure 4 in the fifth column exceeds the figure 2 in the fourth column by 2. Similarly the next two figures give an additional excess of 2 for the odd column, making an excess of 4 to which the last figure 3 is added making a total excess of 7. In case of the third amount on the left side, there is only one figure to consider, *viz.*, the 3 in the fifth column, which is an odd column. The excess figure is therefore 3. In the fifth amount the only figure to consider is a 5 which is in an even column. The excess figure is therefore 11 − 5, or 6.

WRITTEN. 1. Copy the account at the left, below. Write the check figure of each item at the time you write the item. Balance and prove as in the illustrative example. For simplicity, all data are omitted except the amounts.

2. At the right, below, is an account which the bookkeeper ruled and closed as settled in full. It contains an error. Apply the system of checking by casting out 11's, and find the error.

J. G. Cole

$441.16	$250.00
352.65	191.16
710.24	275.00
589.50	77.65
345.27	352.50
107.05	357.74
300.00	150.00
150.00	345.27
243.76	439.50
154.85	107.05
372.94	243.76
463.83	150.00
129.32	150.00
218.43	150.00
149.73	154.85

W. L. Brock

$350.00	$200.00
267.29	150.00
418.50	267.29
232.47	640.97
352.68	729.43
729.43	352.68
280.56	180.56
427.35	527.35
192.83	476.57
283.74	365.56
465.56	354.45
354.45	100.00
263.54	650.00
429.32	42.86
5047.72	5047.72

Written Review Test

(Time, 30 minutes)

Subtract.

1.	2.	3.	4.
\$346.29	\$573.46	\$284.65	\$922.81
159.74	284.57	192.61	748.27

Prove the foregoing by finding the difference between the total of minuends and the total of subtrahends and comparing this with the total of differences.

5. Following is a summary of banking business done by six depositors in a bank on a given day. It shows the six balances at the beginning of the day. Find the balances at the end of the day. Prove the totals.

	Balances	Checks	Deposits	Balances
Smith	\$275.60	\$113.29	\$50.00	
Jones	387.29	75.42	120.50	
Brown	765.60	10.26	35.60	
Green	529.43	129.32		
Gray	624.83		25.00	
White	523.46	284.93	155.50	
Totals				

6. Find the gross profit of each department, the net profit of each department, and the total profits. Prove the totals.

Summary for September, 1918, by Departments

Department	Sales Total	Cost of Goods Sold	Gross Profit	Expense	Net Profit
Furniture	\$4729.60	\$2886.75		\$1189.38	
Dry Goods	6378.75	5635.97		935.07	
Groceries	3811.17	2475.80		764.24	
Clothing	2675.92	1709.78		538.54	
Furnishings	3044.69	1974.37		679.88	
Carpets	6879.54	4311.70		1014.34	
Totals					

CHAPTER III

MULTIPLICATION PRACTICE

The fact that multiplication is a short process for finding the sum of a number of equal addends can be illustrated as follows:

16
16
16
16
64

"4 times 16" means that 16 is to be taken 4 times. The same result is secured by addition as by multiplication.

The following multiplication table, if not already known, should be memorized by the student.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225

The multiplicand and multiplier are the two factors which when multiplied together produce the product. It makes no difference in the result which one is used as multiplier. The product divided by either factor gives the other.

Thus, $4 \times 3 = 12$ or $3 \times 4 = 12$
And $12 \div 4 = 3$ or $12 \div 3 = 4$

ORAL. Multiply the following at sight:

1.	16	29	36	45	82	73	24	56	37	43
	3	2	3	4	5	3	6	5	4	2
2.	61	52	18	63	55	83	75	31	84	57
	3	4	5	2	3	2	2	5	3	3
3.	93	28	85	69	46	41	23	86	79	66
	2	6	4	2	3	5	6	2	2	3
4.	42	32	76	17	87	72	35	26	34	58
	4	5	3	8	4	4	6	3	2	2
5.	33	48	25	88	59	18	71	42	89	91
	4	3	9	4	2	5	2	5	3	9
6.	97	96	47	19	33	94	49	78	65	44
	4	5	3	6	4	2	5	3	6	7
7.	24	51	77	39	98	21	64	53	99	67
	9	7	5	4	2	9	3	4	2	3
8.	59	68	22	95	74	38	81	19	27	36
	4	5	7	6	5	8	8	9	9	4

WRITTEN

Ex. Multiply 364 by 527

364
527
2548
728
1820
191,828

Ex. Multiply 527 by 364

527
364
2108
3162
1581
191,828

Since one solution gives a different set of partial products from the other it is clear that to reverse the factors and multiply again is an excellent method of proof.

WRITTEN. The following problems contain every necessary combination in multiplication.

93245	24953	78106
728	195	372
45293	81760	60718
364	619	548

Multiply the following and add the products. Then add the multiplicands and multiply the total by 562.

1. 2516	**2.** 7384	**3.** 4915	**4.** 6279	**5.** 1832
562	562	562	562	562
6. 3725	**7.** 6813	**8.** 4958	**9.** 4802	**10.** 7913
562	562	562	562	562

What have you just discovered?

Multiply 42,762 by 623. Multiply 25,819 by 623. Subtract the second product from the first. Then subtract the second multiplicand from the first and multiply by 623. What have you just discovered?

Short Methods of Multiplication

Multiplying by 10, 100, 1000, etc.

$$7\times10=70$$
$$7\times100=700$$
$$7\times1000=7000$$

To multiply by 10, 100, 1000, or any number consisting of the figure 1 followed by ciphers, annex to the multiplicand as many ciphers as there are ciphers in the multiplier.

Multiply at sight:

1. 1856×10	**4.** 1738×100	**7.** 1784×10
2. 937×100	**5.** 1156×1000	**8.** 3295×100
3. 57385×1000	**6.** 3158×100	**9.** 1857×1000

Multiplying by 11

EXAMPLE. Multiply 235,816 by 11

$$\begin{array}{r} 235816 \\ \underline{\quad\quad 11} \\ 235816 \\ 235816 \\ \hline 2593976 \end{array}$$

The result is written at sight. From an inspection of the solution as fully set forth here, however, it is evident that the right-hand figure in the product is the same as the right-hand figure in the multiplicand (6); that the next figure is the sum of the 1st and 2d figures at the right in the multiplicand (6+1=7); that the third figure is the sum of the 2d and 3d in the multiplicand (1+8=9); and so on. The fourth figure in the result being 13 (8+5), we write the 3 and carry 1, which we add to the fifth figure in the result.

WRITTEN. Multiply each of the following numbers by 11, writing the results at sight:

1. 2352	**5.** 35,622	**9.** 153,246
2. 4321	**6.** 46,713	**10.** 729,357
3. 5243	**7.** 52,434	**11.** 821,462
4. 6135	**8.** 61,572	**12.** 423,482

Multiplying by 25, 50, and 75

EXAMPLE. Multiply 463 by 25.

$$\begin{array}{r} 4)46{,}300 \\ \hline 11{,}575 \end{array}$$

Since 25 is one-fourth of 100, it is clear that the product when any number is multiplied by 25 is one-fourth of the product secured by multiplying the number by 100. Multiply 463 by 100 by adding two ciphers. Divide by 4.

EXAMPLE. Multiply 671 by 50.

$$\begin{array}{r} 2)67{,}100 \\ \hline 33{,}550 \end{array}$$

Add two ciphers and divide by 2. Why?

EXAMPLE. Multiply 463 by 75.

a	b	c
$\begin{array}{r} 4)46300 \\ \hline 11575 \\ \quad\quad 3 \\ \hline 34725 \end{array}$	$\begin{array}{r} 46300 \\ \quad\quad 3 \\ \hline 4)138900 \\ \hline 34725 \end{array}$	$\begin{array}{r} 4)46300 \\ \hline 11575 \\ \hline 34725 \end{array}$
Add two ciphers, divide by 4, and multiply by 3. Why?	Add two ciphers, multiply by 3, and divide by 4. Why?	Add two ciphers, divide by 4, and subtract. Why?

Multiply 6274 by 25 and by 75. Add the results. Your answer is how many times the multiplicand? Why?

After studying the foregoing, can you formulate rules for your own guidance in multiplying by 250, 500, and 750?

WRITTEN

1.	2.	3.
$3426\times25=?$	$529\times50=?$	$529\times75=?$
$5172\times25=?$	$346\times50=?$	$245\times75=?$
$4834\times25=?$	$138\times50=?$	$317\times75=?$
$2596\times25=?$	$294\times50=?$	$681\times75=?$
$5437\times25=?$	$437\times50=?$	$924\times75=?$
$2148\times25=?$	$629\times50=?$	$305\times75=?$

Prove each one of the above by the method which its arrangement suggests. Explain.

To multiply two numbers of two figures each, when the tens figure in the multiplier is 1.

EXAMPLE. Multiply 17×15.

Full Process	Short Method
17	1 7
15	1 5
85	255
17	
255	

The method is simply a plan for holding a part of the process in mind in order to save the time of writing the partial products. $5\times7=35$. Write the 5 and hold the 3 in mind. $(5\times1)+3=8$. Hold the 8 in mind. $(1\times17)+8=25$. Write the 25 at the left of the 5.

This short method can be used for multiplying any two numbers of two figures each, but it is not easy to apply except when the figure in the tens column of the multiplier is 1.

WRITTEN. The following problems should be solved mentally, and pen or pencil used only to write results.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
14	13	17	18	19	17	24	36	28	43	68	85
16	19	16	15	18	14	18	16	13	13	14	15

13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.
78	69	52	54	29	73	49	67	93	62	47	49
17	16	12	17	18	16	17	14	13	19	13	16

To multiply any two numbers of two figures each.

EXAMPLE. Multiply 46×39.

FIRST STEP

$$\begin{array}{r} 4\ 6 \\ \times\ 3\ 9 \\ \hline 4 \end{array}$$

SECOND STEP

$$\begin{array}{r} 4\ 6 \\ \times\ 3\ 9 \\ \hline 9\ 4 \end{array}$$

THIRD STEP

$$\begin{array}{r} 4\ 6 \\ \times\ 3\ 9 \\ \hline 1794 \end{array}$$

FIRST STEP. $9 \times 6 = 54$. Write the 4 and carry 5.

SECOND STEP. $(9 \times 4) + (3 \times 6) + 5 = 59$. Write the 9 and carry 5.

THIRD STEP. $(3 \times 4) + 5 = 17$. Write the 17.

Multiply by the process just shown:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
27	36	46	54	63	74	87	29	37	46
24	22	28	27	26	23	29	32	35	36
11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
58	67	78	86	92	27	35	46	57	64
33	34	37	38	39	42	46	44	49	43
21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
17	82	89	41	35	72	30	84	21	92
85	57	30	79	21	64	36	46	59	56

To multiply two numbers whose units figures are the same.

$$\begin{array}{r} 46 \\ 76 \\ \hline 3496 \end{array}$$

$6 \times 6 = 36$. Write the 6 and carry the 3. $(7+4) \times 6 + 3 = 69$. Write the 9 and carry the 6. $4 \times 7 + 6 = 34$. Write the 34.

WRITTEN. Find the products by the method just described.

1. 35×65
2. 24×54
3. 19×29
4. 62×32
5. 47×57
6. 23×63
7. 31×41
8. 62×42
9. 56×96

To multiply two numbers whose tens figures are the same.

37
32
1184

$2\times7=14$. Write the 4 and carry the 1. $(7+2)\times3+1=28$. Write the 8 and carry the 2. $3\times3+2=11$. Write the 11.

WRITTEN. Find the products by the method just described.

1. 25×26	**4.** 85×87	**7.** 47×42
2. 51×54	**5.** 34×31	**8.** 18×13
3. 92×96	**6.** 65×69	**9.** 72×77

To multiply by the supplement or complement method.

EXAMPLE. Multiply 324 by 103.

32,400
972
33,372

The multiplier is $100+3$. Multiply 324 by adding two ciphers. Add 3 times 324.

EXAMPLE. Multiply 324 by 97.

32,400
972
31,428

The multiplier is $100-3$. Multiply 324 by 100 by adding two ciphers. Subtract 3 times 324.

WRITTEN. Find the products.

1. 167×98	**4.** 527×104	**7.** 918×97
2. 716×102	**5.** 283×94	**8.** 439×107
3. 354×95	**6.** 845×99	**9.** 692×96

When the number of units is an integral part of the number of tens or hundreds, or vice versa.

EXAMPLE. Multiply 427×248.

427
248
3416
10248
105,896

The number of units in the multiplier (8) is one-third of the number of tens (24). Multiply 427 by 8, which gives 3416 as a partial product. Multiply this partial product by 3 to get the partial product for the tens' column.

The foregoing example and the example given at the top of the next page have little or no practical value. They are interesting, however, because they show what can be done with figures, and they will suggest a new line of thought to the student who has been accustomed to handling figures by formula without thought.

EXAMPLE. Multiply 356 by 728.

356
728
2492
9968
259,168

The number of hundreds in the multiplier (7) is one-fourth of the number of units (28). Multiply 356 by 7 and write the partial product (2492) in the hundreds' column. Multiply this partial product by 4 to get the partial product for the unit's column.

Find the products, using either factor as multiplier, as may appear convenient in each case.

1. 563×124
2. 357×429
3. 637×312
4. 235×126
5. 327×255
6. 754×168
7. 439×183
8. 527×153
9. 724×279

Proofs of multiplication.

The accuracy of any result in multiplication may be tested. There are several good methods:

1. By transposing the factors and multiplying again.
2. By dividing the product by either factor to get the other factor.
3. By casting out the 9's or the 11's.
4. By the factor method.

The first two methods named require no special explanation. Following are explanations of the last two methods.

Proof of multiplication by casting out the nines.

346 − 4
624 − 3
1384 − 12—3
692
2076
215904 − 3

Casting out the 9's from the multiplicand and multiplier gives 4 and 3 as check figures. $4\times3=12$, the check figure of which is 3, which agrees with the check figure secured by casting out the 9's from the product. Since the two check figures agree the result is probably correct. Failure to agree indicates error.

Proof of multiplication by casting out the elevens.

367 − 4
479 − 6
3303 − 24—2
2569
1468
175793 − 2

The proof is the same as that given above except that the 11's are cast out instead of the 9's.

WRITTEN. Multiply the following problems. Prove your work before handing in your results. Your method of proof may be either

(*a*) By reversing multiplicand and multiplier, and multiplying.
(*b*) By dividing the product by one of its factors to get the other.
(*c*) By casting out the 9's or the 11's.

1. 43×567	**5.** 234×567	**9.** 536×8243
2. 56×843	**6.** 618×527	**10.** 327×5209
3. 72×419	**7.** 824×356	**11.** 407×6124
4. 81×325	**8.** 723×461	**12.** 546×4039

Prove each of the following groups of problems by the plan which their arrangement suggests:

13.	**14.**	**15.**
$28\times\ 356=?$	$73\times429=?$	$564\times329=?$
$28\times\ 721=?$	$73\times316=?$	$564\times416=?$
$28\times\ 538=?$	$73\times587=?$	$564\times807=?$
$28\times\ 914=?$	$73\times405=?$	$564\times514=?$
$28\times2529=?$	$73\times\ ?\ =?$	$564\times\ ?\ =?$

Multiplying by the factor method.

When the multiplier can be divided into two factors, the product can be found by multiplying the multiplicand by one of the two factors and multiplying the result so secured by the other factor.

EXAMPLE. Multiply 429 by 63.

429	429
63	7
1287	3003
2574	9
27027	27027

The factors of 63 are 7 and 9. The product secured by multiplying 429 by 63 is the same as that secured by multiplying 429 by 7 and then multiplying by 9 the result so secured. If there are more than two factors, the process is continued until all are used.

Multiplying by the factor method is not economical of time, but it furnishes a convenient method of proof, and a knowledge of it adds to your general conception of the structure of numbers.

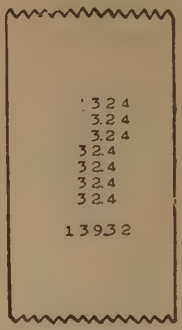
WRITTEN. Multiply. Prove by the factor method.

1. 63×582	**5.** 24×758	**9.** 25×456
2. 21×146	**6.** 32×379	**10.** 36×247
3. 35×297	**7.** 56×951	**11.** 72×848
4. 49×631	**8.** 48×623	**12.** 27×139

Multiplication on the adding machine.

A multiplier may be divided into two or more addends, and the product found by multiplying by the addends separately and adding the results. This principle is adapted to multiplication on the adding machine as follows:

Ordinary Method	Addend Method
324	$3 \times 324 = 972$
43	$40 \times 324 = 12960$
972	$43 \times 324 = 13932$
1296	
13932	



Explanation. The multiplier is divided into two addends, 40 and 3. By the method shown in the middle column (which constitutes a good explanation of the process of multiplication, since it is simply another way of showing what is in the first column) it is apparent that $(3 \times 324) + (40 \times 324)$ gives the same result as 43×324.

On the adding machine 3×324 is represented by 324 listed three times as an addend, and 40×324 is represented by 324 written as tens and listed four times as an addend.

Written. Perform the following multiplications. Prove your results with an adding machine or by listing your addends as they would be listed on an adding machine and finding the total.

1. 496×43	**5.** 239×24	**9.** 836×51
2. 525×33	**6.** 563×21	**10.** 729×24
3. 629×25	**7.** 629×42	**11.** 432×32
4. 823×42	**8.** 729×32	**12.** 987×26

Invoices and Statements

An **invoice** is a written list of the items of a sale. It should show in its heading the names of the parties to the sale, and the date. Its body should show the name, quantity, and price of each article, the total selling price of each item and the total of the sale. Filling in the totals of the various items is called *extending* the invoice; finding the total of the invoice is called *footing* it.

NEW YORK, April 30, 19 ____

Messrs John King & Co.
1133 Broadway, City.

BOUGHT OF MASON & OSGOOD

TERMS: 30 DAYS

32	lbs. Y. H. Tea	.45	14	40	
75	" Java Coffee	.20	15	—	
25	kits Mackerel	1.50	45	—	
48	bas Herring	2.75	13	20	
15	" Val. Raisins	1.40	21	—	
					108 60

Avoid indiscriminate use of the word "bill," which is variously used to indicate an invoice, statement of account, statement of services, piece of paper currency, note, draft, bill of sale, bill of exchange, and a legislative enactment, besides having several uses in legal phraseology. Use the specific word whenever possible. There seems to be no word other than *bill* which is commonly used when a statement of amount due for services is meant, but in all other cases it is better to use the specific term.

Rule the necessary forms and make an invoice for each of the following transactions:

1. On Jan. 2, 1928, Wenly & Cowling bought of Beecher, Peck, and Lewis, Detroit, Mich.: 2 boxes assorted visiting cards @ 75¢; 200 Thanksgiving napkins @ 20¢ a hundred; 1000 napkins @ 75¢ a hundred; 3½ doz. writing pads @ 40¢; 2½ doz. single oak slates @ 68¢; 2 gr. rules @ 94¢; 6 doz. note books @ 35¢; 5 doz. pads drawing paper @ 38¢; 4 doz. paint balls @ $1.86. Terms: Net cash.

2. Butler Bros., Chicago, Ill., sold to John Dallas & Son, Fargo, N. Dakota, on Jan. 3, 1928: 4 doz. work baskets @ 89¢; 3 doz. match safes @ 87¢; 5 doz. cloth sprayers @ 45¢; 6 doz. towel bars @ 78¢; 5 doz. flour sifters @ 84¢; 3 doz. strainers @ $1.84; 2 doz. percolaters @ 89¢; 4 doz. 8 x 10-inch drip pans @ 65¢; 3 doz. wash baskets @ $10.20. Terms: 1/20 n/60.*

* 1% off for cash in 20 days, or net price (without discount) in 60 days.

3. The United Fruit Co., New York, N. Y., sold to Harper Bros., Yonkers, N. Y., on Jan. 5, 1928: 24 bbl. Spy apples @ $2.85; 15 bbl. Greening apples @ $2.60; 15 bx. Florida grape fruit @ $3.10; 12 bx. California lemons @ $5.60; 50 bu. potatoes @ 78¢; 36 bx. Florida oranges @ $2.90. Terms: Net 10 days.

4. Tare is a deduction from total weight to allow for boxes, casks, or other containers. In the following invoice the amount of tare is shown opposite each item. Find the total and net weight of each item; then extend and foot the invoice.

NEW YORK, Jan. 2, 19--.

M WESTERN GROCERY HOUSE,

ADDRESS 45 E. Randolph St., Chicago. Ill.

BOUGHT OF **WESTFELDT BROS.**

IMPORTERS AND JOBBERS OF

COFFEE

SHIPPED BY Pa. Railroad 76 FRONT STREET

TERMS: 60 Days-Net, 2%-Spot Cash on receipt of pro forma invoice.

Bags	Brand	Weight Lbs.				Price	Amount		Total	
			Gross	Tare 1%	Net					
20	Best O. G. Java	10/133, 3/134, 7/132		26		17				
25	Select Golden Rio	14/133, 5/134, 6/132		33		11				
30	Fancy Santos	17/133, 6/134, 7/132		39		9				

5. On Feb. 1, 1928, the Sheboygan Cheese Company sold to the Monarch Grocery Co., 147 E. Water St., Chicago, 30 boxes Am. Cheese weighing as follows: 1-40#, 2-42#, 5-43#, 3-46#, 7-45#, 12-44#, at 11¢ per lb.; and 1 block Swiss Cheese weighing 780# less 105# tare, at 13¢. What was the amount of the invoice?

6.

73 lb. coffee at	39¢
49 lb. tea at	42¢
54 lb. chocolate at	53¢
47 lb. cocoa at	41¢
56 lb. tea at	45¢
67 lb. coffee at	37¢
29 lb. tea at	46¢
37 lb. cocoa at	43¢

7.

129 lb. pecans at	37¢
87 lb. raisins at	16¢
238 lb. sugar at	7¢
89 lb. almonds at	23¢
36 lb. dates at	27¢
147 lb. pecans at	26¢
248 lb. sugar at	6¢
49 lb. almonds at	29¢

8.	47 yd. muslin at	13¢	**9.**	130 bu. wheat at	$1.12
	29 yd. denim at	18¢		213 bu. rye at	1.07
	74 yd. percale at	23¢		127 bu. barley at	1.02
	32 yd. silk at	89¢		47 bu. oats at	.87
	73 yd. flannel at	53¢		97 bu. corn at	.97
	27 yd. dress goods at	94¢		43 bu. wheat at	.99
	34 yd. duck at	28¢		57 bu. oats at	.78
	93 yd. cotton at	9¢		134 bu. barley at	1.01
	45 yd. madras at	52¢		219 bu. corn at	.87

A statement of account is a written list of the totals of invoices to a customer during a certain period, usually a month. This list always shows the dates and amounts of the various invoices and payments during the period, but does not necessarily show the detail of the invoices since that data is given on the separate invoices.

The statement should usually show the charges in one group and the credits in another group. The balance due is the difference between the totals of the two groups. Herewith are model forms of two monthly statements of account. They are made to the same person and show how the balance due at the end of one month is carried to the next month's statement as a beginning balance.

STATEMENT

CHICAGO, ILL. Mar. 31 19—

M. C E Birch

1224 Harrison St, City.

IN ACCOUNT WITH **H. T. RAYNOR**

364 WABASH AVE.

19—						
Mar.	10	To Mdse.	200	25		
	28	" "	72			
					272	75

STATEMENT

CHICAGO, ILL. Apr. 30 19—

M. C E Birch,

1224 Harrison St, City.

IN ACCOUNT WITH **H. T. RAYNOR**

364 WABASH AVE.

19—						
Mar.	31	Stat. rendered.	272	75		
Apr.	10	To Mdse.	66	50		
	28	" "	327	50		
					666	75
		Credits				
Apr.	10	By note	66	50		
	25	" cash	272	75		
					339	25
		Balance due			327	50

TWO OF H. T. RAYNOR'S MONTHLY STATEMENTS TO C. E. BIRCH

10. Prepare a statement dated June 30 from the following facts: On June 1, 1927, R. L. McNeil owed Bowen & Raidner $235.64. His purchases and payments during June were as follows: June 3, invoice, $79.43; June 10, invoice, $83.45; June 12, cash payments, $52.47; June 17, invoice, $91.65; June 21, payment by note, $200.00; June 23, invoice, $127.33; June 27, invoice, $42.69.

11. R. L. McNeil's further transactions on the same account: July 3, invoice, $32.96; July 9, invoice, $43.85; July 13, invoice, $54.74; July 15, cash payment, $50.65; July 20, invoice, $65.63; July 21, cash payment, $32.96; July 24, payment by note, $127.33; July 26, invoice, $76.52. Prepare a statement as of July 31, 1917.

Postal Rates

Problems involving mailing expense, while very simple, are important and teach facts which everyone should know.

Mail matter is divided into four classes arranged in the order of cost of mailing, as follows:

1st class. Written Matter, Sealed or Unsealed: Rate, 2¢ per oz. or fraction; postal cards, 1¢.

2nd class. Newspapers and Magazines: Rate, 1¢ for each 2 oz. or fraction, to 8 oz.

3rd class. Merchandise and Printed Matter: Rates: (1) 1½¢ for each 2 oz. or fraction, to 8 oz., inclusive; (2) Books, catalogues, seeds, etc., 1¢ for each 2 oz. or fraction, to 8 oz., inclusive.

4th class (Parcel Post). Merchandise and Printed Matter weighing more than 8 oz.: Rates: For the purpose of fixing rates readily, each mailing point is assumed to be the center of a system of circles around it having radii of 50, 150, 300, 600, 1000, 1400, and 1800 miles, respectively. Points between each of these circles and the next are said to be within a certain *zone*. Thus all points between 50 and 150 miles away are said to be within the second zone. The same postal rate applies to all parcels mailed to all points within a zone.

Parcels weighing more than 8 oz. up to 50 lb. may be sent to any one of the eight zones, and parcels weighing up to 70 lb. to the first three. See table, on page 57, for rates. "Service charge" was abolished July 1, 1928.

The cheapest rate is on matter intended for delivery at or from the same office at which it is mailed; this is called the local rate. The next lowest rate is for matter mailed to points within a radius of 50 miles, or within the 1st zone. All zones, and the rates applicable thereto, are shown by the table herewith.

WEIGHT	First Zone		2d Zone 50 to 150 Miles	3d Zone 150 to 300 Miles	4th Zone 300 to 600 Miles	5th Zone 600 to 1000 Miles	6th Zone 1000 to 1400 Miles	7th Zone 1400 to 1800 Miles	8th Zone all over 1800 Miles
	Local Rate	Zone Rate 50 miles	Rate	Rate	Rate	Rate	Rate	Rate	Rate
1 pound....	$0.07	$0.07	$0.07	$0.08	$0.08	$0.09	$0.10	$0.12	$0.13
2 pounds....	.08	.08	.08	.10	.12	.15	.18	.22	.25
3 pounds....	.08	.09	.09	.12	.16	.21	.26	.32	.37
4 pounds....	.09	.10	.10	.14	.20	.27	.34	.42	.49
5 pounds....	.09	.11	.11	.16	.24	.33	.42	.52	.61
6 pounds....	.10	.12	.12	.18	.28	.39	.50	.62	.73
7 pounds....	.10	.13	.13	.20	.32	.45	.58	.72	.85
8 pounds....	.11	.14	.14	.22	.36	.51	.66	.82	.97
9 pounds....	.11	.15	.15	.24	.40	.57	.74	.92	1.09
10 pounds....	.12	.16	.16	.26	.44	.63	.82	1.02	1.21
11 pounds....	.12	.17	.17	.28	.48	.69	.90	1.12	1.33
12 pounds....	.13	.18	.18						
13 pounds....	.13	.19	.19						
14 pounds....	.14	.20	.20						
15 pounds....	.14	.21	.21						
16 pounds....	.15	.22	.22						
17 pounds....	.15	.23	.23						
18 pounds....	.16	.24	.24						
19 pounds....	.16	.25	.25						
20 pounds....	.17	.26	.26						

Parcels must not be greater in size than 84 inches in length and girth combined nor in form or kind likely to injure the person of any postal employe or damage the mail equipment or other mail matter and not of a character perishable within a period reasonably required for transportation and delivery.

Limit of weight for delivery within first, second, and third zones, 70 pounds; to all other zones, 50 pounds.

Note. The table may be continued by adding for each additional pound as follows: 1st and 2nd zone, 1¢; 3rd zone, 2¢; 4th, 4¢; 5th, 6¢; 6th, 8¢; 7th, 10¢; 8th, 12¢. Local zone, 1¢ for each 2 lbs.

Parcel post charges are prepaid by affixing stamps, as to most other packages sent through the mails.

1. What will be the cost of sending an article weighing 5 lb. into the third zone? One weighing 9 lb.?

2. Into what zone was an article mailed if its weight was 11 lb. and the cost of postage was $1.13? 28¢?

3. A package weighing 16 lb. is sent 75 mi. What will it cost?

4. A package weighing 10 lb. is sent from New York City to Seattle, Wash. What is the amount of postage required?

5. A department store wishes to send 20 parcels weighing 3 lb. each into the second zone. How much postage is required?

6. A mail order house made the following shipments in one day:

148 parcels, 8 lb. each, to the 4th zone.
352 parcels, 11 lb. each, to the 1st zone.
210 parcels, 9 lb. each, to the 2d zone.
174 parcels, 10 lb. each, to the 6th zone.
55 parcels, 4 lb. each, to the 8th zone.

What was the total cost of postage?

7. Into what zone may a parcel weighing 18 lb. be sent for 24¢?

8. J. D. Burnett & Co. sent out the following parcels. How much did the stamps cost them for all?

WEIGHT	DISTANCE TO GO	WEIGHT	DISTANCE TO GO
10 lb.	100 miles	$5\frac{1}{2}$ lb.	1120 miles
8 lb.	175 miles	6 lb.	4th zone
$4\frac{1}{2}$ lb.	20 miles	10 lb.	29 miles
9 lb.	local	$3\frac{3}{4}$ lb.	2d zone
3 lb.	487 miles	$7\frac{1}{2}$ lb.	local
$6\frac{3}{4}$ lb.	392 miles	2 lb.	5th zone
$1\frac{1}{2}$ lb.	29 miles	7 lb.	1st zone
4 lb. 1 oz.	1001 miles	11 lb.	3d zone
4 lb. 15 oz.	1399 miles	$10\frac{1}{2}$ lb.	6th zone
5 lb. 2 oz.	453 miles	$6\frac{1}{2}$ lb.	1805 miles

9. The five packages listed below were sent out C.O.D. by Barclay and Barrett. Find the amount of the money order collected in payment for each parcel and the total amount of money returned to Barclay and Barrett, if postage was in each case included in the bills to the customers.

NO. OF ARTICLES	WEIGHT PER UNIT	PRICE PER UNIT	WEIGHT OF WRAPPER	FEE FOR MONEY ORDER	DISTANCE TO BE CARRIED
20	8 oz.	$0.50	12 oz.	$0.08	625 mi.
36	$4\frac{1}{2}$ oz.	.20	2 lb.	.08	76 mi.
4	2 lb.	10.50	$1\frac{1}{2}$ lb.	.18	83 mi.
12	13 oz.	5.70	6 oz.	.20	429 mi.
15	$\frac{1}{2}$ lb.	3.50	1 lb.	.18	362 mi.

Written Review Test

(Time, 30 minutes)

Multiply

1. 427	**2.** 518	**3.** 417	**4.** 258	**5.** 639
346	269	384	615	274

Prove each of the above by inverting the factors and multiplying.

6. 504	**7.** 916	**8.** 720	**9.** 183	**10.** 427
146	237	381	295	456

Prove each of the last five problems by casting out 9's.

11. 621	**12.** 257	**13.** 318	**14.** 436	**15.** 594
275	381	296	407	318

Prove each of the last five problems by casting out 11's.

16. 248	**17.** 362	**18.** 517	**19.** 130	**20.** 759
63	48	64	72	49

Prove each of the last five problems by the factor method.

Write the answers only to the following. Solve by inspection.

21. Multiply 427 by 10
22. Multiply 634 by 25
23. Multiply 533 by 50
24. Multiply 248 by 75
25. Multiply 53 by 73
26. Multiply 3476 by 11
27. Multiply 463 by 1000
28. Multiply 529 by 100
29. Multiply 17 by 19
30. Multiply 24 by 32

31. Perform the five multiplications and add the products. Add the multiplicands and prove the whole problem by one multiplication.

3127	6543	2782	3642	5901
247	247	247	247	247

CHAPTER IV

DIVISION PRACTICE

The fact that division is a short method of finding how many times one number contains another can be proved by a series of subtractions, thus:

How many times does 109 contain 35?

109
35
74
35
39
35
4

Subtracting 35 as many times as possible, we find that it is contained in 109 three times, with a remainder of 4. The process here illustrated is called continuous subtraction, and is said to be required frequently in civil service examinations. The same result may be secured more quickly by division.

WRITTEN. By continuous subtraction determine how many times

1. 136 contains 25
2. 83 contains 12
3. 147 contains 17
4. 976 contains 120
5. 3720 contains 470
6. 739 contains 150

Division is the reverse of multiplication and calls for a knowledge of the same tables. When the divisor is a number with the multiplication table of which you are familiar, use the short division method; if not, use the long division method.

SHORT DIVISION	LONG DIVISION
4405	4405
9)39645	18)79290
	72
	72
	72
	90
	90

Never use the long division method when the short division method can be used.

ORAL

1. How many times is 11 contained in 352? 495? 132? 451?
2. How many times is 14 contained in 294? 448? 434? 728?
3. How many times is 15 contained in 765? 945? 360? 465?

4. How many times is 16 contained in 320? 336? 496? 336?
5. How many times is 9 contained in 828? 225? 513? 423?
6. How many times is 12 contained in 144? 1728? 216? 252?
7. Divide 720 by 2; by 3; by 4; by 5; by 6; by 8; by 9; by 12.
8. Divide 462 by 3; by 7; by 11; by 2; by 21; by 22; by 14.
9. Divide 390 by 2; by 3; by 5; by 6; by 13; by 10; by 15.
10. Divide 660 by 11; by 6; by 10; by 22; by 33; by 15; by 12.

WRITTEN. Using the short division method

1. Divide 83,962 by 7
2. Divide 72,841 by 4
3. Divide 73,416 by 8
4. Divide 82,143 by 3
5. Divide 52,631 by 11
6. Divide 26,183 by 12
7. Divide 24,578 by 7
8. Divide 57,319 by 5
9. Divide 24,694 by 11
10. Divide 73,156 by 9

Divide the following by the long division method:

11. 264)43,662
12. 317)19,125
13. 281)81,727
14. 653)24,535
15. 628)49,253
16. 471)36,818
17. 249)57,464
18. 595)67,918
19. 437)29,387

Dividing by 10, 100, 1000, etc.

$$70 \div 10 = 7$$
$$700 \div 100 = 7$$
$$7000 \div 1000 = 7$$

You have learned that to multiply by 10, 100, or 1000 it is only necessary to annex to the multiplicand as many ciphers as there are ciphers in the multiplier. It follows that to divide by 10, 100, or 1000 it is only necessary to take off the ciphers. Rule:

To divide by 10, 100 or any number consisting of 1 followed by ciphers, point off in the dividend as many places as there are ciphers in the divisor. Thus to divide 7000 by 1000, point off three places in the dividend. The result is 7. Divide

1. 4560 by 10
2. 38,900 by 100
3. 46,060 by 10
4. 37,800 by 100
5. 473,000 by 1000
6. 12,860 by 10
7. 9700 by 100
8. 12,500 by 10
9. 16,300 by 100
10. 37,850 by 10
11. 19,780 by 10
12. 20,000 by 1000
13. 45,050 by 10
14. 13,900 by 100
15. 26,370 by 10

Dividing by 25, 50, or 75.

EXAMPLE. Divide 11,575 by 25.

11,575
4
46,3ØØ

Under multiplication you learned that to multiply by 25 you should add two ciphers and divide by 4. To divide by 25, therefore, reverse the process — that is, multiply by four and strike off the two ciphers (or other figures) at the right. Another way of stating the case is this: If a given number is 25 times a required number, 4 times the given number is 100 times the required number.

EXAMPLE. Divide 33,550 by 50.

33,550
2
67,1ØØ

Multiply by 2 and strike off two ciphers or divide by 100. Why?

EXAMPLE. Divide 34,725 by 75.

3)34,725
11,575
4
46,3ØØ

Divide by 3, multiply by 4, and divide by 100. Why?

Formulate rules of your own for dividing by 250, 500, and 750.

WRITTEN. Divide

1. 2375 by 25
2. 3225 by 25
3. 4950 by 50
4. 1300 by 50
5. 3175 by 25
6. 7950 by 75
7. 3675 by 75
8. 1825 by 25
9. 3175 by 25
10. 95,250 by 750
11. 27,500 by 500
12. 62,250 by 250

Proofs of Division

There are several methods of proving division:

1. Multiply the quotient by the divisor, adding to this result the remainder if any, to get the dividend. This is as satisfactory a method as any and more generally used than others.
2. Divide the dividend by the quotient to get the divisor.
3. Prove by casting out the 9's or the 11's.
4. Separate the divisor into factors. Divide by these in turn.

Of the four methods of proof suggested above, the first two need no description. Following are descriptions of the other two:

Proof by casting out the 9's.

```
        455
327)148,785
    130 8
     17 98
     16 35
      1 635
      1 635
```

Casting out the 9's from 455 leaves 5. Casting out the 9's from 327 leaves 3. $5 \times 3 = 15$. Casting out the 9 from 15 leaves 6 as an excess number. Casting out the 9's from 148785 leaves 6, which agrees with the excess number previously secured. Therefore the division is probably correct.

The proof by casting out the 11's is similar to the proof by casting out the 9's, and as you are already familiar with the method of finding the excess numbers by casting out the 11's, no further explanation is necessary.

Proof by the factor method.

```
      2057          7)129,636
63)129,636          9)18 519 − 3
   126                 2 057 − 6
    3 63
    3 15     6×7+3=45, rem.
     486
     441
      45
```

Instead of dividing by 63, divide first by 7 and then by 9, these being the factors of 63. Show the remainders as in the illustration. The result, 2057, is the same as found by dividing by 63. The remainder is found as follows: After dividing by 7, 3 units were left over. The first quotient 18519 represented 18519 *sevens*. Hence the remainder of 6 represents *sevens*. 6 sevens = 42 units. $42 + 3 = 45$, the remainder.

WRITTEN. Divide. Prove by the factor method.

1. 1044 by 36	**13.** 1449 by 63	**25.** 1381 by 36
2. 1512 by 54	**14.** 2448 by 72	**26.** 1280 by 27
3. 1540 by 28	**15.** 1664 by 64	**27.** 1642 by 33
4. 756 by 21	**16.** 2176 by 49	**28.** 1582 by 35
5. 1680 by 35	**17.** 2997 by 81	**29.** 1738 by 45
6. 1377 by 51	**18.** 2112 by 66	**30.** 1471 by 21
7. 972 by 18	**19.** 1470 by 42	**31.** 1186 by 42
8. 815 by 27	**20.** 1176 by 24	**32.** 1473 by 18
9. 1881 by 33	**21.** 1835 by 28	**33.** 1247 by 22
10. 1305 by 45	**22.** 1947 by 32	**34.** 2643 by 54
11. 1248 by 48	**23.** 1483 by 24	**35.** 1370 by 24
12. 1519 by 49	**24.** 1179 by 25	**36.** 1185 by 64

Written Review Test

(Time, 20 minutes)

1.	2.	3.	4.	5.
$12\overline{)75,852}$	$9\overline{)65,916}$	$7\overline{)52,871}$	$11\overline{)13,574}$	$8\overline{)137,652}$

Prove the above by multiplication.

6.	7.	8.	9.
$532\overline{)27,664}$	$426\overline{)37,914}$	$713\overline{)41,354}$	$496\overline{)52,576}$

10.	11.	12.	13.
$841\overline{)68,121}$	$489\overline{)79,218}$	$315\overline{)80,640}$	$527\overline{)91,698}$

Prove problems 6 to 13 by dividing the dividend by the quotient in each case. Is the remainder the same?

General Review Problems

1. A publisher had readers on hand as follows on April 1, 1927:

	Primers	First Readers	Second Readers	Third Readers	Fourth Readers	Fifth Readers	Sixth Readers	Higher Readers
In stock room.....	1269	2456	3524	829	1529	2162	1564	2132
At bindery.....	4000	7500	8295	3000	6500	3000	4800	3550

During the fiscal year ensuing he printed 25,000 of each of the first three books and 15,000 of each of the others. On April 1, 1928, he had readers on hand as follows:

	Primers	First Readers	Second Readers	Third Readers	Fourth Readers	Fifth Readers	Sixth Readers	Higher Readers
In stock room.....	627	1454	1569	2456	1246	2054	3027	3129
At bindery.....	2560	3240	3500	2060	3427	4360	5000	6550

What were his sales of each book for the fiscal year?

2. Prepare an invoice from the following facts: J. C. Jones, 1249 Cabanne Ave., St. Louis, Mo., bought from H. P. Harkness & Co. 529 S. Main St., Peoria, Ill., on June 5, 1926, 6 tubs of lard at 14¢, 4 bbl. hams at 19¢, and 3 bbl. ribs at 16¢. Weights and tare were as follows: Lard 69-15, 70-14, 71-15, 70-15, 69-14, 70-14; hams 415-69, 418-70, 412-68, 416-69; ribs 411-66, 419-68, 415-67. Terms 2/10, n/60.

3. From the following information, compute the profit for the year in each department, and the total profit.

	On Hand May 1, 1917	Purchases for Year	On Hand May 1, 1918	Cost of Goods Sold	Sales	Profit
Groceries	2563.92	10624.50	2489.65		12864.29	
Hardware	3417.65	8542.93	3219.46		10946.60	
Dry Goods	3678.23	9346.72	2943.82		9529.43	
Clothing	5285.64	12554.89	4857.21		15629.82	
TOTALS						

4. Find the balance due each of the following depositors and the total balances at the close of the day.

Depositor	Balance	Checks		Total Checks	Dep.	Balance
		10.	25.32			
Henry Adams	278.54	5.50	1.69		50.00	
		13.87	27.60			
George Black	529.86	15.00	5.49		145.69	
		2.25				
William Carson	23.49	5.	1.07		27.82	
Charles Deere	456.27	158.63			300.00	
TOTAL						

5. In 1920 the population of New York State exceeded that of Pennsylvania by 1,665,210. Illinois was 2,234,737 smaller than Pennsylvania and 1,677,238 more than double Iowa in population. If the population of Iowa was 2,404,021, what was the population of New York? Illinois?

6. Construct a problem in addition as follows: Write 3167 as the first addend and 2584 as the second. For the third addend write the sum of the first and second addends; for the fourth, the sum of the second and third; and so continue until there are ten addends. Add. Multiply by 6. Multiply the product by 7. Multiply the product by 9. Divide by 4158. Subtract the third addend three times and the second addend twice.

7. A Month's Business

On Jan. 1, 1926, I invested in business $1000 in cash.

Jan. 1. Paid rent for 1 month in cash, $35.00.

Jan. 2. Bought merchandise for cash, $300.50.

Jan. 3. Sold merchandise for cash, $50.65.

Jan. 4. Sold merchandise to John Smith on account, $45.20.

Jan. 6. Paid cash for expenses, $12.75.

Jan. 10. Bought merchandise from Henry Black on account, $65.75.

Jan. 13. Paid cash for expenses, $13.30.

Jan. 15. Sold merchandise for cash, $47.50.

Jan. 18. Sold merchandise to John Smith on account, $37.28.

Jan. 19. Sold merchandise for cash, $52.69.

Jan. 20. Paid cash for expenses, $13.72.

Jan. 22. Received cash from John Smith on account, $32.50.

Jan. 25. Paid cash to Henry Black on account, $43.65.

Jan. 27. Paid cash for expenses, $13.47.

Jan. 28. Sold merchandise to John Smith on account, $13.75.

Jan. 30. Sold merchandise for cash, $22.45.

(*a*) How much cash did I have on hand Jan. 31? How much did John Smith owe me? If I had merchandise on hand valued at $287.32, what was the total of my assets on Jan. 31?

(*b*) How much did I owe Henry Black on Jan. 31? Subtracting this from my total assets, how much was my business worth on Jan. 31?

(*c*) What were my total purchases of merchandise during the month? Deducting the value of the merchandise on hand, what is left as the cost of the goods sold? What was the total of the merchandise sales? What was the profit on merchandise?

(*d*) What was the total cost of rent and other expenses? Deducting this from the profit on merchandise, how much is my net profit for the month? Adding this net profit to my original investment, what is shown to be the worth of my business? Does this agree with your answer to (*b*)?

General Review

1. Add each of the following groups, adding downwards, and add the results for a grand total.

a	b	c	d	e
$234.68	$347.26	$948.26	$654.23	$815.16
729.03	623.41	95.83	254.29	535.47
624.51	61.97	772.58	44.30	696.09
752.93	575.03	66.84	32.92	103.18
124.86	76.78	109.21	198.13	670.21
250.60	183.92	92.65	391.75	974.43
526.92	624.61	47.82	880.56	62.57
341.62	35.10	981.46	21.37	786.18
253.29	307.64	108.03	450.48	51.02
674.38	42.03	25.14	46.97	48.93

Prove by adding upwards.

2. Subtract the following:

a	b	c	d	e
$924.68	$834.56	$922.15	$764.31	$812.42
632.49	567.82	435.79	129.76	675.35

Prove by adding the minuends and adding the subtrahends, then testing whether the difference between the totals equals the sum of the five results.

3.
$329 \times 49 = ?$
$265 \times 49 = ?$
$417 \times 49 = ?$
$608 \times 49 = ?$
$431 \times 49 = ?$
$236 \times 49 = ?$
$? \times 49 = ?$

4.
$328 \times 63 = ?$
$471 \times 63 = ?$
$613 \times 63 = ?$
$795 \times 63 = ?$
$526 \times 63 = ?$
$408 \times 63 = ?$
$? \times 63 = ?$

5.
$543 \times 56 = ?$
$716 \times 56 = ?$
$825 \times 56 = ?$
$397 \times 56 = ?$
$204 \times 56 = ?$
$618 \times 56 = ?$
$? \times 56 = ?$

6. 621)43,582

7. 185)20,473

8. 397)69,474

Problems without Figures

1. If you sold a customer several bills of goods and the customer made several payments, how could you determine the amount still due?

2. Henry Williams made six different deposits in a bank at various times and has issued ten different checks drawing money out of the bank. How could you tell the balance in the bank, and what convenient arrangement would show him the facts?

3. If you know the balance in the bank at the end of May, the amount deposited and the amount withdrawn during the month of May, how can you find the balance in the bank at the beginning of May?

4. If you know the total cost of six articles and the individual cost of five of them, how would you find the cost of the remaining one?

5. James Bronson had a certain amount of cash at the beginning of April. You know how much he received during April and how much he has at the end of April. How can you find the amount he spent during April?

6. Williams Brothers bought a certain amount of goods during the year 1927. At the end of the year you know the cost of the goods unsold. How will you find the cost of the goods that they sold?

7. During the year 1928, Hannan & Co. made a profit from selling goods and a profit from renting part of their store. They were at an expense in hiring clerks, advertising their goods, paying for fuel, light, etc. If you knew the amounts in each of the above items, how would you find the net profits?

8. If you knew the amount of money with which James Kenny began business and knew how much he had gained and how much he had lost, how would you find what he is worth now?

9. If you knew the weight, separately, of several horses, how would you find the average weight?

10. If you knew the distance from St. Louis to New Orleans and knew how far a gallon of gasoline would run an automobile, and knew the cost per gallon, how would you estimate the cost of gasoline in running from one of these cities to the other?

CHAPTER V

FRACTIONS

Preliminary Considerations

Factoring

The **factors** of a number are the numbers which when multiplied together make the number. Thus 3 and 5 are the factors of 15. Factoring a number is the process of determining its factors.

A **composite number** is one that is composed of two or more integral factors (besides itself and 1). Thus, 15 is a composite number.

A **prime** number is a number which has no factors besides itself and 1. Thus 13 is a prime number. *Prime factors* are therefore factors which cannot themselves be factored. Thus, 4 and 5 are factors of 20, but are not its prime factors. The prime factors of 20 are 2, 2, and 5. When the word *factors* is used, prime factors are usually understood.

ORAL EXERCISE

1. Name the prime numbers from 1 to 100.

2. Name the composite numbers from 1 to 100, giving at least two factors for each.

Explanation: In naming the factors of a number you must name numbers which when multiplied together produce it, naming them all. If asked to name two factors of 30, your only proper answer is either "6 and 5" or "3 and 10" or "2 and 15." It would be improper to say that 3 and 5 are factors of 30 because *factors* (from *facio* = to *make*) are *makers*, and 3 and 5 multiplied together do not make 30. The factors of 30 are 2, 3, and 5.

3. Name the prime factors of

18	24	16	20	28	44	52
27	45	63	81	72	36	50
12	48	54	32	60	56	66
78	40	76	90	30	98	75
8	70	96	88	45	80	99

Divisibility of Numbers. A knowledge of the following facts regarding divisibility of numbers will prove of great assistance in factoring.

All even numbers are divisible by 2.

A number is even if its last digit is 2, 4, 6, 8, or 0.

A number is divisible by 3 if the sum of its digits is divisible by 3.

Thus we can tell at a glance that 471 is divisible by 3, because $4+7+1=12$, which is divisible by 3.

A number is divisible by 4 if its two digits at the right are ciphers or express a number which is divisible by 4.

Thus we can tell at a glance that 1324 is divisible by 4, because 24 is divisible by 4.

A number is divisible by 5 if its right hand figure is 0 or 5.

Thus, 5, 10, 15, 20, etc., are divisible by 5.

All even numbers the sum of whose digits is divisible by 3, are divisible by 6.

Thus, 354 is seen to be divisible by 6 because it is an even number and because $3+5+4=12$, which is divisible by 3.

This is a combination of the two facts of divisibility first stated. If a number is divisible by 2 and is also divisible by 3, it is clear that it is divisible by 6.

A number is divisible by 8 if its three digits at the right are ciphers or express a number which is divisible by 8.

Thus, 14,128 is seen to be divisible by 8, because 128 is divisible by 8.

A number is divisible by 9 if the sum of its digits is divisible by 9.

Thus, 2439 is seen to be divisible by 9, because $2+4+3+9=18$, which is divisible by 9.

All numbers ending in 0 are divisible by 10.

Thus, 1280 is divisible by 10.

A number is divisible by 11 if the sum of the digits in the even places equals the sum of the digits in the odd places, or if the difference between these two sums is 11 or a multiple of 11.

Thus, 72,919 is seen to be divisible by 11 because $9+9+7=25$, $1+2=3$, and $25-3=22$, which is a multiple of 11.

A number is divisible by 25 if its last two digits are ciphers or express a number which is divisible by 25.

Thus, 1925 is divisible by 25.

A number is divisible by 125 if its last three digits are ciphers or express a number which is divisible by 125.

Thus, 12,625 is divisible by 125.

EXAMPLE. Find the prime factors of 2730.

$$\begin{array}{r} 2)2730 \\ 3)1365 \\ 5)455 \\ 7)91 \\ 13 \end{array}$$

Applying the methods for determining divisibility, find some one prime factor, preferably the smallest possible (in this case, 2). Divide by this factor. Find another prime factor and divide in the same way. Continue until no further division can be performed. When the last quotient cannot be factored, but is itself a prime number, it and the other prime numbers which were used as divisors, are the prime factors of the number. In this case the prime factors of 2730 are 2, 3, 5, 7, and 13.

Proof: $2\times3\times5\times7\times13=2730$

WRITTEN. What are the prime factors of the following numbers?

1. 64	7. 128	13. 1270	19. 642	25. 7875
2. 81	8. 154	14. 975	20. 813	26. 6325
3. 96	9. 175	15. 624	21. 8572	27. 4025
4. 72	10. 144	16. 1560	22. 6964	28. 6525
5. 115	11. 1142	17. 1900	23. 3260	29. 8045
6. 114	12. 976	18. 1150	24. 9100	30. 2254

Greatest Common Divisor

A common divisor of two or more numbers is a number which will divide each of them an integral number of times. Thus, 6 is a common divisor of 18 and 24.

To find the greatest common divisor of two or more numbers, first separate each into its prime factors. The product of all common prime factors is the greatest common factor or greatest common divisor.

EXAMPLE. Find the greatest common factor of 192, 144, and 168.

Reasoning Method

2)192	2)144	2)168
2)96	2)72	2)84
2)48	2)36	2)42
2)24	2)18	3)21
2)12	3)9	7
2)6	3	
3		

By inspection of the above it is apparent that 2, 2, 2, and 3 are common to all three of the above numbers. The greatest common divisor of the three numbers, therefore, is $2\times2\times2\times3=24$.

Formula Method

2)192—144—168	Arrange the numbers which are to be factored, as shown, and divide by some number which will divide each of them. Continue doing this until no number can be found which will divide each of the remainders. Multiply together the numbers which have been used as divisors. The product (in this case $2\times2\times2\times3$) is the greatest common divisor.
2)96— 72— 84	
2)48— 36— 42	
3)24— 18— 21	
8— 6— 7	

WRITTEN. Find the greatest common divisor of

1. 96, 72, and 84
2. 27, 45, and 63
3. 42, 56, 72, and 144
4. 721, 88, and 40
5. 84, 144, and 126
6. 140, 168, 91, and 126
7. 48, 36, 90, and 96
8. 120, 192, 136, and 24
9. 320, 192, 144, and 216
10. 51, 85, 136, and 187
11. 76, 152, 57, and 133
12. 92, 115, 46, and 138

Least Common Multiple

A **multiple** of a number is any number that is determined by taking the given number an integral number of times. The multiple will therefore contain the given number without a remainder.

Thus, 15, 25, 50, 75, etc., are multiples of 5, and each will contain 5 without a remainder.

A **common multiple** of two or more numbers is a number that is a multiple of each.

Thus, 72 is a common multiple of 2, 3, 6, 9, etc., and contains each without a remainder.

The **least common multiple** of two or more numbers is the smallest number that is a multiple of each.

Thus, 18 is the least common multiple of 2, 3, 6, and 9.

EXAMPLE. Find the least common multiple of 4, 8, 6, and 15.

Reasoning Method

The prime factors of 4 are 2 and 2.
The prime factors of 8 are 2, 2, and 2.
The prime factors of 6 are 2 and 3.
The prime factors of 15 are 3 and 5.

The least common multiple must contain every factor that is contained in each of the four numbers. Inspection of the above shows that these are a 2, a second 2, a third 2, a 3, and a 5. The least common multiple is therefore $2\times2\times2\times3\times5=120$.

Formula Method

$$\begin{array}{r} 2\overline{)4—8—6—15} \\ 2\overline{)2—4—3—15} \\ 3\overline{)1—2—3—15} \\ 1—2—1—\ 5 \end{array}$$

Write the four numbers as shown and divide by any prime number that is seen to be a factor of any two of the numbers. Dividing by 2, we get 2, 4, and 3 for quotients in the first three instances. In the fourth instance, we find that 2 is not an exact divisor, so we bring down the 15. Continue this process until all the numbers on the bottom line are prime to each other. Then multiply together all divisors and the quotients remaining on the bottom line, thus: $2\times2\times3\times2\times5=120$, least common multiple. It is not necessary to use 1 as a factor, so that has been omitted. It will be seen that the factors of the least common multiple are the same as by the reasoning method first shown.

WRITTEN. Find the least common multiple of the following:

1. 8, 12, 18, 24	**6.** 6, 36, 80, 52	**11.** 62, 39, 15, 142
2. 15, 24, 12, 30	**7.** 18, 20, 16, 48	**12.** 125, 225, 105, 35
3. 16, 18, 9, 24	**8.** 27, 54, 18, 9	**13.** 120, 114, 160, 72
4. 24, 32, 18, 30	**9.** 24, 72, 120, 18	**14.** 42, 84, 126, 252
5. 16, 72, 80, 12	**10.** 15, 50, 75, 30	**15.** 23, 46, 64, 128

Cancellation

Cancellation is the omission of the same factor from both dividend and divisor. Sometimes cancellation may be used to shorten the work of division.

EXAMPLE. Divide $15\times20\times32$ by $12\times24\times6$.

First arrange the numbers so that the factors of the dividend will be above a horizontal line and the factors of the divisor will be below the line.

This is a fraction. The problem is to eliminate equal factors from dividend and divisor, which, as previously noted, will change the fraction to lower terms without changing its value.

(*a*) $\dfrac{\overset{5}{\cancel{15}}\times20\times32}{\underset{4}{\cancel{12}}\times24\times\ 6}$

(*a*) It is evident by inspection that the 15 above the line and the 12 below are each divisible by 3. Cancel the 15 and the 12, writing in place of them the quotients secured by dividing by 3.

(*b*) $\dfrac{\overset{5}{\cancel{15}}\times\overset{5}{\cancel{20}}\times32}{\underset{1}{\cancel{4}}\times24\times\ 6}$ (*c*) $\dfrac{\overset{5}{\cancel{15}}\times\overset{5}{\cancel{20}}\times\overset{4}{\cancel{32}}}{\underset{1}{\cancel{4}}\times\underset{3}{\cancel{24}}\times\ 6}$ (*d*) $\dfrac{\overset{5}{\cancel{15}}\times\overset{5}{\cancel{20}}\times\overset{2}{\cancel{32}}}{\underset{1}{\cancel{4}}\times\underset{3}{\cancel{24}}\times\underset{3}{\cancel{6}}}$

(*b*) Now note that the 20 above the line and the 4 below the line are each divisible by 4. Cancel and divide.

(*c*) Next note that 32 and 24 are divisible by 8, and cancel.

(*d*) Next note that 4 and 6 are divisible by 2, and cancel.

Since no further cancellation can be performed, it is evident that the fraction is in its lowest terms. Multiply the factors of the dividend together, and multiply the factors of the divisor together, thus:

$$\frac{5\times5\times2}{1\times3\times3}=\frac{50}{9}=5\tfrac{5}{9}$$

WRITTEN. Using cancellation, divide the following:

1. $6\times12\times20$ by $4\times8\times5$.
2. $3\times8\times15$ by $4\times5\times3$.
3. $15\times24\times30$ by $8\times10\times12$.
4. $18\times6\times9$ by $15\times36\times2$.
5. $32\times150\times5$ by $75\times4\times9$.
6. $56\times28\times32$ by $20\times42\times128$.
7. $27\times14\times30$ by $9\times15\times7$.
8. $16\times24\times18$ by $36\times6\times4$.
9. $8\times12\times10$ by $6\times15\times24$.
10. $48\times10\times15$ by $24\times20\times2$.

CHAPTER VI

FRACTIONS — Continued

A **fraction** is one or more of the equal parts of a unit.

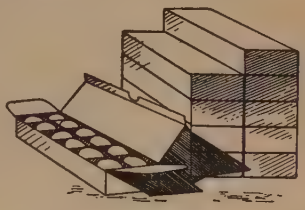
A **fractional unit** is one of the equal parts of a unit, as *one third*, *one fourth*.

Assuming $1 as the unit, 50¢, 25¢, 10¢, 20¢, 80¢, 1¢ are what parts of it?

The term *unit*, as applied in commercial calculations, may mean a group of similar things such as 1 dozen, 1 gross, 1 hundred, 1 thousand, or 1 ton. Prices are often based upon such units. When such units are used in figuring costs, they are called cost units.

Assuming 1 dozen as the unit, what fractional part of 1 dozen would the following numbers be: 1, 3, 4, 6, 8, 9?

Assuming 1 hundred as the unit, what fractional part of 100 would the following numbers be: 1, 10, 20, 25, 50, 75?



Units of 1 Dozen

Fractions may be written in either of two forms, the common fraction form or the decimal fraction form.

The **common fraction form** is that in which the numerator is written above, and the denominator below, a horizontal line, as in $\frac{7}{10}$. The **decimal fraction form** is that in which the digits of the numerator are written at the right of a point, called the decimal point, and the denominator is not written, but is considered to be 1 with as many ciphers annexed as there are places to the right of the decimal point.

Thus in the fraction .7, the numerator is 7 and the denominator is 10 as indicated by the fact that there is only one decimal place.

The name decimal is from the Latin *decem*, meaning ten. The indicated denominator of any decimal is either 10 or some power of 10.

Common fractions and decimal fractions are merely different forms for expressing fractional values.

A common fraction is said to be an indicated division. Thus the fraction $\frac{3}{4}$ is an indicated division of 3 by 4. Were this division performed, the form of the fraction would be changed from $\frac{3}{4}$, the common fraction form, to .75, the decimal fraction form.

Read the following:

1. .8	11. $.6\frac{1}{4}$	21. $16.00\frac{2}{3}$	31. 100.006
2. .18	12. $.16\frac{2}{3}$	22. $9.00\frac{3}{4}$	32. .106
3. .865	13. $.463\frac{1}{3}$	23. .63722	33. 125000.
4. .1645	14. $.0062\frac{1}{2}$	24. $.387\frac{1}{2}$	34. .125
5. .20485	15. .04875	25. $.00\frac{1}{2}$	35. 12000.012
6. .0125	16. $.36\frac{2}{3}$	26. $.0\frac{1}{2}$	36. 510000.
7. .625	17. .362455	27. $.\frac{1}{2}$	37. .510
8. .875	18. *26.475	28. $16.0\frac{3}{8}$	38. 500.010
9. .12	19. 38.754	29. $1.00\frac{2}{3}$	39. $40.040\frac{2}{3}$
10. .6305	20. 2.03	30. $14.000\frac{5}{8}$	40. $3169.005\frac{1}{3}$

Write the following:

1. Two hundred and nine thousandths.
2. Two hundred nine thousandths.
3. Three hundred forty-three thousand.
4. Three hundred forty-three thousandths.
5. Three hundred and forty three thousandths.
6. Fifty-nine thousand and fifty-nine thousandths.
7. Six hundred ten thousand.
8. Six hundred ten thousandths.
9. Six hundred and ten thousandths.
10. Six hundred ten thousand and six hundred ten thousandths.
11. Six hundred ten thousand and six hundred ten-thousandths.
12. Seven hundred forty-three and 43 hundredths.

*In reading, insert the word "and" only between the integer and the fraction. Thus, "one hundred *and* six thousandths" means 100.006, while "one hundred six thousandths" means .106.

Write the following in figures, using a common fraction to represent the remainder in each case:

13. Express one-half thousandth in thousandths.

14. Express one-half hundredth in hundredths.

15. Express one-half tenth in tenths.

16. Express thirty and one-sixth thousandths in thousandths.

17. Express forty and five-eighths ten-thousandths in ten-thousandths.

Changing Fractions to Lower or Higher Terms

The *terms* of a fraction are its numerator and denominator. Since the denominator indicates the size of the fractional unit used, it is clear that to increase the denominator without changing the numerator lessens the value of the fraction.

Thus, to multiply the denominator of the fraction $\frac{3}{4}$ by 2 lessens the value of the fraction one-half. ($\frac{3}{8}=\frac{1}{2}$ of $\frac{3}{4}$.)

Since the numerator indicates the number of fractional units taken, it is clear that to increase the numerator without changing the denominator increases the value of the fraction.

Thus, to multiply the numerator of the fraction $\frac{3}{4}$ by 2 doubles the value of the fraction. ($\frac{6}{4}=2\times\frac{3}{4}$.)

Since multiplying the numerator by 2 doubles the value of the fraction and multiplying the denominator by 2 divides the value of the fraction by 2, it is clear that if the numerator be multiplied by 2 and the denominator be multiplied by 2 at the same time, the value of the fraction is not changed.

The foregoing can be proved true, also, of any number other than 2. It can also be proved by the same process that if numerator and denominator be divided by the same number the value of the fraction is not changed. Therefore we have the rule:

> The numerator and denominator of a fraction may be multiplied by the same number or may be divided by the same number, without changing the value of the fraction.

Thus $\frac{3}{4}=\frac{3\times2}{4\times2}=\frac{6}{8}$ or $\frac{6}{8}=\frac{6\div2}{8\div2}=\frac{3}{4}$

Since a fourth is twice an eighth, 3 fourths exactly equal 6 eighths.

Dividing both terms of a fraction by the same number is equivalent to canceling the same factor from both terms, which method was studied in the last chapter.

Expressing a fraction in lower terms, or in higher terms, without changing its value, is called *reduction*.

To Express a Fraction in its Lowest Terms

EXAMPLE. Express $\frac{1680}{1890}$ in its lowest terms.

$$\frac{1680}{1890}=\frac{1680\div 10}{1890\div 10}=\frac{168}{189}=\frac{168\div 3}{189\div 3}=\frac{56}{63}=\frac{56\div 7}{63\div 7}=\frac{8}{9}$$

Both terms are divided by the same number, and this is repeated until the terms are prime to each other. In the above case this is equivalent to canceling, successively, the factors 10, 3, and 7 from both terms.

ORAL. Express the following fractions in their lowest terms.

1.	$\frac{8}{12}$	$\frac{12}{18}$	$\frac{9}{12}$	$\frac{15}{18}$	$\frac{12}{15}$	$\frac{21}{28}$	$\frac{15}{20}$	$\frac{10}{12}$	$\frac{25}{40}$	$\frac{14}{21}$
2.	$\frac{10}{15}$	$\frac{15}{21}$	$\frac{21}{24}$	$\frac{18}{30}$	$\frac{30}{36}$	$\frac{30}{45}$	$\frac{27}{36}$	$\frac{35}{50}$	$\frac{49}{70}$	$\frac{81}{90}$
3.	$\frac{8}{16}$	$\frac{9}{27}$	$\frac{32}{48}$	$\frac{40}{60}$	$\frac{36}{60}$	$\frac{72}{96}$	$\frac{18}{27}$	$\frac{12}{21}$	$\frac{15}{27}$	$\frac{26}{39}$
4.	$\frac{39}{65}$	$\frac{42}{48}$	$\frac{54}{81}$	$\frac{49}{70}$	$\frac{44}{66}$	$\frac{45}{75}$	$\frac{28}{35}$	$\frac{28}{40}$	$\frac{21}{49}$	$\frac{51}{68}$
5.	$\frac{6}{16}$	$\frac{24}{36}$	$\frac{27}{45}$	$\frac{28}{42}$	$\frac{17}{51}$	$\frac{34}{51}$	$\frac{26}{65}$	$\frac{34}{85}$	$\frac{85}{95}$	$\frac{80}{96}$

WRITTEN. Express the following fractions in their lowest terms:

1. $\frac{345}{375}$	9. $\frac{315}{420}$	17. $\frac{15120}{25920}$	25. $\frac{2312}{3400}$
2. $\frac{735}{840}$	10. $\frac{240}{360}$	18. $\frac{6944}{13020}$	26. $\frac{7128}{9240}$
3. $\frac{2520}{2835}$	11. $\frac{210}{252}$	19. $\frac{6600}{14400}$	27. $\frac{8064}{9600}$
4. $\frac{1320}{1440}$	12. $\frac{396}{468}$	20. $\frac{25494}{54630}$	28. $\frac{9120}{9975}$
5. $\frac{1080}{1152}$	13. $\frac{918}{972}$	21. $\frac{3175}{7620}$	29. $\frac{16275}{17205}$
6. $\frac{2024}{2112}$	14. $\frac{2069}{2156}$	22. $\frac{4779}{8496}$	30. $\frac{6260}{9752}$
7. $\frac{1674}{1728}$	15. $\frac{2940}{3024}$	23. $\frac{3604}{7420}$	31. $\frac{38632}{84288}$
8. $\frac{2870}{2940}$	16. $\frac{16830}{17160}$	24. $\frac{2046}{4464}$	32. $\frac{18524}{45468}$

A decimal, unless it ends in a cipher, is in its lowest terms.

To Express a Fraction in Higher Terms.

A fraction may be expressed in higher terms by multiplying both of its terms by the same number.

EXAMPLE. Express $\frac{3}{5}$ in 60ths.

$60 \div 5 = 12$

$$\frac{3}{5} = \frac{3 \times 12}{5 \times 12} = \frac{36}{60}$$

60 will be the new denominator of the fraction. To obtain 60 as a denominator, the old denominator, 5, must be multiplied by 12. To keep the fraction at the same value, the numerator also must be multiplied by 12. The result will be $\frac{36}{60}$.

A decimal fraction may be expressed in higher terms without changing its value by the same process as that used for common fractions. Multiply both of its terms by the same number. In the case of a decimal fraction, however, the number multiplied by must be some power of 10.

Thus $$.6 = \frac{6}{10} = \frac{6 \times 10}{10 \times 10} = \frac{60}{100} = .60$$

It is not common to express decimal fractions in any but their lowest terms, except in writing U. S. money, in which case tenths are never used.

Thus: .6 is seldom written .60, though \$.6 is always written \$.60.

Express the following in higher terms as indicated:

1. $\frac{2}{3}$ in 15ths.
2. $\frac{3}{4}$ in 24ths.
3. $\frac{3}{8}$ in 16ths.
4. $\frac{4}{7}$ in 28ths.
5. $\frac{5}{6}$ in 30ths.
6. $\frac{4}{9}$ in 45ths.
7. $\frac{7}{12}$ in 48ths.
8. $\frac{5}{16}$ in 64ths.
9. $\frac{7}{9}$ in 72ds.
10. $\frac{3}{4}$ in 100ths.
11. .5 in hundredths.
12. $\frac{3}{20}$ in hundredths.
13. .9 in hundredths.
14. $\frac{4}{5}$ in hundredths.
15. $\frac{3}{7}$ in 63ds.
16. $\frac{3}{4}$ in 16ths.
17. .8 in hundredths.
18. $\frac{3}{5}$ in 100ths.
19. $\frac{7}{16}$ in 48ths.
20. $\frac{11}{12}$ in 60ths.
21. $\frac{9}{11}$ in 44ths.
22. $\frac{16}{17}$ in 68ths.
23. $\frac{17}{18}$ in 72ds.
24. $\frac{13}{24}$ in 96ths.
25. Change to 10ths.: $\frac{1}{2}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$.
26. Change to 100ths.: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{3}{10}$, $\frac{3}{5}$, $\frac{7}{10}$, $\frac{4}{5}$, $\frac{9}{10}$
27. Change to 1000ths.: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{3}{20}$, $\frac{4}{5}$, $\frac{5}{8}$ $\frac{7}{25}$, $\frac{9}{10}$, $\frac{7}{8}$.

To Express an Improper Fraction as an Integer or Mixed Number.

A *proper fraction* is a fraction the value of which is less than 1, *i.e.*, the numerator of which is smaller than the denominator, as $\frac{3}{4}$.

An *improper fraction* is one having a numerator greater than its denominator, and whose value is greater than 1, as $\frac{4}{3}$.

A *mixed number* is an integer and a fraction expressed as one number, as $1\frac{1}{3}$.

An improper fraction can always be expressed as an integer or as a mixed number.

EXAMPLE. Express $\frac{25}{4}$ as a mixed number.

$25 \div 4 = 6\frac{1}{4}$ Since $\frac{4}{4} = 1$, in $\frac{25}{4}$ there will be as many units as the number of times 25 contains 4, which is $6\frac{1}{4}$.

ORAL. Express as integers or mixed numbers.

1.	$\frac{45}{8}$	$\frac{72}{9}$	$\frac{63}{15}$*	$\frac{56}{11}$	$\frac{17}{4}$	$\frac{43}{6}$	$\frac{58}{7}$	$\frac{72}{14}$	$\frac{132}{12}$
2.	$\frac{42}{9}$	$\frac{90}{15}$	$\frac{15}{4}$	$\frac{72}{9}$	$\frac{69}{5}$	$\frac{102}{12}$	$\frac{42}{13}$	$\frac{85}{17}$	$\frac{66}{13}$
3.	$\frac{63}{7}$	$\frac{64}{12}$	$\frac{17}{5}$	$\frac{23}{4}$	$\frac{52}{13}$	$\frac{44}{7}$	$\frac{85}{9}$	$\frac{87}{15}$	$\frac{26}{9}$
4.	$\frac{91}{8}$	$\frac{57}{11}$	$\frac{27}{8}$	$\frac{92}{11}$	$\frac{87}{12}$	$\frac{45}{8}$	$\frac{58}{14}$	$\frac{78}{9}$	$\frac{68}{15}$
5.	$\frac{86}{12}$	$\frac{52}{9}$	$\frac{13}{5}$	$\frac{19}{4}$	$\frac{17}{3}$	$\frac{57}{6}$	$\frac{63}{11}$	$\frac{68}{13}$	$\frac{44}{7}$

WRITTEN. Express as integers or mixed numbers,

1. $\frac{145}{28}$	7. $\frac{180}{15}$	13. $\frac{1254}{24}$	19. $\frac{378}{12}$
2. $\frac{142}{39}$	8. $\frac{494}{114}$	14. $\frac{362}{72}$	20. $\frac{416}{9}$
3. $\frac{282}{47}$	9. $\frac{328}{24}$	15. $\frac{2625}{250}$	21. $\frac{3120}{104}$
4. $\frac{391}{28}$	10. $\frac{1625}{125}$	16. $\frac{3840}{54}$	22. $\frac{3475}{96}$
5. $\frac{862}{124}$	11. $\frac{1980}{45}$	17. $\frac{3760}{126}$	23. $\frac{1446}{12}$
6. $\frac{476}{96}$	12. $\frac{3250}{125}$	18. $\frac{1946}{72}$	24. $\frac{1642}{324}$

*Always express your result in its lowest terms: thus, $4\frac{1}{5}$, not $4\frac{3}{15}$.

To express a mixed number as an improper fraction

EXAMPLE. Express $7\frac{3}{5}$ as an improper fraction.

$7\frac{3}{5}$
5
35
3
$38 = \frac{38}{5}$

In 1 there are 5 fifths. Hence, in 7 there are 7×5 fifths or 35 fifths. $\frac{35}{5} + \frac{3}{5} = \frac{38}{5}$.

Multiply the integer by the denominator of the fraction, and add to the product the numerator of the fraction. The number thus secured is the numerator of the improper fraction. The denominator of the improper fraction is the same as the denominator of the fractional part of the mixed number.

ORAL. Express as improper fractions.

1. $5\frac{2}{3}$ $6\frac{3}{8}$ $7\frac{4}{5}$ $9\frac{3}{4}$ $8\frac{1}{3}$ $7\frac{5}{8}$ $9\frac{7}{12}$ $4\frac{5}{6}$ $12\frac{4}{5}$
2. $6\frac{5}{7}$ $7\frac{2}{3}$ $8\frac{3}{4}$ $5\frac{7}{8}$ $9\frac{3}{8}$ $6\frac{7}{8}$ $8\frac{3}{12}$ $4\frac{7}{8}$ $16\frac{5}{7}$
3. $4\frac{5}{9}$ $9\frac{3}{8}$ $7\frac{2}{9}$ $6\frac{2}{3}$ $9\frac{2}{5}$ $8\frac{5}{6}$ $11\frac{3}{8}$ $7\frac{1}{9}$ $2\frac{5}{16}$
4. $8\frac{1}{12}$ $5\frac{3}{4}$ $5\frac{1}{2}$ $8\frac{4}{5}$ $6\frac{1}{7}$ $5\frac{7}{11}$ $6\frac{2}{13}$ $12\frac{3}{7}$ $4\frac{7}{20}$
5. $7\frac{3}{9}$ $4\frac{6}{7}$ $6\frac{3}{8}$ $9\frac{1}{8}$ $7\frac{3}{4}$ $12\frac{4}{5}$ $7\frac{7}{12}$ $4\frac{5}{14}$ $8\frac{5}{11}$

WRITTEN. Express as improper fractions.

1. $27\frac{3}{7}$
2. $38\frac{4}{9}$
3. $18\frac{5}{7}$
4. $29\frac{9}{11}$
5. $45\frac{7}{12}$
6. $63\frac{7}{8}$
7. $115\frac{5}{7}$
8. $174\frac{8}{9}$
9. $139\frac{3}{16}$
10. $627\frac{3}{5}$
11. $439\frac{3}{8}$
12. $724\frac{4}{5}$
13. $683\frac{7}{8}$
14. $946\frac{4}{9}$
15. $236\frac{5}{8}$
16. $532\frac{1}{5}$
17. $627\frac{8}{9}$
18. $132\frac{3}{5}$
19. $65\frac{2}{3}$
20. $82\frac{6}{7}$
21. $64\frac{8}{9}$
22. $55\frac{4}{5}$
23. $87\frac{5}{7}$
24. $28\frac{9}{11}$
25. $73\frac{11}{12}$
26. $38\frac{5}{8}$
27. $76\frac{3}{4}$
28. $46\frac{12}{17}$
29. $91\frac{3}{5}$
30. $75\frac{5}{6}$

Interchangeability of Common Form and Decimal Form

Fractions are expressed in two forms, the common form and the decimal form.

In the common form any number may be used as a denominator.
In the decimal form, the denominator is always 10 or some power of 10.

Any common fraction has its equivalent decimal form, and any decimal fraction has its equivalent common fraction form.

Thus, $\frac{1}{2}$ may be written .5, or .5 may be written $\frac{1}{2}$.

To change a fraction from a common form to a decimal form,

divide the numerator by the denominator.

EXAMPLE. Change $\frac{7}{8}$ to its decimal form.

$$8)\overline{7.000}\quad .875$$

Divide the numerator, 7, by the denominator, 8, and continue to so divide until there is no remainder, annexing as many ciphers at the right of the decimal point in the numerator as required. Point off as many places in the quotient as there are decimal places in the dividend.

If the division should continue indefinitely, carry the decimal as far as desired and either omit the remainder, write it in the common form, or if it is more than half of the last decimal unit, increase the last decimal unit by 1.

EXAMPLE. Change $\frac{2}{3}$ to a decimal form.

$$3)\overline{2.0000}\quad .6666\tfrac{2}{3}$$

This division would continue indefinitely; therefore the decimal is carried to four places and the remainder is either written in the common form as shown, or omitted (giving .6666 as the result) or counted as 1 whole number in the last decimal order, which would give .6667 as the result. Such fractions are called interminate fractions.

WRITTEN. Change the following common fractions to the decimal form, extending them to three places, if necessary, and writing the remainders in the common form.

1. $\frac{1}{2}$, $\frac{1}{14}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{5}{16}$, $\frac{9}{16}$, $\frac{11}{16}$, $\frac{15}{16}$, $\frac{17}{64}$, $\frac{27}{64}$, $\frac{45}{64}$, $\frac{59}{64}$.
2. $\frac{1}{3}$, $\frac{1}{6}$, $\frac{4}{6}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{11}{12}$, $\frac{5}{12}$, $\frac{11}{24}$, $\frac{17}{24}$, $\frac{19}{24}$, $\frac{5}{24}$, $\frac{1}{24}$.
3. $\frac{1}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{2}{9}$, $\frac{4}{9}$, $\frac{5}{9}$, $\frac{7}{9}$, $\frac{8}{9}$, $\frac{2}{11}$, $\frac{5}{11}$.

EXAMPLE. Change $\frac{13}{25}$ to hundredths.

$$\frac{13}{25}=\frac{13\times 4}{25\times 4}=\frac{52}{100}=.52$$

To obtain 100 for the denominator it was necessary to multiply 25 by 4. Multiply 13 by 4 for the new numerator. The required result is $\frac{52}{100}$, which when written as a decimal is .52.

1. $\frac{3}{4}=\frac{?}{100}$ $\quad \frac{1}{2}=\frac{?}{10}$ $\quad \frac{7}{8}=\frac{?}{1000}$ $\quad \frac{4}{5}=\frac{?}{100}$
2. $\frac{2}{3}=\frac{?}{10}$* $\quad \frac{3}{8}=\frac{?}{100}$ $\quad \frac{5}{6}=\frac{?}{100}$ $\quad \frac{7}{12}=\frac{?}{100}$
3. $\frac{4}{7}=\frac{?}{100}$ $\quad \frac{4}{5}=\frac{?}{1000}$ $\quad \frac{1}{3}=\frac{?}{100}$ $\quad \frac{3}{7}=\frac{?}{100}$
4. $\frac{5}{16}=\frac{?}{100}$ $\quad \frac{7}{16}=\frac{?}{100}$ $\quad \frac{5}{9}=\frac{?}{100}$ $\quad \frac{3}{8}=\frac{?}{10}$

*The multiplier need not be an integer. In this case it is $3\frac{1}{3}$.

WRITTEN. Express the following as decimals, extending each to three places if necessary. If there is a remainder, write it as a common fraction:

1. $\frac{3}{4}$	5. $\frac{2}{3}$	9. $\frac{13}{20}$	13. $\frac{47}{50}$
2. $\frac{7}{8}$	6. $\frac{11}{12}$	10. $\frac{17}{30}$	14. $\frac{9}{16}$
3. $\frac{5}{6}$	7. $\frac{7}{16}$	11. $\frac{5}{12}$	15. $\frac{23}{40}$
4. $\frac{7}{12}$	8. $\frac{18}{25}$	12. $\frac{37}{40}$	16. $\frac{17}{20}$

To change a fraction from a decimal form to a common fraction form.

Write the figures of the decimal for a numerator, and for a denominator write 1, annexing as many ciphers to the right of it as there are decimal places in the decimal form. Express the common fraction in its lowest terms.

EXAMPLE. Change .325 to the common fraction form.

$.325=\frac{325}{1000}$ or $\frac{13}{40}$. 325 is written as the numerator of the common form. The denominator is obtained by writing 1 and annexing to it three ciphers, since there are three places in the decimal form.

EXAMPLE. Change $.83\frac{1}{3}$ to the common fraction form.

$.83\frac{1}{3}=\frac{83\frac{1}{3}}{100}=\frac{250}{300}$

$\frac{250}{300}=\frac{5}{6}$

Following the rule above given, write the decimal fraction as a common fraction. Then multiply both terms of the common fraction by three, to clear of fractions. Then divide both terms by 50, to express the fraction in its lowest terms.

ORAL. Express the following decimals in the common fraction form. Always express the result in its lowest terms.

1. .375	.25	.56	.875	.75	.625
2. .0015	.005	.015	.15	.045	.45
3. .02	.04	.225	.24	.275	.425
4. .055	.065	.075	.725	.705	.35
5. .095	.28	.425	.05	.85	.1125
6. .95	.925	.195	.18	.54	.345

WRITTEN. Express as common fractions in lowest terms:

1. $.44\frac{4}{9}$	7. $.7\frac{1}{7}$	13. $.018\frac{3}{4}$	19. .1375	25. .0325	31. .0128
2. $.083\frac{1}{3}$	8. $.0028\frac{4}{7}$	14. $.283\frac{1}{3}$	20. $.001\frac{1}{3}$	26. $.083\frac{1}{3}$	32. $.133\frac{1}{3}$
3. $.012\frac{1}{2}$	9. .1875	15. .0064	21. $.22\frac{2}{9}$	27. $.03\frac{3}{7}$	33. .325
4. $.11\frac{1}{9}$	10. $.003\frac{1}{3}$	16. $.006\frac{2}{3}$	22. $.166\frac{2}{3}$	28. $.093\frac{3}{4}$	34. $.333\frac{1}{3}$
5. $.06\frac{1}{4}$	11. $.009\frac{3}{8}$	17. .1625	23. .025	29. $.006\frac{2}{3}$	35. $.666\frac{2}{3}$
6. $.16\frac{2}{3}$	12. $.001\frac{1}{4}$	18. $.04\frac{4}{9}$	24. $.229\frac{2}{9}$	30. .08125	36. $.444\frac{4}{9}$

Least Common Denominator

Fractions having the same denominator (which is called a common denominator) can be added without change of their terms. If fractions which are to be added have not a common denominator, they must be expressed in terms of some common denominator before they can be added.

Thus, $\frac{1}{2}$ and $\frac{1}{3}$ cannot be added until they are expressed as $\frac{3}{6}$ and $\frac{2}{6}$, respectively, or expressed with some other common denominator.

The **least common denominator** of several fractions is the least common multiple of their denominators.

Fractions having a common denominator are called *similar* fractions.

EXAMPLE. Change $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{2}$ to similar fractions having the least common denominator.

The l.c.m. of 3, 5, 6, and 2 is 30. $\frac{1}{3}=\frac{10}{30}$; $\frac{1}{5}=\frac{6}{30}$; $\frac{1}{6}=\frac{5}{30}$; $\frac{1}{2}=\frac{15}{30}$.

ORAL

1. $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{4}$.
2. $\frac{1}{6}$, $\frac{1}{9}$, $\frac{1}{3}$.
3. $\frac{3}{4}$, $\frac{2}{5}$, $\frac{3}{7}$.
4. $\frac{3}{4}$, $\frac{2}{3}$, $\frac{5}{6}$.
5. $\frac{3}{8}$, $\frac{2}{3}$, $\frac{1}{2}$.
6. $\frac{3}{5}$, $\frac{3}{4}$, $\frac{5}{6}$.
7. $\frac{2}{3}$, $\frac{7}{8}$, $\frac{5}{12}$.
8. $\frac{3}{8}$, $\frac{4}{7}$, $\frac{3}{14}$.

WRITTEN

1. $\frac{7}{12}$, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{5}{6}$.
2. $\frac{7}{16}$, $\frac{5}{12}$, $\frac{2}{3}$, $\frac{11}{24}$.
3. $\frac{3}{5}$, $\frac{4}{7}$, $\frac{1}{2}$, $\frac{7}{10}$.
4. $\frac{6}{7}$, $\frac{9}{14}$, $\frac{3}{4}$, $\frac{1}{2}$.
5. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{12}$.
6. $\frac{1}{8}$, $\frac{6}{15}$, $\frac{2}{3}$, $\frac{1}{6}$.
7. $\frac{17}{18}$, $\frac{23}{24}$, $\frac{15}{16}$, $\frac{2}{3}$.
8. $\frac{9}{14}$, $\frac{3}{7}$, $\frac{21}{28}$, $\frac{1}{2}$.

Addition of Fractions

Since the denominator indicates the size of the fractional unit, and the numerator indicates the number of such units, it is clear that if two fractions have the same denominator they are added by adding the numerators.

To add common fractions express each in terms of the least common denominator of the fractions. Add the numerators, and write the total over the common denominator.

EXAMPLE. Add $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{1}{2}$.

The l.c.m. of 3, 4, and 2 is 12.

$\frac{1}{3}=\frac{4}{12}$; $\frac{3}{4}=\frac{9}{12}$; $\frac{1}{2}=\frac{6}{12}$

$$\frac{4}{12}+\frac{9}{12}+\frac{6}{12}=\frac{4+9+6}{12}=\frac{19}{12}=1\frac{7}{12}$$

Common Form

$$\begin{array}{r} 12ths \\ \frac{1}{3} - 4 \\ \frac{3}{4} - 9 \\ \frac{1}{2} - 6 \\ \hline 19 \end{array}$$

$\frac{19}{12}=1\frac{7}{12}$, *Ans.*

Note. Always express the result, if more than 1, as a mixed number.

ORAL.

1. $\frac{2}{3}+\frac{3}{4}$
2. $\frac{1}{2}+\frac{4}{7}$
3. $\frac{3}{4}+\frac{2}{3}$
4. $\frac{1}{8}+\frac{2}{5}$
5. $\frac{1}{3}+\frac{1}{6}$
6. $\frac{3}{14}+\frac{2}{5}$
7. $\frac{2}{3}+\frac{3}{8}$
8. $\frac{1}{2}+\frac{1}{3}$
9. $\frac{1}{2}+\frac{3}{5}$
10. $\frac{2}{3}+\frac{7}{8}$
11. $\frac{3}{5}+\frac{3}{4}$
12. $\frac{4}{7}+\frac{3}{8}$
13. $\frac{3}{14}+\frac{1}{2}$
14. $\frac{2}{5}+\frac{3}{8}$
15. $\frac{1}{3}+\frac{1}{4}$
16. $\frac{1}{4}+\frac{2}{3}$
17. $\frac{5}{8}+\frac{1}{2}$
18. $\frac{2}{9}+\frac{5}{6}$
19. $\frac{5}{6}+\frac{7}{12}$
20. $\frac{9}{14}+\frac{2}{7}$
21. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$
22. $\frac{2}{3}+\frac{5}{6}+\frac{1}{12}$
23. $\frac{4}{7}+\frac{3}{14}+\frac{1}{7}$
24. $\frac{2}{3}+\frac{4}{9}+\frac{1}{6}$

WRITTEN

1. $\frac{1}{3}+\frac{1}{4}+\frac{5}{12}$
2. $\frac{3}{5}+\frac{8}{15}+\frac{2}{3}$
3. $\frac{3}{5}+\frac{1}{2}+.7$†
4. $\frac{3}{8}+\frac{4}{7}+\frac{5}{14}$
5. $\frac{4}{7}+\frac{3}{5}+.875$
6. $\frac{4}{9}+\frac{3}{4}+\frac{5}{6}$
7. $\frac{2}{5}+\frac{5}{6}+.9$
8. $\frac{6}{7}+\frac{3}{8}+\frac{9}{14}$
9. $\frac{3}{4}+\frac{7}{12}+\frac{11}{14}$
10. $\frac{4}{9}+\frac{1}{4}+\frac{2}{3}$
11. $\frac{3}{4}+\frac{5}{7}+\frac{11}{14}$
12. $\frac{3}{4}+\frac{5}{9}+\frac{7}{11}$
13. $\frac{1}{13}+\frac{3}{11}+\frac{9}{22}+\frac{5}{6}$
14. $\frac{4}{5}+\frac{7}{8}+\frac{1}{4}+.375$
15. $\frac{7}{8}+\frac{2}{3}+\frac{1}{2}+.6$
16. $\frac{7}{12}+\frac{2}{3}+\frac{3}{4}+\frac{3}{8}$
17. $\frac{2}{3}+\frac{4}{9}+\frac{11}{15}+.25$
18. $\frac{1}{2}+\frac{3}{4}+.33\frac{1}{3}$

† In this exercise change decimals to common fractions before adding.

19. $\frac{1}{5}+\frac{7}{12}+\frac{7}{8}+.625$
20. $\frac{5}{12}+.125+\frac{1}{3}+.6$
21. $\frac{4}{9}+.66\frac{2}{3}+.42\frac{6}{7}$
22. $\frac{1}{9}+.16\frac{2}{3}+\frac{1}{2}+\frac{1}{3}$
23. $.06\frac{2}{3}+\frac{4}{5}+\frac{4}{9}+\frac{1}{3}$
24. $\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}$
25. $\frac{5}{6}+.14\frac{2}{7}+.3\frac{1}{3}$
26. $\frac{7}{8}+\frac{11}{24}+\frac{1}{3}$
27. $\frac{3}{5}+\frac{7}{8}+\frac{3}{4}$
28. $\frac{4}{6}+\frac{5}{9}+.33\frac{1}{3}$
29. $\frac{7}{8}+\frac{2}{3}+.25$
30. $\frac{4}{5}+\frac{1}{2}+.875$
31. $\frac{5}{7}+\frac{1}{2}+\frac{11}{14}$
32. $\frac{6}{11}+\frac{1}{2}+\frac{1}{3}$
33. $\frac{3}{8}+\frac{3}{9}+\frac{11}{24}$
34. $\frac{5}{6}+\frac{2}{3}+.83\frac{1}{3}$
35. $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}+.08\frac{1}{3}$
36. $\frac{11}{24}+\frac{13}{48}+\frac{29}{72}+.41\frac{2}{3}$

To Add Fractional Units.

$\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$

When the fractions added are fractional units, it is most convenient to use the product of their denominators for their common denominator.

When two fractions are thus changed to similar fractions, the new numerator of each fraction is always the same figure as the old denominator of the other fraction. Thus $\frac{1}{2}=\frac{3}{6}$, and $\frac{1}{3}=\frac{2}{6}$.

Hence the rule: To add two fractional units, multiply together the old denominators to obtain the new denominator, and add the old denominators to obtain the new numerator.

ORAL

1. $\frac{1}{3}+\frac{1}{4}$
2. $\frac{1}{6}+\frac{1}{5}$
3. $\frac{1}{4}+\frac{1}{5}$
4. $\frac{1}{6}+\frac{1}{7}$
5. $\frac{1}{8}+\frac{1}{3}$
6. $\frac{1}{5}+\frac{1}{7}$
7. $\frac{1}{3}+\frac{1}{7}$
8. $\frac{1}{4}+\frac{1}{7}$
9. $\frac{1}{8}+\frac{1}{5}$
10. $\frac{1}{2}+\frac{1}{5}$
11. $\frac{1}{3}+\frac{1}{10}$
12. $\frac{1}{8}+\frac{1}{9}$
13. $\frac{1}{9}+\frac{1}{5}$
14. $\frac{1}{4}+\frac{1}{9}$
15. $\frac{1}{7}+\frac{1}{9}$
16. $\frac{1}{2}+\frac{1}{9}$
17. $\frac{1}{10}+\frac{1}{7}$
18. $\frac{1}{10}+\frac{1}{9}$

ORAL. In solving each of these problems, first add the two fractional units. Then add the third fraction to the result secured.

1. $\frac{1}{3}+\frac{1}{4}+\frac{5}{6}$
2. $\frac{1}{6}+\frac{1}{5}+\frac{2}{3}$
3. $\frac{1}{4}+\frac{1}{5}+\frac{7}{10}$
4. $\frac{1}{6}+\frac{1}{7}+\frac{5}{14}$
5. $\frac{1}{8}+\frac{1}{3}+\frac{5}{12}$
6. $\frac{1}{5}+\frac{1}{7}+\frac{1}{2}$
7. $\frac{1}{3}+\frac{1}{7}+\frac{3}{4}$
8. $\frac{1}{4}+\frac{1}{7}+\frac{9}{14}$
9. $\frac{1}{8}+\frac{1}{5}+\frac{3}{4}$
10. $\frac{1}{2}+\frac{1}{5}+.875$
11. $\frac{1}{3}+\frac{1}{10}+.625$
12. $\frac{1}{8}+\frac{1}{9}+.33\frac{1}{3}$
13. $\frac{1}{9}+\frac{1}{5}+.44\frac{4}{9}$
14. $\frac{1}{4}+\frac{1}{9}+.08\frac{1}{3}$
15. $\frac{1}{7}+\frac{1}{9}+.22\frac{2}{9}$
16. $\frac{1}{2}+\frac{1}{9}+.83\frac{1}{3}$
17. $\frac{1}{10}+\frac{1}{7}+.6$
18. $\frac{1}{10}+\frac{1}{9}+.66\frac{2}{3}$

To Add Decimals

EXAMPLE. Add .05, .8, .015, .3675, .004.

.05
.8
.015
.3675
.004
1.2365

In the addition of decimals, the decimal points should be kept under each other, and each decimal order under the corresponding order above. That is, tenths under tenths, hundredths under hundredths, etc.

By arranging as suggested above, the sum will be given in its common denominator automatically. Why is this so?

WRITTEN

1. Add 123.786, 24.5203, 1035.001, 5.20079, 1436.5, 1002.2001, 34.6901, and 35.0078.

2. Add 153.472, 15.3472, 1534.72, 1500.3472, 1500.3472, 200.002, 521.052, and 634.0207.

3. Add two hundred four and two hundred four thousandths, two hundred and four thousandths, two hundred four thousandths, six hundred and fifty four ten-thousandths, one hundred fifty and one hundred forty-one thousandths, three hundred fifty three and four hundred sixty three ten-thousandths.

4. Add three thousand one hundred forty-three and two hundred sixty-seven thousandths, four hundred sixty-five hundred-thousandths, five thousand one and one thousand five ten thousandths, four hundred sixty five thousand and eighty-nine thousandths, one hundred one and one hundredth, three thousand four hundred and sixty five ten-thousandths.

To Add Mixed Numbers

EXAMPLE. Add $12\frac{3}{4}$, $8\frac{5}{6}$, $7\frac{2}{3}$.

$12\frac{3}{4}$
$8\frac{5}{6}$
$7\frac{2}{3}$
$29\frac{1}{4}$

Arrange the numbers under each other as in the illustration. First add the fractions, then the integers. By inspection it is found that 12 is the least common denominator of the fractions, and that $9+10+8=27$, the total of the numerators. $\frac{27}{12}=2\frac{1}{4}$. Write the $\frac{1}{4}$ as the fraction in the total. Then add the 2 with the other integers and the total is 29. Therefore $29\frac{1}{4}$ is the required result. If it is impossible to add the fractions mentally write this part of the solution at the right, using the form of solution shown on page 85.

WRITTEN

1.	2.	3.	4.	5.	6.
$538\frac{1}{2}$	$364\frac{1}{2}$	$237\frac{5}{6}$	$473\frac{1}{7}$	$246\frac{1}{10}$	$489\frac{1}{6}$
$624\frac{1}{3}$	$715\frac{3}{4}$	$469\frac{1}{3}$	$608\frac{3}{4}$	$371\frac{7}{30}$	$605\frac{3}{4}$
$271\frac{5}{6}$	$589\frac{5}{16}$	$521\frac{1}{6}$	$391\frac{1}{4}$	$410\frac{2}{15}$	$951\frac{7}{9}$
$309\frac{3}{4}$	$202\frac{1}{8}$	$183\frac{1}{9}$	$295\frac{3}{8}$	$706\frac{3}{5}$	$878\frac{2}{3}$

When there are several addends, some being expressed in the common fraction form and some in decimal form, it is necessary to express all in the same form before adding.

EXAMPLE. Add $16\frac{2}{3}$, $.123\frac{1}{3}$, $9\frac{3}{5}$, 6.24.

$$\begin{array}{r} 16.666\frac{2}{3} \\ .123\frac{1}{3} \\ 9.6 \\ 6.24 \\ \hline 32.630 \end{array}$$

In this case it is more convenient to change the common fractions to decimals. Change each common fraction to its decimal form, carrying any common fractions that will not reduce to pure decimals to as many places as the longest decimal has, with the remainder annexed as a common fraction. Thus the $\frac{2}{3}$ following 16 is changed to $.666\frac{2}{3}$, because the next number, having three decimal places, is the longest decimal. Write the numbers so the decimal points will be under each other and add. The result is 32.630.

WRITTEN. Express all fractions in the decimal form before adding. Carry to four places.

1. $42.274+37\frac{2}{3}+4.7+587.01\frac{3}{4}$
2. $609.007+45.072+52\frac{7}{8}+700.16\frac{2}{3}$
3. $1740.105+47.06\frac{1}{4}+73\frac{4}{9}+84\frac{1}{3}$
4. $188.89\frac{1}{9}+52\frac{3}{5}+75\frac{5}{8}+123\frac{4}{5}$
5. $92.49\frac{3}{8}+50.50+64\frac{7}{9}+29\frac{1}{7}$
6. $84\frac{7}{8}+503.29\frac{1}{4}+6\frac{1}{4}+16\frac{2}{3}$
7. $33\frac{1}{3}+16\frac{3}{4}+.4275$
8. $84.129+19\frac{1}{8}+192\frac{3}{7}$
9. $945.62+23\frac{1}{3}+47\frac{3}{11}$
10. $828.54+37\frac{1}{6}+19\frac{3}{7}$
11. $25.46\frac{1}{4}+48\frac{5}{6}+38\frac{2}{3}$
12. $61.07\frac{3}{4}+57\frac{2}{3}+209.09$

13. Smith borrowed \$948.50. In 1 yr. 1 mo. 12 days he paid it back, plus \$56.91 for the use of it for 1 yr., \$4.7425 for 1 month's use, and \$1.897 for 12 days' use. What did he pay in all?

14. If the use of \$840. costs \$42.30 for 1 yr., \$21.15 for 6 mo., \$7.05 for 2 mo., \$3.525 for 1 mo., and \$1.175 for 10 days, what is the cost for the full time?

WRITTEN. Add the following. In each problem express all fractions in the common form or in the decimal form, as may be most convenient.

1. $6\frac{1}{2}$, $5\frac{1}{3}$, $4\frac{1}{4}$.
2. $8\frac{2}{3}$, $9\frac{1}{4}$, $7\frac{1}{5}$.
3. $5\frac{1}{6}$, $7\frac{3}{8}$, $8\frac{1}{4}$.
4. $9\frac{3}{5}$, $3\frac{1}{3}$, $7\frac{1}{4}$.
5. $6\frac{1}{3}$, $8\frac{3}{4}$, $2\frac{1}{2}$.
6. $9\frac{1}{8}$, $8\frac{4}{5}$, $6\frac{3}{4}$.
7. $12\frac{4}{9}$, $18\frac{1}{3}$, $14\frac{5}{6}$.
8. $16\frac{3}{5}$, $19\frac{1}{3}$, $22\frac{1}{2}$.
9. $18\frac{3}{8}$, $27\frac{1}{2}$, $16\frac{2}{3}$.
*10. 44^3, 45^1, 46^2.
11. 47^1, 46^1, 44^3.
12. $27\frac{3}{4}$, $41\frac{2}{3}$, $28\frac{5}{6}$.
13. 16, $3\frac{1}{3}$, $46\frac{3}{4}$, $29\frac{3}{8}$.
14. .627, $84\frac{5}{9}$, $68\frac{7}{8}$.
15. $172\frac{3}{5}$, $197\frac{3}{8}$, 216.625.
16. $196\frac{4}{9}$, $126\frac{2}{3}$, $178.08\frac{1}{3}$.
17. $176\frac{1}{2}$, $147\frac{1}{3}$, $29.83\frac{1}{3}$.
18. $316\frac{7}{8}$, 41, $127.66\frac{2}{3}$.
19. $176\frac{2}{3}$, $137\frac{4}{9}$, $68.16\frac{2}{3}$.
20. $46\frac{7}{12}$, $176\frac{3}{4}$, 79.375.
21. $178\frac{3}{5}$, 196.8, 112.25.
22. $61\frac{8}{9}$, $147\frac{1}{4}$, $183.08\frac{1}{3}$.
23. $43\frac{4}{5}$, $67\frac{1}{3}$, $96.06\frac{2}{3}$.
24. $38\frac{3}{4}$, $71\frac{4}{9}$, $37.22\frac{3}{9}$.

*In expressing yards, the number of 4ths is often indicated by a small raised figure following the integer.

Oral Review Problems

1. Count to 100 by $2\frac{1}{2}$; by $3\frac{1}{3}$; by $6\frac{1}{4}$; by $6\frac{2}{3}$; by $8\frac{1}{3}$; by $12\frac{1}{2}$; by $16\frac{2}{3}$.

2. Wm. Brown and Henry James own 640 acres of land in one tract. Mr. Brown owns 400 acres and Mr. James owns the remainder. What fractional part of the land does each own?

3. A certain job of work required 48 days. Frank Jones worked 18 days on it and Harry Black 30 days. What fractional part of the time did each work?

4. In hauling a train 450 miles three engines are used. The first one hauls it 150 miles; the second 180 miles; and the third, the remainder of the distance. What fractional part of the distance does each one haul it?

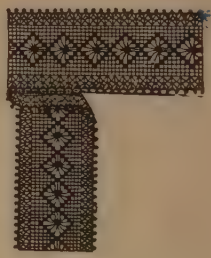
5. A, B and C are partners. Their total investment was $15,000. A invested $6,000, B invested $5,000, and C the remainder. What fractional part did each invest?

6. X owes $2400 and has $900. What fractional part of his debts can he pay? What fractional part of each dollar can he pay?

7. Ten men were to receive $180. It was reported that one of them had died, and the money was divided among nine. Later, the man reported as dead came to claim his share. What part of the amount received by him should each of the nine pay?

Written Review Problems

1. A picture frame is $14\frac{3}{4}$ in. by $11\frac{1}{4}$ in. How many inches of molding are used in it?



2. If twice the width of the lace used must be allowed for turning a square corner, how much lace will be needed to finish a handkerchief 10 in. square with $\frac{5}{8}$-inch lace?

3. A dresser cover 18 inches by 45 inches is to be finished with $1\frac{3}{4}$ inch lace on the two ends and front. A pin cushion, 4 in. x 15 in. is also finished with the lace. How many inches of lace will be required?

4. The tax rate for the state of Ohio in 1914 for state purposes was as follows: Sinking Fund, 2.5 mills; University Fund, .0925 mill; State Common School Fund, .055 mill; State Highway Improvement Fund, .3 mill. What is the total state levy?

5. Add three hundred twenty seven and twenty seven thousandths, one thousand four hundred six and forty-seven ten-thousandths, five hundred seven and five hundred seven thousandths, six hundred thirty four and seven eighths, eight thousand and eight thousandths.

6. Add 439.05, 526.89$\frac{1}{4}$, 62$\frac{5}{8}$, 333$\frac{1}{3}$, 729.06$\frac{1}{4}$, 3003.003, 124$\frac{1}{16}$.

7. Add 84$\frac{3}{7}$, 579$\frac{2}{3}$, 62$\frac{5}{9}$, 123$\frac{1}{21}$, 347$\frac{23}{27}$.

8.	9.	10.	11.
$347\frac{1}{3}$	$485\frac{2}{3}$	$541\frac{1}{7}$	$152\frac{2}{9}$
$568\frac{2}{9}$	$569\frac{2}{9}$	$716\frac{3}{28}$	$317\frac{3}{4}$
$912\frac{3}{4}$	$213\frac{11}{27}$	$250\frac{1}{2}$	$349\frac{1}{3}$
$675\frac{2}{3}$	$624\frac{1}{3}$	$428\frac{3}{14}$	$802\frac{4}{27}$
$984\frac{1}{2}$	$705\frac{13}{18}$	$309\frac{4}{7}$	$564\frac{4}{9}$

Written Review Test

(Time, 50 minutes.)

1. Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{3}{16}$, and $\frac{5}{24}$.
2. Add $\frac{2}{3}$, $\frac{5}{6}$, $\frac{4}{9}$, $\frac{1}{2}$, and $\frac{11}{18}$.
3. Add $2\frac{3}{4}$, $5\frac{7}{8}$, $3\frac{1}{2}$, $52\frac{4}{7}$, $13\frac{11}{28}$, and $7\frac{3}{7}$.
4. $342\frac{3}{4}+458\frac{5}{6}+5\frac{2}{9}+1003\frac{13}{18}+5201.875=?$
5. $6104.3005+5293\frac{7}{8}+4384.702+5326\frac{7}{16}+25.423+1001\frac{15}{16}=?$

6.	7.	8.	9.
2001.354	$3243.78\frac{3}{4}$	743.962	896.425
350.6721	890.5	$84.534\frac{1}{5}$	107.358
8403.72	63.0225	$6350.07\frac{1}{8}$	$25.3\frac{7}{8}$
45.0625	7.5067	500.005	$1573\frac{3}{16}$
372.015	849.03	32.5043	$58.12\frac{1}{4}$
580.506	50.0001	854.1	743.003

10.	11.	12.	13.
$627\frac{1}{6}$	$245\frac{3}{4}$	$233\frac{7}{9}$	$546\frac{1}{4}$
$153\frac{1}{4}$	$129\frac{9}{16}$	$429\frac{1}{2}$	$738\frac{3}{5}$
$846\frac{3}{4}$	$834\frac{1}{2}$	$751\frac{1}{9}$	$628\frac{7}{10}$
$519\frac{1}{2}$	$525\frac{5}{8}$	$634\frac{1}{3}$	$571\frac{1}{2}$
$707\frac{2}{3}$	$606\frac{5}{16}$	$502\frac{5}{6}$	$903\frac{2}{5}$
$694\frac{3}{4}$	$715\frac{7}{8}$	$146\frac{8}{9}$	$819\frac{3}{4}$
$181\frac{2}{3}$	$843\frac{7}{16}$	$780\frac{2}{3}$	$246\frac{3}{10}$

14. Add each of the following. Then add the four results for a grand total.

(a)	(b)	(c)	(d)
$428\frac{1}{2}$	$537\frac{5}{16}$	$463\frac{1}{9}$	$517\frac{1}{3}$
$618\frac{1}{3}$	$429\frac{1}{8}$	$292\frac{1}{3}$	$129\frac{1}{4}$
$537\frac{3}{4}$	$601\frac{3}{8}$	$318\frac{4}{9}$	$302\frac{1}{2}$
$829\frac{2}{3}$	$382\frac{5}{8}$	$174\frac{1}{3}$	$430\frac{3}{4}$
$362\frac{1}{4}$	$754\frac{5}{16}$	$506\frac{4}{9}$	$846\frac{1}{3}$

Subtraction of Fractions

ORAL. Perform the following subtractions:

1. $\frac{3}{4}-\frac{1}{4}$
2. $\frac{4}{5}-\frac{2}{5}$
3. $\frac{6}{7}-\frac{3}{7}$
4. $\frac{8}{9}-\frac{2}{9}$
5. $\frac{7}{8}-\frac{3}{8}$
6. $\frac{9}{16}-\frac{7}{16}$
7. $\frac{11}{24}-\frac{5}{24}$
8. $\frac{13}{28}-\frac{10}{28}$
9. $\frac{14}{35}-\frac{2}{35}$
10. $\frac{16}{24}-\frac{11}{24}$
11. $.47-.25$
12. $.378-.142$
13. $.346-\frac{79}{1000}$
14. $.9-\frac{3}{10}$
15. $.523-\frac{3}{8}$
16. $.478-\frac{61}{1000}$
17. $\frac{3163}{10000}-.1432$
18. $.615-.029$
19. $.392-.007$
20. $\frac{76}{1000}-.014$
21. $\frac{13}{16}-\frac{17}{24}$

Similar fractions are subtracted by subtracting the numerator of the smaller one from the numerator of the larger. If the two fractions are not similar, they must be made so.

To subtract one proper fraction from another.

EXAMPLE. From $\frac{7}{8}$ take $\frac{1}{5}$.

	40*ths*
$\frac{7}{8}$	35
$\frac{1}{5}$	8
	$\frac{27}{40}$

The l.c.m. of 8 and 5 is 40. $\frac{7}{8}=\frac{35}{40}$. $\frac{1}{5}=\frac{8}{40}$. $\frac{35}{40}-\frac{8}{40}=\frac{27}{40}$. Expressed as a rule, this is: Express minuend and subtrahend as similar fractions. Write the difference between the two numerators as the numerator of the result. Write the common denominator as the denominator of the result.

ORAL

1. $\frac{3}{4}-\frac{1}{2}$
2. $\frac{5}{8}-\frac{1}{5}$
3. $\frac{2}{3}-\frac{1}{4}$
4. $\frac{3}{4}-\frac{2}{3}$
5. $\frac{5}{6}-\frac{3}{4}$
6. $\frac{7}{8}-\frac{5}{6}$
7. $\frac{7}{8}-\frac{1}{4}$
8. $\frac{3}{8}-\frac{1}{5}$
9. $\frac{3}{5}-\frac{1}{4}$
10. $\frac{7}{12}-\frac{1}{3}$
11. $\frac{11}{12}-\frac{2}{3}$
12. $\frac{3}{4}-\frac{1}{5}$
13. $\frac{3}{8}-\frac{2}{9}$
14. $\frac{2}{3}-\frac{9}{12}$
15. $\frac{3}{5}-\frac{4}{15}$
16. $\frac{3}{4}-\frac{1}{8}$
17. $\frac{7}{9}-\frac{2}{3}$
18. $\frac{11}{12}-\frac{2}{5}$
19. $\frac{5}{8}-\frac{1}{4}$
20. $\frac{6}{7}-\frac{1}{5}$
21. $\frac{3}{4}-\frac{5}{12}$
22. $\frac{7}{8}-\frac{2}{5}$
23. $\frac{9}{10}-\frac{3}{4}$
24. $\frac{7}{10}-\frac{1}{3}$

WRITTEN

1. $\frac{7}{8}-\frac{2}{3}$
2. $\frac{11}{16}-\frac{1}{3}$
3. $\frac{13}{20}-\frac{2}{5}$
4. $\frac{5}{32}-\frac{1}{8}$
5. $\frac{3}{4}-\frac{13}{25}$
6. $\frac{4}{9}-\frac{3}{19}$
7. $\frac{5}{8}-\frac{3}{25}$
8. $\frac{9}{13}-\frac{4}{7}$
9. $\frac{8}{9}-\frac{5}{12}$
10. $\frac{7}{12}-\frac{5}{13}$
11. $\frac{11}{21}-\frac{7}{24}$
12. $\frac{13}{16}-\frac{11}{24}$
13. $\frac{15}{27}-\frac{17}{36}$
14. $\frac{27}{48}-\frac{13}{32}$
15. $\frac{35}{72}-\frac{19}{48}$
16. $\frac{43}{80}-\frac{23}{48}$
17. $\frac{13}{24}-\frac{13}{32}$
18. $\frac{17}{25}-\frac{7}{15}$
19. $\frac{11}{21}-\frac{13}{28}$
20. $\frac{17}{18}-\frac{25}{27}$

Subtraction of Decimal Fractions

EXAMPLE. From .864 subtract .5206.

.8640
.5206

Write tenths below tenths, hundredths below hundredths, etc. If there are fewer places in minuend than in the subtrahend add a cipher, or ciphers. Subtract as in ordinary subtraction. Place a decimal point in the answer just beneath the decimal points in the minuend and subtrahend.

WRITTEN

1. Subtract .752 from .9348.
2. From .0629 subtract .05.
3. From .8 subtract .7826.
4. From ninety-eight hundredths subtract .0098.
5. Find the difference between .946 and seventy-five hundredths.
6. Find the difference between ninety-six thousandths and .1.
7. Find the difference between .0016 and .016.
8. Find the difference between one hundredth and one ten-thousandth.

When common fractions and decimal fractions are mixed.

EXAMPLE. From .775 take $\frac{3}{5}$.

.775
.6
.175

First, change $\frac{3}{5}$ to a decimal ($\frac{3}{5} = .6$). Then write .6 under .775, with the decimal point directly under the one above and each order under its corresponding order. Subtract as in integers. The result is .175.

Sometimes it is shorter to change the decimal fraction to a common fraction, and sometimes it is shorter to change the common fraction to a decimal fraction. Each problem should be examined to determine which is the better plan of procedure.

WRITTEN. Find the value of:

1. $\frac{5}{6} - \frac{1}{2}$
2. $\frac{7}{8} - \frac{3}{4}$
3. $\frac{7}{12} - \frac{1}{3}$
4. $\frac{9}{16} - \frac{1}{4}$
5. $\frac{11}{18} - \frac{1}{9}$
6. $\frac{8}{15} - \frac{1}{5}$
7. $\frac{11}{24} - \frac{1}{6}$
8. $\frac{17}{32} - \frac{1}{8}$
9. $.642 - \frac{3}{8}$
10. $\frac{17}{64} - .125$
11. $.66\frac{2}{3} - \frac{1}{4}$
12. $.762 - \frac{3}{4}$
13. $.978 - \frac{7}{8}$
14. $.563 - \frac{2}{5}$
15. $\frac{11}{12} - .138$
16. $.793 - \frac{1}{8}$
17. $\frac{3}{8} - .016$
18. $.298 - \frac{1}{4}$
19. $.673 - \frac{1}{2}$
20. $\frac{17}{24} - \frac{2}{3}$
21. $\frac{13}{48} - \frac{1}{12}$
22. $\frac{13}{16} - \frac{5}{8}$
23. $.475 - \frac{2}{5}$
24. $.835 - \frac{3}{8}$

To subtract one mixed number from another.

EXAMPLE. What is the difference between $18\frac{1}{3}$ and $9\frac{1}{2}$?

WRITTEN

$$\begin{array}{r} 18\frac{1}{3} \\ 9\frac{1}{2} \\ \hline 8\frac{5}{6} \end{array}$$

MENTAL

$18\frac{1}{3} = 17\frac{4}{3}$ $\quad 17\frac{4}{3} - 9\frac{1}{2} = 8\frac{5}{6}$

It is apparent that $\frac{1}{2}$ cannot be subtracted from $\frac{1}{3}$. $18\frac{1}{3} = 17\frac{4}{3}$. The problem then becomes: $\frac{1}{2}$ from $\frac{4}{3}$ leaves $\frac{5}{6}$, and 9 from 17 leaves 8.

EXAMPLE. What is the difference between 16.138 and $11\frac{3}{4}$?

$$\begin{array}{r} 16.138 \\ 11.75 \\ \hline 4.388 \end{array}$$

Change the common fraction to a decimal and subtract, or change the decimal to a common fraction and subtract. In writing mixed decimals for subtraction be sure to get the decimal point in the subtrahend beneath that in the minuend, and in the difference place the decimal point in the same column.

WRITTEN. Solve the following problems:

1. $22\frac{1}{2} - 17\frac{5}{8}$
2. $39\frac{2}{3} - 22\frac{5}{6}$
3. $41\frac{1}{3} - 27\frac{1}{2}$
4. $54\frac{3}{8} - 29\frac{3}{5}$
5. $37\frac{4}{9} - 28\frac{2}{3}$
6. $46\frac{1}{3} - 28\frac{5}{9}$
7. $27\frac{1}{4} - 18\frac{3}{5}$
8. $64.8 - 47\frac{1}{2}$
9. $54.3 - 37.8$
10. $68\frac{3}{4} - 43.7$
11. $45.392 - 28\frac{3}{8}$
12. $28\frac{4}{5} - 19.75$
13. $73.46 - 48.875$
14. $85.3 - 27\frac{4}{9}$
15. $139\frac{7}{8} - 97.6$
16. $274\frac{5}{12} - 137.25$
17. $197.68 - 85\frac{3}{4}$
18. $26.839 - 19\frac{3}{5}$
19. $386.7 - 170\frac{4}{5}$
20. $260.18 - 138\frac{1}{5}$
21. $176\frac{7}{8} - 93.354$

WRITTEN. Find the value of:

1. $\frac{1}{2} + \frac{1}{3} - \frac{2}{3}$
2. $\frac{1}{4} + \frac{1}{3} - \frac{5}{12}$
3. $\frac{1}{6} + \frac{1}{3} - \frac{1}{2}$
4. $\frac{1}{8} + \frac{1}{9} - \frac{5}{24}$
5. $\frac{1}{4} + \frac{1}{7} - \frac{3}{14}$
6. $\frac{1}{3} + \frac{1}{8} - \frac{5}{24}$
7. $\frac{1}{7} + \frac{1}{6} - \frac{3}{14}$
8. $\frac{1}{5} + \frac{1}{9} - \frac{2}{9}$
9. $\frac{1}{4} + \frac{1}{7} - .025$
10. $\frac{1}{3} + \frac{1}{5} - .06\frac{2}{3}$
11. $\frac{1}{2} + \frac{1}{5} - .038$
12. $\frac{1}{7} + \frac{1}{5} - .0139$
13. $\frac{3}{4} + \frac{9}{11} - .62$
14. $\frac{1}{8} + \frac{3}{5} - .376$
15. $\frac{1}{5} + \frac{1}{4} - .35$
16. $\frac{1}{6} + \frac{1}{8} - .013$
17. $16\frac{2}{3} + 8\frac{3}{4} - 12.41$
18. $37\frac{3}{8} + 14\frac{4}{5} - 8.375$
19. $29\frac{5}{6} + 17\frac{3}{8} - 21.17$
20. $71\frac{5}{12} + 39\frac{3}{4} - 48.33$
21. $14\frac{3}{5} + 87\frac{1}{4} - 63.49$
22. $76\frac{1}{4} + 43 - 8.875$
23. $49.14 + 17.6 - 27\frac{1}{2}$
24. $36\frac{3}{8} + 12\frac{4}{5} - 16\frac{3}{4}$

Oral Review

1. Begin with 75 and subtract $6\frac{1}{4}$ again and again until nothing remains.

2. Begin with 60 and subtract $2\frac{1}{2}$ again and again until nothing remains.

3. Subtract $2\frac{3}{4}$ again and again, beginning with each of the following numbers, until nothing remains: 66; $82\frac{1}{2}$; 110; 55; $104\frac{1}{2}$; $27\frac{1}{2}$; $46\frac{3}{4}$.

4. From the sum of $27\frac{3}{4}$ and $16\frac{3}{8}$ take $8\frac{1}{2}$.

5. A carpenter has a board 16 ft. long. After cutting from this board two lengths, each $6\frac{3}{8}$ ft., how much remains?

6. A builder owns a lot $57\frac{1}{2}$ ft. wide. He wishes to construct upon it a building that will measure $48\frac{1}{3}$ ft. The building is to be erected $4\frac{1}{2}$ ft. from one lot line. How far will it be from the lot line on the other side?

7. If 9 be added to each term of $\frac{2}{3}$, by how much is the fraction increased or decreased?

8. Four is subtracted from each term of $\frac{7}{8}$; does it increase or decrease the value of the fraction, and how much?

Written Review Test

(Time, 25 minutes)

1. $\frac{43}{66}-\frac{19}{44}$
2. $\frac{14}{24}-\frac{3}{16}$
3. $\frac{29}{32}-\frac{23}{48}$
4. $\frac{13}{18}-\frac{7}{12}$
5. $\frac{31}{42}-\frac{19}{28}$
6. $\frac{47}{48}-\frac{31}{32}$

7. Find the difference between one hundred and six ten-thousandths and one hundred six ten-thousandths.

8. Find the difference between $\frac{4}{7}$ and .729.

9. Find the difference between $12\frac{5}{8}$ and 52.8364.

10. $125.3267-39\frac{3}{4}$
11. $3482.125-437\frac{7}{8}$
12. $149\frac{2}{3}-27.333\frac{1}{3}$
13. $270.0701-135\frac{13}{16}$
14. $129.007-\frac{213}{1000}$
15. $42.042-\frac{7}{12}$

16. Add $12\frac{3}{4}$, $247\frac{1}{8}$, $693\frac{5}{12}$, and $546\frac{3}{8}$.

17. Add $324\frac{2}{3}$, $516\frac{5}{6}$, $298\frac{4}{9}$, and $1277\frac{1}{3}$.

18. Add $536\frac{2}{7}$, $328\frac{3}{14}$, $743\frac{19}{21}$, and $486\frac{2}{3}$.

Multiplication of Fractions

Since the numerator of a fraction indicates the number of fractional units in it, a fraction is multiplied by multiplying its numerator without changing its denominator.

Thus $\frac{3}{8}$ is doubled by multiplying its numerator by 2. $\frac{3}{8}\times 2=\frac{6}{8}$ or $\frac{3}{4}$

Since the denominator of a fraction indicates the size of the fractional units of which the fraction consists, a fraction is multiplied by dividing its denominator without changing its numerator.

Thus $\frac{3}{8}$ is multiplied by 2 by dividing its denominator by 2. $\frac{3}{8}\times 2=\frac{3}{4}$

To multiply an integer by a common fraction.

EXAMPLE. Multiply 16 by $\frac{4}{5}$.

$$16\times\tfrac{4}{5}=\frac{16\times 4}{5}=\tfrac{64}{5}=12\tfrac{4}{5}$$

Multiply the integer by the numerator of the fraction, and divide by the denominator. It makes no difference whether the fraction be the multiplier or the multiplicand, as these may be interchanged without affecting the result.

Apply the principle of cancellation to problems in multiplication of fractions when possible.

ORAL

1. $8\times\frac{6}{7}$	**6.** $8\times\frac{3}{7}$	**11.** $\frac{7}{8}\times 9$	**16.** $7\times\frac{3}{8}$
2. $9\times\frac{7}{8}$	**7.** $16\times\frac{3}{5}$	**12.** $9\times\frac{3}{5}$	**17.** $14\times\frac{7}{9}$
3. $14\times\frac{3}{7}$	**8.** $24\times\frac{2}{3}$	**13.** $32\times\frac{7}{8}$	**18.** $44\times\frac{3}{4}$
4. $16\times\frac{3}{8}$	**9.** $72\times\frac{5}{8}$	**14.** $\frac{3}{4}\times 24$	**19.** $\frac{3}{8}\times 72$
5. $27\times\frac{4}{9}$	**10.** $84\times\frac{3}{4}$	**15.** $\frac{5}{8}\times 48$	**20.** $\frac{5}{9}\times 7$

WRITTEN

1. $26\times\frac{4}{9}$	**6.** $39\times\frac{5}{9}$	**11.** $\frac{2}{7}\times 360$	**16.** $\frac{31}{32}\times 12864$
2. $71\times\frac{5}{8}$	**7.** $167\times\frac{5}{12}$	**12.** $\frac{3}{7}\times 374$	**17.** $\frac{7}{12}\times 8460$
3. $86\times\frac{3}{7}$	**8.** $186\times\frac{12}{17}$	**13.** $\frac{4}{7}\times 168$	**18.** $\frac{15}{16}\times 14416$
4. $29\times\frac{7}{8}$	**9.** $396\times\frac{4}{7}$	**14.** $\frac{11}{12}\times 3963$	**19.** $\frac{31}{36}\times 10872$
5. $61\times\frac{2}{3}$	**10.** $\frac{5}{6}\times 139$	**15.** $\frac{11}{15}\times 6450$	**20.** $\frac{31}{32}\times 9664$

Multiplication of Decimal Fractions

EXAMPLE. Multiply 56 by .125.

$.125 = \frac{1}{8}$
$56 \times \frac{1}{8} = 7$

Multiplication may be accomplished by changing the decimal fraction to the common fraction form and then multiplying. This is often convenient.

EXAMPLE. Multiply 3.4 by .64.

.6 4
3.4
2.1 7 6

When the multiplier or multiplicand is a decimal fraction, or both are decimal fractions, multiply in the usual way and point off in the product as many places as there are in both factors.

When integers are multiplied	When decimals are multiplied
$10 \times 10 = 100$	tenths × tenths = hundredths
$10 \times 100 = 1000$	tenths × hundredths = thousandths
$100 \times 100 = 10{,}000$	hundredths × hundredths = ten-thousandths
$100 \times 1000 = 100{,}000$	hundredths × thousandths = hundred-thousandths

ORAL

1. $54 \times .3$	6. $35 \times .4$	11. $4 \times .12$	16. $34 \times .05$
2. $24 \times .8$	7. $27 \times .33\frac{1}{3}$	12. $15 \times .12$	17. $19 \times .07$
3. $36 \times .25$	8. $36 \times .66\frac{2}{3}$	13. $12 \times .09$	18. $60 \times .75$
4. $70 \times .2$	9. $38 \times .3$	14. $56 \times .4$	19. $58 \times \frac{3}{10}$
5. $40 \times .75$	10. $25 \times .6$	15. $32 \times .25$	20. $18 \times \frac{7}{10}$

WRITTEN

1. $81 \times .33\frac{1}{3}$	6. $38 \times .875$	11. $.78 \times 1.74$	16. $18.5 \times .19$
2. $24 \times .625$	7. $96 \times .66\frac{2}{3}$	12. 4.53×1.8	17. $.647 \times .014$
3. $85 \times .056$	8. $70 \times .14\frac{2}{7}$	13. $39.2 \times .54$	18. $4.63 \times .21$
4. $97 \times .045$	9. $.16 \times 34.5$	14. 13.6×8.2	19. 17.4×12
5. $62 \times .28$	10. 3.96×4.6	15. $395 \times .12$	20. $4.73 \times .16$

To multiply a fraction by a fraction.

EXAMPLE. Multiply $\frac{3}{8}$ by $\frac{3}{5}$.

$\frac{3}{8} \times \frac{3}{5} = \frac{9}{40}$

Multiply the numerators together for the numerator of the product, and multiply the denominators together for the denominator of the product.

The work should be shortened by cancellation when possible.

ORAL

1. $\frac{4}{5}\times\frac{3}{8}$	7. $\frac{6}{7}\times\frac{11}{14}$	13. $\frac{6}{7}\times\frac{11}{13}$	19. $\frac{35}{38}\times\frac{2}{7}$
2. $\frac{4}{7}\times\frac{7}{16}$	8. $\frac{15}{22}\times\frac{11}{30}$	14. $\frac{5}{8}\times\frac{3}{7}$	20. $\frac{4}{9}\times\frac{3}{8}$
3. $\frac{3}{9}\times\frac{2}{3}$	9. $\frac{9}{28}\times\frac{14}{45}$	15. $\frac{17}{24}\times\frac{6}{9}$	21. $\frac{5}{7}\times\frac{14}{15}$
4. $\frac{6}{25}\times\frac{5}{12}$	10. $\frac{7}{8}\times\frac{3}{4}$	16. $\frac{3}{12}\times.48$	22. $\frac{8}{9}\times\frac{27}{32}$
5. $\frac{4}{9}\times\frac{7}{16}$	11. $\frac{3}{7}\times\frac{4}{5}$	17. $\frac{27}{52}\times\frac{4}{9}$	23. $\frac{3}{4}\times.72$
6. $\frac{3}{8}\times\frac{5}{9}$	12. $\frac{4}{11}\times\frac{8}{9}$	18. $\frac{21}{32}\times\frac{4}{7}$	24. $\frac{5}{6}\times\frac{18}{25}$

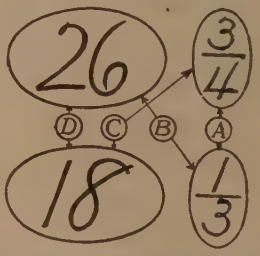
WRITTEN

1. $\frac{5}{12}\times.45$	7. $.306\times.042$	13. $1.35\times.83\frac{1}{3}$	19. 56.2×1.47
2. $\frac{7}{12}\times.625$	8. $.375\times4.2$	14. $672\times.66\frac{2}{3}$	20. $50.9\times\frac{9}{10}$
3. $\frac{14}{15}\times.83\frac{1}{3}$	9. $.625\times.27$	15. 8.91×21.06	21. $8.2\times.7$
4. $\frac{15}{16}\times.66\frac{2}{3}$	10. $83\times.44\frac{4}{9}$	16. $\frac{13}{16}\times\frac{64}{75}$	22. $\frac{4}{5}\times8.55$
5. $.156\times.12\frac{1}{2}$	11. $7.07\times.14\frac{2}{7}$	17. $\frac{8}{9}\times.819$	23. $\frac{4}{9}\times\frac{36}{41}$
6. $.39\times471$	12. $.65\times.403$	18. $\frac{7}{9}\times\frac{27}{28}$	24. 18.2×1.82

To multiply mixed numbers.

Example. Multiply $26\frac{3}{4}$ by $18\frac{1}{3}$.

$26\frac{3}{4}$
$18\frac{1}{3}$
$\frac{1}{4}$
$8\frac{2}{3}$
$13\frac{1}{2}$
208
26
$490\frac{5}{12}$



By a study of the diagram, it will be seen that the multiplier is divided into two parts — a fraction and a whole number; and that the multiplicand is similarly divided. The solution consists in multiplying each of the two parts of the multiplicand by each of the two parts of the multiplier, making four operations as shown by the diagram, and adding the partial products so obtained.

First multiply the fraction by the fraction: $\frac{3}{4}\times\frac{1}{3}=\frac{1}{4}$. Write $\frac{1}{4}$ as a partial product. Next multiply the whole number in the multiplicand by the fraction in the multiplier. $\frac{1}{3}\times26=8\frac{2}{3}$. Write $8\frac{2}{3}$ as a partial product. Next multiply the fraction in the multiplicand by the whole number in the multiplier, and write the result, $13\frac{1}{2}$, as a partial product. Next multiply the whole numbers and write the partial products as shown. Add all partial products. The result is $490\frac{5}{12}$.

If either the multiplicand or the multiplier is an integer, without a fractional part, two of the above steps will be eliminated. Similarly, if either is a fraction without an integral part, two of the steps will be eliminated.

In case of mixed numbers when the integers are small, it is sometimes easier to express both multiplier and multiplicand as improper fractions and multiply.

Decimal fractions and common fractions are interchangeable in form. The form of either may be changed to the other if the change will make solution easier.

WRITTEN

1. $16\frac{3}{8}\times36\frac{2}{3}$
2. $41\frac{1}{3}\times26\frac{3}{4}$
3. $62\frac{1}{5}\times12\frac{3}{8}$
4. $24\frac{3}{5}\times16\frac{3}{4}$
5. $29\frac{4}{7}\times21\frac{1}{2}$
6. $34\frac{2}{3}\times17\frac{3}{8}$
7. $25\frac{2}{5}\times18\frac{3}{5}$
8. $27\frac{1}{2}\times20\frac{3}{8}$
9. $26\frac{2}{7}\times35\frac{4}{9}$
10. $42\frac{5}{6}\times18\frac{2}{3}$
11. $42\frac{3}{8}\times4.15$
12. $29\frac{2}{3}\times16.4$
13. $14\frac{3}{5}\times46\frac{3}{4}$
14. $58\frac{3}{8}\times45\frac{1}{4}$
15. $65\frac{2}{3}\times58\frac{1}{2}$
16. $73.4\times8.16\frac{2}{3}$
17. $63.5\times7.08\frac{1}{3}$
18. $86\frac{3}{4}\times9.83\frac{1}{3}$
19. $59\frac{5}{8}\times17.33\frac{1}{3}$
20. $37.25\times26.44\frac{4}{9}$
21. $23.4\times18.04\frac{4}{9}$
22. $16.24\times1.38\frac{2}{3}$
23. $16.04\times8.03\frac{1}{3}$
24. 62.5×1.254
25. 13.625×29.875
26. $26.83\frac{1}{3}\times137.33\frac{1}{3}$
27. 26.487×13.251
28. 163.75×19.476
29. 38.78×63.7
30. $492.3\frac{1}{3}\times78.125$

WRITTEN

1. $24\times16\frac{3}{8}$
2. $36\times12\frac{1}{2}$
3. $45\times18\frac{4}{9}$
4. $39\times22\frac{1}{3}$
5. $46\times27\frac{3}{4}$
6. $51\times24\frac{2}{3}$
7. $48\times26\frac{3}{8}$
8. $56\times19\frac{4}{7}$
9. $37\times21\frac{2}{3}$
10. $63\times27\frac{4}{5}$
11. $35\times21\frac{1}{5}$
12. $42\times18\frac{5}{6}$
13. $54\times19\frac{3}{4}$
14. $47\times21\frac{1}{3}$
15. $55\times27\frac{2}{11}$
16. $39\times22\frac{2}{3}$
17. 62×4.125
18. 39×3.035
19. 74×2.12
20. 86×3.16
21. $75\times9.06\frac{2}{3}$
22. $84\times5.12\frac{1}{2}$
23. $194\times4.37\frac{3}{4}$
24. $145\times8.16\frac{2}{3}$

WRITTEN

1. $\frac{3}{4}\times27\frac{3}{8}$
2. $\frac{3}{5}\times46\frac{2}{3}$
3. $\frac{3}{8}\times96\frac{1}{2}$
4. $\frac{4}{5}\times42\frac{1}{3}$
5. $\frac{2}{9}\times38\frac{1}{2}$
6. $\frac{4}{7}\times28\frac{3}{5}$
7. $\frac{3}{7}\times24\frac{2}{3}$
8. $\frac{11}{14}\times41\frac{5}{6}$
9. $4.624\times.25$
10. $12.18\times.015$
11. $13.5\times\frac{3}{4}$
12. $14.15\times\frac{7}{8}$

13. $16.04 \times \frac{3}{5}$ **16.** $12\frac{7}{8} \times \frac{3}{7}$ **19.** $268.3 \times \frac{4}{7}$ **22.** $460.3 \times \frac{2}{3}$

14. $127.8 \times \frac{5}{6}$ **17.** $34\frac{3}{5} \times .17$ **20.** $193.14 \times \frac{2}{3}$ **23.** $19.6 \times \frac{3}{7}$

15. $396.7 \times \frac{4}{5}$ **18.** $\frac{1}{2}\frac{3}{9} \times .25$ **21.** $363.76 \times \frac{5}{9}$ **24.** $27.47 \times \frac{3}{8}$

WRITTEN. Extend and foot the following bills:

1.
41 yd. @ $27\frac{1}{2}¢$
46 yd. @ $9\frac{3}{4}¢$
42 yd. @ $7\frac{1}{4}¢$
43 yd. @ $8\frac{3}{4}¢$

2.
$29\frac{1}{2}$ ft. @ $1\frac{3}{4}¢$
$46\frac{3}{4}$ ft. @ $3\frac{1}{2}¢$
$82\frac{1}{4}$ ft. @ $8\frac{1}{3}¢$
$96\frac{1}{2}$ ft. @ $12\frac{1}{2}¢$

3.
$7\frac{7}{8}$ lb. @ $22\frac{1}{2}¢$
$14\frac{1}{2}$ lb. @ $19\frac{3}{4}¢$
$22\frac{3}{4}$ lb. @ $28\frac{1}{4}¢$
$20\frac{3}{8}$ lb. @ $23\frac{1}{2}¢$

4.
126 doz. @ $\$1.86\frac{1}{2}$
49 doz. @ $\$2.12\frac{1}{2}$
73 doz. @ $\$4.37\frac{3}{4}$
68 doz. @ $\$ .97\frac{1}{4}$

5.
61.8 oz. @ $53\frac{1}{2}¢$
162.4 oz. @ $54\frac{1}{4}¢$
93.5 oz. @ $53\frac{3}{4}¢$
71.7 oz. @ $52\frac{3}{4}¢$

6.
$126\frac{1}{2}$ rd. @ \$1.65
$270\frac{3}{4}$ rd. @ \$1.47
$86\frac{1}{4}$ rd. @ \$1.18
$92\frac{1}{2}$ rd. @ \$2.09

7.
146.4 gal. @ $19\frac{1}{2}¢$
73.9 gal. @ $19\frac{3}{4}¢$
137.4 gal. @ $18\frac{1}{4}¢$
76.3 gal. @ 20¢

8.
$16\frac{1}{2}$ gross @ $28\frac{3}{4}¢$
$161\frac{3}{4}$ gross @ $27\frac{1}{2}¢$
$85\frac{1}{4}$ gross @ $31\frac{1}{4}¢$
$67\frac{3}{4}$ gross @ 36¢

9. Copy the following invoice, make extensions, and foot.

THE WAUKESHA CANNING CO.

GROWERS AND PACKERS OF
FANCY CANNED GOODS
WAUKESHA, WIS.

INVOICE NO. 76
S. O. NO. 1
CONTRACT NO. 10
DATE Jan. 2, 19--.

SOLD TO WESTERN GROCERY HOUSE,
VIA Wis. Cent. 45 E. Randolph St., Chicago, Ill.
CAR INITIAL W. C. CAR NO. 72874 F. O. B. Waukesha.

BRANDS:
MENU
OLD ABE
PEWAUKEE
LAKE
SPARKLING WAUKESHA MINERAL SPRING WATER USED EXCLUSIVELY IN PREPARING THESE GOODS

TERMS { 60 DAYS ACCEPTANCE / 1% CASH 10 DAYS

120	Cs	Menu 2# Peas, 240 doz. at	.82½			
103	"	Old Abe 2# Corn, 206 doz. at	.77½			
81	"	Pewaukee Lake 2# String Beans, 162 doz. at	.80			
55	"	" " 2# Tomatoes, 110 doz. at	.78½			
75	"	" " 3# " 150 doz. at	1.07½			
65	"	Menu 2# Pumpkins, 130 doz. at	1.12½			
75	"	Old Abe 2# Succotash, 150 doz. at	.82½			

Oral Review Problems

1. What is $\frac{1}{2}$ of 32? 78? 96? 156? 170? 196?

2. 12 is $\frac{1}{3}$ of what number? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{9}$? $\frac{1}{10}$?

3. If each of the following numbers be multiplied by $1\frac{1}{2}$, what will the result be: 12, 16, 29, 31, 45, 60, 80?

4. How much weather stripping would be required for three outside doors of a house if each door is $2\frac{2}{3}$ ft. by $6\frac{2}{3}$ ft. and the strip is to go on one side and one end of each door?

5. If each term of $\frac{4}{5}$ be multiplied by 3, what effect will this have upon the value of the fraction?

6. $\frac{3}{4}\times 72$

7. $\frac{5}{6}\times\frac{12}{25}$

8. $\frac{4}{5}\times\frac{35}{36}$

9. $.8\times\frac{25}{32}$

10. $\frac{2}{7}\times\frac{14}{15}$

11. $2\frac{3}{4}\times\frac{2}{11}$

12. $\frac{2}{3}\times 2\frac{1}{4}$

13. $\frac{5}{8}\times\frac{16}{25}$

14. $\frac{7}{9}\times\frac{18}{49}$

Written Review Problems

1. Two men do a certain piece of work at a contract price of \$178.50. One works 36 days and the other works 15 days. If their work is paid for at the same rate, how much shall each receive?

2. A tomato can is $3\frac{3}{8}$ in. in diameter and $4\frac{1}{2}$ in. high. What must be the inside width, length, and height of a box to contain 4 doz. cans if they are put in 4 rows of 6 cans each, on the bottom, and the box contains 2 tiers?

3. A tailor bought a piece of goods containing $41\frac{1}{4}$ yd. If $3\frac{1}{3}$ yards are required to make a suit, how many yards will remain after 11 suits have been made from the goods?

4. In a certain factory the employees are allowed "time and one half" for overtime; *i.e.*, for all time they work beyond the regular hours they receive $1\frac{1}{2}$ times the regular wage. An employee works $10\frac{3}{4}$ hours in one day, when 8 hours are considered a day, at 35¢ per hour. What is his pay for the day?

5. In the State of Ohio the state levy for highway improvement for 1914 was .3 mill. If a farmer owned 193.8 acres of land assessed at \$42.50 per acre how much tax would he pay for this purpose?

6. A woman bought a turkey weighing $11\frac{1}{2}$ lb. @ $26\frac{1}{2}$¢ per lb. How much change should she receive out of a \$5.00 bill offered in payment? \$1.95

7. The following bill was purchased at a grocery store: $3\frac{1}{4}$ lb. steak at 26¢; $6\frac{1}{2}$ lb. ham at 22¢; 12 lb. lard at $17\frac{1}{2}$¢; 50 lb. sugar at $7\frac{1}{2}$¢; and 1 doz. cans corn at 2 cans for 25¢. How much change should be returned out of a $10 bill?

8. Each section of a radiator is $2\frac{1}{2}$ in. thick. What will be the length of a radiator having 17 such sections? Suppose each section of this radiator contains $3\frac{3}{4}$ sq. ft. of radiation; how many square feet of radiation does the radiator contain?

9. A boiler contains 92.8 sq. ft. of grate surface and burns $9\frac{3}{4}$ lb. of coal per hour per sq. ft. of grate surface. How much will it burn in $24\frac{1}{2}$ hours?

10. How much coke will three fires burn in 27 days, allowing $7\frac{1}{2}$ hr. per day and $8\frac{1}{4}$ lb. of coke per hour for each fire?

11. A steam piston has an area of 78.15 sq. in. If the cylinder is filled with steam at a pressure of 150 lb. per sq. in., what is the total force exerted on the piston?

12. Navy brass is composed of .62 copper, .01 tin and .37 zinc. The weight of a certain brass casting is 2785 lb. How much of each kind of metal was used to produce the casting?

13. Henry Lane is bankrupt and a statement of his affairs shows the following condition:

Assets		Liabilities	
Cash	$ 139.50	Notes and Accounts Payable	$20,000
Accounts and Notes Receivable	1028.00		
Real Estate	6600.00		
Personal Property not exempt	570.00		

A sale of his assets is ordered on behalf of his creditors and upon liquidation cash is realized as follows: $\frac{7}{8}$ of the face value of accounts and notes receivable; $\frac{1}{1\frac{1}{2}}$ of the book value of the real estate; and $\frac{4}{5}$ of the book value of his personal property. If the expenses of the bankruptcy proceedings are $295.50, what part of what is due them do his creditors realize? Express your answer in both common and decimal form.

14. A certain railroad company has 3,069.34 miles of track. The main track consists of 1,680.95 miles; the second track, 338.63 miles. The remainder is sidings and yards. How many miles of trackage in sidings and yards?

15. The total tax levy in a certain village in Ohio was 13.60 mills. The State levy was .45 mills and the county levy 3.05 mills. The balance was for village purposes; what was the levy for the village?

16. The average income on the investments of a certain estate is \$.09375 per dollar. The taxes average 15.38 mills per dollar. What is the income on the investment after taxes have been allowed for?

Written Review Test

(Time, 25 minutes.)

1. $28\times\frac{3}{7}=$	2. $56\times\frac{7}{8}=$	3. $75\times\frac{4}{15}=$
$52\times\frac{3}{9}=$	$43\times\frac{5}{9}=$	$\frac{2}{7}\times29=$
$33\times\frac{5}{8}=$	$64\times\frac{3}{5}=$	$\frac{4}{9}\times37=$
$47\times\frac{2}{3}=$	$28\times\frac{2}{3}=$	$\frac{5}{8}\times60=$
$56\times\frac{5}{8}=$	$57\times\frac{5}{6}=$	$\frac{3}{7}\times32=$
4. $162\times.66\frac{2}{3}=$	5. $328\times.875=$	6. $3.7\times37.01=$
$48\times6.25=$	$192\times.33\frac{1}{3}=$	$4.92\times5.802=$
$170\times.028=$	$270\times.28\frac{4}{7}=$	$.63\times1.16\frac{2}{3}=$
$194\times.091=$	$69\times3.125=$	$59.2\times29.5=$
$6.3\times.208=$	$48\times.83\frac{1}{3}=$	$6.4\times.375=$
7. $18\frac{1}{3}\times34\frac{3}{8}=$	8. $27\frac{2}{5}\times53\frac{3}{9}=$	9. $16\frac{3}{8}\times20\frac{4}{5}=$
$27\frac{3}{4}\times43\frac{2}{9}=$	$36\frac{4}{15}\times45\frac{3}{4}=$	$39\frac{6}{9}\times72\frac{14}{15}=$
$36\frac{5}{6}\times52\frac{2}{3}=$	$64\frac{8}{9}\times72\frac{9}{16}=$	$38\frac{8}{9}\times14\frac{2}{3}=$
$45\frac{1}{16}\times64\frac{3}{5}=$	$54\frac{3}{9}\times81\frac{3}{5}=$	$56\frac{4}{5}\times74\frac{5}{4}=$
$56\frac{9}{10}\times90\frac{7}{8}=$	$90\frac{1}{3}\times24\frac{7}{10}=$	$58\frac{2}{3}\times\ 9\frac{3}{4}=$
10. 47^{3} yd. at $32\frac{1}{4}$¢ =	11. $39\frac{3}{4}$ ft. at $5\frac{3}{4}$¢ =	12. $48\frac{1}{2}$ doz. at \$$1.87\frac{1}{2}$ =
49^{1} yd. at $11\frac{3}{4}$¢ =	$46\frac{1}{2}$ ft. at $7\frac{1}{2}$¢ =	$52\frac{3}{4}$ doz. at $1.12\frac{1}{2}$ =
51^{2} yd. at $17\frac{1}{2}$¢ =	$87\frac{3}{8}$ ft. at $8\frac{3}{4}$¢ =	$63\frac{5}{12}$ doz. at $1.37\frac{1}{2}$ =
84^{3} yd. at $7\frac{5}{8}$¢ =	$17\frac{3}{4}$ ft. at $23\frac{3}{4}$¢ =	$72\frac{1}{2}$ doz. at $1.42\frac{1}{2}$ =
92^{1} yd. at $13\frac{1}{2}$¢ =	$29\frac{1}{4}$ ft. at $25\frac{1}{2}$¢ =	$43\frac{3}{4}$ doz. at 1.63 =

Division of Fractions

Dividing a fraction by an integer.

Since the numerator of a fraction indicates the number of fractional units of which it consists, it is clear that a fraction is divided if its numerator is divided without changing its denominator.

Thus $\frac{6}{8}$ is divided by 2 by dividing its numerator by 2.

$$\frac{6}{8} \div 2 = \frac{6 \div 2}{8} = \frac{3}{8}$$

Since the denominator of a fraction indicates the size of the fractional units of which it consists, it is clear that a fraction is divided if its denominator is multiplied without changing the numerator.

Thus $\frac{6}{8}$ is divided by 2 by multiplying its denominator by 2.

$$\frac{6}{8} \div 2 = \frac{6}{8 \times 2} = \frac{6}{16} = \frac{3}{8}$$

Hence, to divide a fraction by an integer, divide its numerator or multiply its denominator by that integer.

ORAL. In solving these problems, divide the numerator if it contains the divisor an integral number of times; if not, multiply the denominator.

1. $\frac{6}{7} \div 2$	6. $\frac{5}{6} \div 2$	11. $\frac{16}{25} \div 3$	16. $\frac{19}{23} \div 3$
2. $\frac{8}{11} \div 4$	7. $\frac{5}{8} \div 3$	12. $\frac{25}{32} \div 5$	17. $\frac{27}{28} \div 9$
3. $\frac{9}{13} \div 3$	8. $\frac{7}{8} \div 7$	13. $\frac{25}{32} \div 3$	18. $\frac{32}{35} \div 3$
4. $\frac{2}{3} \div 3$	9. $\frac{8}{15} \div 4$	14. $\frac{24}{25} \div 16$	19. $\frac{45}{49} \div 3$
5. $\frac{3}{4} \div 3$	10. $\frac{16}{15} \div 4$	15. $\frac{29}{30} \div 2$	20. $\frac{49}{50} \div 7$

EXAMPLE. Divide .3728 by 4.

$$4\overline{)}.3728$$
$$.0932$$

The numerator is .3728, and the denominator 10,000. Divide the numerator by 4. Point off in the result the same number of places as in the dividend, thus observing the principle that in the division of fractions the denominator remains unchanged.

ORAL

1. $.4624 \div 4$	4. $47.34 \div 9$	7. $1.152 \div 6$	10. $5.292 \div 6$
2. $2.510 \div 5$	5. $3.693 \div 3$	8. $4.963 \div 7$	11. $8.496 \div 12$
3. $3.429 \div 9$	6. $3.843 \div 3$	9. $8.168 \div 4$	12. $35.299 \div 11$

13. $39.668 \div 13$	**15.** $7.875 \div 15$	**17.** $35.625 \div 5$	**19.** $51.121 \div 7$
14. $48.78 \div 9$	**16.** $8.2161 \div 9$	**18.** $91.584 \div 6$	**20.** $71.20 \div 8$

WRITTEN

1. $7.515 \div 15$	**6.** $473.92 \div 16$	**11.** $386.475 \div 625$	**16.** $97.616 \div 32$
2. $16.032 \div 14$	**7.** $397.25 \div 25$	**12.** $1728.34 \div 24$	**17.** $48.376 \div 19$
3. $9.375 \div 25$	**8.** $37.0125 \div 75$	**13.** $21.6212 \div 36$	**18.** $52.345 \div 17$
4. $7.625 \div 125$	**9.** $186.47 \div 24$	**14.** $34.575 \div 15$	**19.** $84.625 \div 23$
5. $30.075 \div 15$	**10.** $1278.8 \div 45$	**15.** $76.95 \div 45$	**20.** $921.65 \div 19$

To divide by a fraction.

EXAMPLE. Divide 15 by $\frac{3}{4}$.

$15 \div \frac{3}{4} = 15 \times \frac{4}{3} = 20$

To divide 15 by 3 would give 5 as a result. Since 3 is 4 times $\frac{3}{4}$, 15 contains $\frac{3}{4}$ four times as many times as it contains 3. Multiply 5 by 4. The result is 20. Since you have divided by 3 and multiplied by 4, you have in effect inverted the divisor and multiplied.

EXAMPLE. Divide $\frac{1}{8}$ by $\frac{3}{4}$.

$\frac{1}{8} \div \frac{3}{4} = \frac{1}{8} \times \frac{4}{3} = \frac{1}{6}$

To divide $\frac{1}{8}$ by 3 would give $\frac{1}{24}$ as a result. Since 3 is 4 times $\frac{3}{4}$, $\frac{1}{8}$ contains $\frac{3}{4}$ 4 times as many times as it contains 3. Multiply $\frac{1}{24}$ by 4. The result is $\frac{4}{24}$ or $\frac{1}{6}$. Since you have divided by 3 and multiplied by 4 you have in effect inverted the divisor and multiplied.

From the two solutions given above, it is clear that whether the dividend is an integer or a fraction, the rule for dividing by a fraction is

> Invert the divisor and multiply.

WRITTEN

1. $\frac{3}{4} \div \frac{3}{5}$	**6.** $\frac{4}{5} \div \frac{1}{5}$	**11.** $\frac{7}{8} \div \frac{4}{7}$	**16.** $\frac{1}{3} \div \frac{1}{4}$
2. $\frac{5}{8} \div \frac{3}{8}$	**7.** $\frac{1}{5} \div \frac{4}{5}$	**12.** $\frac{8}{9} \div \frac{1}{3}$	**17.** $\frac{1}{4} \div \frac{1}{5}$
3. $\frac{2}{9} \div \frac{2}{3}$	**8.** $\frac{3}{5} \div \frac{3}{5}$	**13.** $\frac{3}{11} \div \frac{3}{22}$	**18.** $\frac{2}{3} \div \frac{3}{4}$
4. $\frac{11}{15} \div \frac{3}{5}$	**9.** $\frac{7}{8} \div \frac{1}{9}$	**14.** $\frac{3}{22} \div \frac{3}{11}$	**19.** $\frac{3}{4} \div \frac{5}{6}$
5. $\frac{1}{8} \div \frac{1}{4}$	**10.** $\frac{3}{4} \div \frac{1}{4}$	**15.** $\frac{1}{2} \div \frac{1}{3}$	**20.** $\frac{7}{8} \div \frac{3}{4}$

To divide a mixed number by an integer.

EXAMPLE. Divide $8\frac{2}{5}$ by 6.

$8\frac{2}{5} = \frac{42}{5}$

$\frac{42}{5} \div 6 = \frac{7}{5}$ or $1\frac{2}{5}$

Change the mixed number to an improper fraction and divide. This is a good method when the integer of the mixed number is small.

EXAMPLE. Divide $1247\frac{1}{5}$ by 4.

$4)1247\frac{1}{5}$

$311\frac{3\frac{1}{5}}{4} = 311\frac{1}{20}$

$311\frac{1}{20} = 311\frac{4}{5}$

When the integer of the dividend is a large number reducing the dividend to an improper number is too cumbersome a plan. Divide as in whole numbers. The remainder is a complex fraction which is simplified by multiplying both numerator ($3\frac{1}{5}$) and denominator (4) by the denominator of the fractional part of the numerator (5).

WRITTEN. Divide, exercising your best judgment in each case as to which of the above methods to use.

1. $6\frac{2}{3} \div 8$	**7.** $1386\frac{1}{2} \div 8$	**13.** $3\frac{3}{5} \div 9$	**19.** $15\frac{3}{5} \div 13$
2. $5\frac{1}{2} \div 12$	**8.** $268\frac{1}{2} \div 12$	**14.** $6\frac{2}{3} \div 5$	**20.** $16\frac{3}{8} \div 4$
3. $3\frac{3}{4} \div 5$	**9.** $128\frac{1}{2} \div 4$	**15.** $5\frac{3}{5} \div 9$	**21.** $52\frac{13}{16} \div 13$
4. $16\frac{1}{4} \div 8$	**10.** $648\frac{2}{3} \div 12$	**16.** $2846\frac{1}{2} \div 8$	**22.** $7962\frac{7}{8} \div 4$
5. $36\frac{2}{3} \div 7$	**11.** $842\frac{3}{4} \div 4$	**17.** $342\frac{3}{4} \div 4$	**23.** $312\frac{5}{6} \div 9$
6. $4\frac{3}{4} \div 12$	**12.** $3978\frac{3}{5} \div 12$	**18.** $1879\frac{1}{7} \div 7$	**24.** $1461\frac{3}{5} \div 27$

To divide a mixed number by a fraction or by a mixed number.

When the numbers are small, they can be changed readily into improper fractions. The division is then performed as in dividing a fraction by a fraction. If they cannot be changed into improper fractions readily, the division can be performed as follows:

EXAMPLE. Divide $1428\frac{1}{3}$ by $13\frac{1}{2}$.

$13\frac{1}{2})1428\frac{1}{3}$

$\frac{6}{81}$ $\frac{6}{)8570(105\frac{65}{81}}$

$\frac{81}{470}$

$\frac{405}{65}$

Arrange the two numbers as for ordinary long division. Multiply both dividend and divisor by the least common denominator of the two fractions. In multiplying both dividend and divisor by the same number, both are cleared of fractions, but their relation to each other is not changed. Divide as in ordinary long division.

WRITTEN

1. $378\frac{2}{3} \div 12\frac{1}{2}$
2. $176\frac{3}{5} \div 7\frac{1}{2}$
3. $139\frac{1}{4} \div 13\frac{1}{3}$
4. $187\frac{3}{7} \div 8\frac{1}{2}$
5. $367\frac{4}{5} \div 16\frac{2}{3}$
6. $263\frac{3}{8} \div 14\frac{2}{3}$
7. $196\frac{1}{5} \div 24\frac{2}{3}$
8. $187\frac{3}{5} \div 21\frac{4}{5}$
9. $278\frac{1}{2} \div 26\frac{2}{3}$
10. $375\frac{1}{5} \div 22\frac{4}{5}$
11. $678\frac{2}{3} \div 35\frac{3}{4}$
12. $725\frac{3}{8} \div 64\frac{1}{4}$
13. $164\frac{1}{4} \div 12\frac{3}{8}$
14. $392\frac{3}{4} \div 8\frac{3}{16}$
15. $726\frac{1}{8} \div 6\frac{1}{2}$
16. $741\frac{3}{8} \div 15\frac{7}{12}$
17. $147\frac{3}{5} \div 4\frac{7}{10}$
18. $465\frac{9}{26} \div 3\frac{1}{2}$

After solving the above problems, solve problem 1 by expressing both dividend and divisor as improper fractions and proceeding as in division of fractions. Compare this solution with your first solution. You will note that the long division method is shorter.

Division of Decimals

(a) When both dividend and divisor have the same number of decimal places and the division is exact, without annexing ciphers, the quotient is a whole number.

EXAMPLE. Divide .45 by .09.

$.09\overline{).45}$
$\underline{5}$

45 hundredths divided by 9 hundredths = 5.

When the dividend has fewer decimal places than the divisor, ciphers must be added to the dividend until both dividend and divisor have the same number of decimal places.

(b) When the divisor is an integer, point off as many places in the quotient as there are in the dividend. This has already been explained.

When there are more decimal places in the dividend than in the divisor, move the decimal point in both dividend and divisor as many places to the right as there are decimal places in the divisor, thus making the divisor an integer, and divide.

WRITTEN. If the dividend has fewer than four places, annex ciphers to make four places, if necessary to complete the division. Show remainders, if any, as fractions.

1. $52.869 \div 2.3$
2. $38.764 \div 1.24$
3. $46.257 \div .325$
4. $63.472 \div .923$
5. $738.25 \div .34$
6. $847.36 \div .125$
7. $901.27 \div .0234$
8. $8134.6 \div 27.25$
9. $3.4624 \div 98.72$
10. $57.864 \div 921.3$
11. $756.43 \div 8.214$
12. $346.3 \div 9.564$
13. $85643 \div 3.65$
14. $9614.5 \div 71.6$
15. $145.27 \div 8.59$
16. $39.378 \div .302$
17. $4.1632 \div 2.31$
18. $528\ 67 \div 54.4$

Review Problems

ORAL

1. What numbers must be multiplied by $2\frac{1}{2}$ to produce the following numbers: 40, 60, 80, 90, 30, 50?

2. $\frac{1}{4}$ of 20 is $\frac{1}{8}$ of what number?

3. $\frac{4}{5}$ of 30 is $\frac{3}{8}$ of what number?

4. What will 2 cords of wood cost if 9 cords cost \$40.50?

5. $\frac{2}{3}$ is what part of $\frac{5}{6}$?

6. By what number must $\frac{3}{4}$ be multiplied to produce $\frac{5}{6}$?

7. 5 tons of coal cost \$36.25. At the same rate what will $\frac{3}{5}$ of a ton cost?

8. A boy sold a horse for \$72. This was $\frac{3}{4}$ of what the horse cost. How much did the horse cost?

9. A farmer bought a certain amount of wheat for \$150. He sold it at a profit of $\frac{1}{2}$ of the cost. If the selling price was \$1.50 per bu., how many bushels were there?

10. At 3 for 5 cents how many peaches may be bought for 30 cents?

11. At 3 for 5 cents what will 2 dozen peaches cost?

12. A farmer lost $\frac{3}{4}$ of his peach trees and had 58 left. How many had he at first?

13. If 8 castings cost \$12.80, what will 5 cost?

14. A grocer buys pineapples at 97¢ per dozen and sells them two for a quarter. Does he gain or lose and how much?

15. At $\$4\frac{1}{4}$ per day, how much money will a man earn in $4\frac{1}{2}$ days?

16. A certain book sells for \$2.25, $\frac{2}{5}$ of which is profit. What is the profit, and how much did the book cost?

17. 30 apples are bought at 2 for 5¢ and 30 are bought at 3 for 5¢. They are all sold at 5 for 10¢. Does the dealer gain or lose and how much?

18. This year Mr. Williams increased his apple orchard by $\frac{3}{4}$ of what it was last year. He now has 84 trees. How many did he have last year?

19. A house and lot are worth \$4500. The lot is worth $\frac{1}{4}$ as much as the house. Find the value of each.

20. A farmer sold $\frac{3}{5}$ of his land for \$1500. He has 20 acres remaining. What is it worth per acre at the same valuation and what was the value of the entire farm?

21. $15 \div \frac{1}{3}$	**26.** $28 \div \frac{4}{7}$	**31.** $875 \div 25$	**36.** $.96 \div .5\frac{1}{3}$
22. $64 \div \frac{1}{2}$	**27.** $42 \div \frac{1}{3}$	**32.** $625 \div .005$	**37.** $161 \div \frac{7}{8}$
23. $15 \div \frac{1}{6}$	**28.** $87 \div \frac{1}{2}$	**33.** $26 \div \frac{3}{4}$	**38.** $95 \div \frac{5}{12}$
24. $21 \div \frac{3}{4}$	**29.** $16 \div \frac{1}{9}$	**34.** $48 \div \frac{1}{3}$	**39.** $32 \div \frac{8}{9}$
25. $16 \div \frac{2}{3}$	**30.** $14 \div \frac{1}{7}$	**35.** $1.856 \div .004$	**40.** $48 \div \frac{6}{7}$

WRITTEN

1. $\frac{1}{4}$ of Henry Brown's property is in land and $\frac{1}{3}$ is in bonds. The remainder, \$1560.93, is cash. What is the value of his estate?

2. My profits for 1925 were $\frac{1}{2}$ more than my profits for the previous year. My profits for 1926 decreased $\frac{1}{3}$ from what they were in 1925. The profits for 1927 show an increase over 1926 of $\frac{2}{3}$. Profits for 1927 were \$2040; what were they each preceding year?

3. A cubic foot of a certain grade of coal weighs 51.75 lb. How many cubic feet of bunker space will be required to hold 56.5 tons? (Give answer to nearest integer.)

4. One cubic inch of a certain grade of cast iron weighs .2607 lb. How many cubic inches are in a casting that weighs 42.25 lb.?

5. The heating surface of a boiler is 3785.45 sq. ft., and the grate surface is 77.5 ft. How many square feet of heating surface are there to each square foot of grate surface?

6. During March, 1916, prices on paper were unusually high. A publisher made the following purchases during the month:

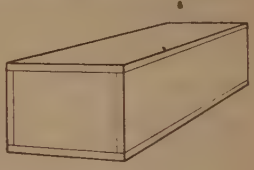
3167 lb. enamel writing at $9\frac{3}{4}$¢; normal price, $7\frac{3}{4}$¢
4011 lb. Eureka bond at $18\frac{1}{4}$¢; normal price, 12¢
23,000 white envelopes at \$2.40 per M.; normal price, \$1.85
5000 lb. special plate finish at \$6.60 per cwt.; normal price, \$4.40
100.05 M. white envelopes at 95¢; normal price, 68¢
4040 lb. B B halftone at $11\frac{1}{2}$¢; normal price, $6\frac{1}{4}$¢
408 lb. mellocoated book at $9\frac{1}{2}$¢; normal price, $7\frac{3}{4}$¢
$46\frac{1}{4}$ rm. mellocoated book, 120 lb. to the ream, at $10\frac{1}{4}$¢; normal price, $7\frac{3}{4}$¢
255 rm. M. F. book, 178 lb. to the ream, at $8\frac{3}{4}$¢; normal price, $5\frac{7}{8}$¢
147 rm. 300 sh. S. C. book, 185 lb. to the ream, at $9\frac{3}{4}$¢, normal price, $6\frac{1}{8}$¢.

How much did the publisher lose because of advances?

7. Condensed milk is sold in circular cans $2\frac{7}{8}''$ in diameter and $4\frac{3}{4}''$ in height, outside dimensions. What must be the size of a packing case that will contain 4 dozen such cans, placed in two tiers of 2 dozen each, 4 cans wide and 6 cans long?

8. The glass for a picture frame is $4\frac{1}{2}$ in. by $7\frac{1}{4}$ in. How many such glasses can be cut from a pane 24 in. by 36 in.? Illustrate by a diagram the most economical way of cutting the glass.

9. William wishes to make a tool box from six boards 1 inch thick. The inside dimensions must be: Height, $12\frac{1}{2}''$; width, $14\frac{3}{4}''$; length, $31\frac{3}{8}''$. The edges are to fit together squarely. The seams will be as shown in the accompanying diagram. Find the dimensions of each board, also the length of a piece of strap iron to go around the sides and ends of the box.



10. A man left an estate of $39,680. In his will it was stated that $5000 was to be donated to a certain charity; after the expenses of the estate were settled, his wife was to receive one-half of what remained, and the balance was to be divided equally among his five children. If the expenses of the estate were $580, what was the share of each child?

11. The following quotations record the transactions in certain stocks on the New York Curb April 10, 1926.

	Sales	Open	High	Low	Close
Am. Marconi	100	$3\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{2}$
Am. Zinc Pfd	770	$74\frac{1}{2}$	$75\frac{1}{2}$	$74\frac{1}{2}$	$75\frac{1}{2}$
Can. Copper	1000	$1\frac{7}{8}$	$1\frac{7}{8}$	$1\frac{3}{4}$	$1\frac{3}{4}$
St. L. R. M. & P	300	37	$37\frac{1}{4}$	37	$37\frac{1}{4}$
Tobacco Prod	1550	43	45	43	45
World Film	500	$1\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$

How much did I pay for 125 shares of American Zinc, Preferred, if I bought at the opening price quoted on that day?

12. Write an invoice from the following data: Ebeling & Reuss Philadelphia, Pa. sold to Cornelius & Ring, Wichita, Kans., on Jan. 1, 1927: $2\frac{1}{2}$ doz. toy tea sets @ $2 per doz.; $2\frac{1}{3}$ doz. celery dishes @ $4 per doz.; $3\frac{1}{4}$ doz. ash trays @ 90¢ per doz.; $4\frac{1}{2}$ doz. oatmeal dishes @ 45¢ per doz.; $2\frac{1}{2}$ gr. white egg cups @ $2.20 per

gr.; $3\frac{1}{2}$ doz. spoon trays @ \$2.10 per doz.; $3\frac{1}{2}$ doz. mugs @ 38¢ per doz. Terms: 2% off for cash in 30 days, or 90 days net.

13. Herewith is a newspaper clipping showing the Chicago Board of Trade quotations on wheat, corn, and oats April 10, 1926.

RANGE OF ACTIVE FUTURES

WHEAT

	Open	High	Low	Close Apr. 10	Close Apr. 8
May	$1.16\frac{3}{8}$	$1.18\frac{5}{8}$	$1.15\frac{3}{8}$	$1.17\frac{3}{4}$	$1.16\frac{1}{8}$
July	$1.14\frac{1}{8}$	1.16	$1.13\frac{1}{4}$	$1.15\frac{1}{8}$	$1.14\frac{1}{8}$
Sept	1.12	$1.13\frac{7}{8}$	$1.11\frac{5}{8}$	$1.13\frac{1}{8}$	$1.12\frac{1}{4}$

CORN

May	$74\frac{3}{4}$	$75\frac{3}{8}$	$74\frac{1}{4}$	$75\frac{1}{8}$	75
July	$76\frac{1}{8}$	$76\frac{5}{8}$	$75\frac{3}{4}$	$76\frac{3}{8}$	$76\frac{1}{4}$
Sept	$75\frac{3}{4}$	$76\frac{3}{8}$	$75\frac{5}{8}$	$76\frac{1}{4}$	$76\frac{1}{8}$

OATS

May	$44\frac{1}{2}$	$44\frac{7}{8}$	$44\frac{1}{8}$	$44\frac{3}{4}$	$44\frac{3}{4}$
July	$43\frac{1}{4}$	$43\frac{5}{8}$	43	$43\frac{1}{2}$	$43\frac{3}{8}$
Sept	$40\frac{1}{4}$	$40\frac{5}{8}$	$39\frac{7}{8}$	$40\frac{1}{4}$	$40\frac{1}{2}$

How much higher was May wheat at the close of the day than it was at the opening? How much lower did July wheat get in the course of the day than it was at the opening?

How much would an operator's profit have been if he had bought 1250 bushels of July corn at its lowest point and sold the same quantity at its highest point in the course of the day?

14. What is the total of the following bill?

4 pcs. Lonsdale cambric muslin, 53, 55^{2}*, 56^{1}, 54 ..... *** yd. at $10\frac{1}{2}$¢

5 pcs. 45-in. White French lawn, 35 yd. each ..... *** yd. at $12\frac{1}{2}$¢

4 pcs. 67-in. French organdie, $16\frac{3}{8}$ yd., each ..... *** yd. at 30¢

4 pcs. Falcon percale, 29, 28^{2}, 30^{1}, 28^{3} ..... *** yd. at 7¢

10 pcs. 36-in. handkerchief linen, sheer, 12 yd. each ..... *** yd. at $27\frac{1}{2}$¢

8 pcs. 32-in. Swiss mull, 24 yd. each ..... *** yd. at 15¢

* Quarter yards are indicated by the small raised figures written at the right.

15. On Sept. 4 Carson, Pirie, Scott & Co. sold to Strawbridge & Clothier the following items of dry goods:

6 pcs. Dependon LL brown sheeting, 50, 49^2, 51, 50^2, 49, 50. . *** yd. at $5\frac{1}{2}$¢

10 pcs. Crown cambric, 44^2, 43^3, 52, 50, 52^1, 51, 50, 50^2, 49^2, 46. . *** yd. at $4\frac{3}{4}$¢

20 gro. Farwell Brand needles, at $1.05

4 pcs. Berkeley 300 cambric muslin, 46, 44^2, 45, 43^3. . . . . *** yd. at $21\frac{1}{8}$¢

4 pcs. 6-in. Androscoggin pillow tubing, 41, 40^2, 39, 40^3. . . . . *** yd. at $14\frac{3}{8}$¢

6 pcs. Pequot 10-4 sheeting, brown, 40, 41^2, 41, 40^3, 46^1, 44. . . . *** yd. at $26\frac{1}{2}$¢

What was the total amount of the bill?

16. On Jan. 4, 1928, the Delamater Hardware Co., Detroit, Mich. sold to Isbister Bros., Atlanta, Ga.: 8 doz. heavy wire potato mashers @ 58¢; $\frac{3}{4}$ gr. No. 9, I. C. deep pie plates @ $5.00; $\frac{3}{4}$ gr. No. 10 I. C. deep pie plates @ $6.40; 2 gr. No. 7 I. C. shallow pie plates @ $2.95; 3 gr. No. 6 I. C. shallow pie plates @ $2.45; 6 doz. one sheet square pans @ 73¢; 5 doz. large bread pans @ $1.25; 3 doz. 10″ x 14″ drip pans @ 95¢; $2\frac{1}{2}$ doz. 10″ x 17″ drip pans @ $1.10. Terms: Net 30 days. Write an invoice.

17. $129\frac{3}{4} \div 26\frac{1}{3}$

18. $236\frac{5}{8} \div 40\frac{3}{4}$

19. $543\frac{4}{9} \div 36\frac{2}{3}$

20. $531\frac{1}{4} \div 26\frac{2}{9}$

21. $453\frac{3}{8} \div 36\frac{2}{3}$

22. $729\frac{1}{4} \div 62\frac{2}{9}$

23. $847\frac{3}{8} \div 64\frac{2}{7}$

24. $345\frac{6}{7} \div 42\frac{2}{3}$

25. $549\frac{3}{4} \div 82\frac{4}{9}$

Written Review Test

(Time, 30 minutes)

1. $150.3 \div 30$

2. $32.064 \div 28$

3. $1.975 \div 25$

4. $142.5 \div .125$

5. $601.545 \div .015$

6. $6\frac{3}{8} \div 7\frac{2}{3}$

7. $4\frac{5}{9} \div 8\frac{1}{4}$

8. $5\frac{2}{5} \div 6\frac{3}{4}$

9. $8\frac{7}{8} \div 3\frac{1}{3}$

10. $15\frac{3}{4} \div 2\frac{4}{7}$

11. $623\frac{3}{5} \div 4$

12. $1423\frac{1}{4} \div 8$

13. $321\frac{3}{4} \div 3$

14. $729\frac{3}{8} \div 5$

15. $654\frac{2}{3} \div 6$

16. $2874\frac{2}{3} \div 17\frac{1}{2}$

17. $349\frac{5}{6} \div 28\frac{1}{4}$

18. $476\frac{5}{7} \div 32\frac{1}{3}$

19. $593\frac{4}{9} \div 73\frac{3}{8}$

20. $1562\frac{2}{3} \div 84\frac{1}{4}$

21. $375\frac{3}{5} \div 43\frac{3}{5}$

22. $556\frac{1}{2} \div 26\frac{2}{3}$

23. $375\frac{1}{5} \div 44\frac{8}{15}$

24. $339\frac{1}{3} \div 71\frac{1}{2}$

25. $752\frac{3}{5} \div 32\frac{1}{8}$

26. $328\frac{1}{2} \div 28\frac{3}{4}$

27. $196\frac{3}{8} \div 16\frac{3}{8}$

28. $363\frac{5}{16} \div 3\frac{1}{4}$

29. $370\frac{11}{16} \div 31\frac{1}{6}$

30. $295\frac{1}{5} \div 9\frac{4}{10}$

Problems without Figures

1. If you know the sum of two fractions and know what one of them is, how can you find the other fraction?

2. If you know what part of a farm is planted in corn and how many acres are in corn, how can you find the number of acres in the farm?

3. If you know how many days it takes Henry to do a certain piece of work and how many days it takes John to do the same work, how can you find the amount they will both do in one day?

4. If you know how much each of three persons invests and know the profits of their business, how can you determine the amount of the profits each should have?

5. If the denominators of two fractions are the same but the numerators are different, which is the larger fraction?

6. If you multiply or divide both terms of a fraction by the same number, what effect does it have upon the value of the fraction?

7. If you multiply the numerator, only, by a certain number, what effect does it have upon the value of the fraction? If you multiply the denominator only, what effect does it have?

8. If the numerators of two fractions are the same but the denominators are not the same, which is the larger fraction?

9. If you know what part of an hour it takes for an automobile to travel a mile, how can you find the number of miles that it will travel in a certain number of hours?

10. If you add the same number to both numerator and denominator, what effect will it have on the value of the fraction? If you add to one and not to the other what effect will it have?

11. If you know the number of students in school today and what part of those enrolled are absent, how can you find the number of students enrolled?

12. If you know the number of days it takes for two men to plow a certain field and the number of days it would take for one of them to plow it, how can you find the number of days in which the other man ought to plow it?

13. If you subtract the same number from both the numerator and denominator of a fraction, what effect will it have upon the value of the fraction?

CHAPTER VII

ALIQUOT PARTS

An *aliquot part* of a number is a number which will divide it without a remainder. Thus, 2 is an aliquot part of 4; $2\frac{1}{2}$ is an aliquot part of 10; $33\frac{1}{3}$ is an aliquot part of 100; $166\frac{2}{3}$ is an aliquot part of 1000.

Aliquot parts of 100 are more commonly used than any other. Aliquot parts of \$1.00 form the basis of an important short-cut in multiplication that is much used by billing clerks. The following illustrative example will show its application.

Although any number which will divide another number an integral number of times is an aliquot part of it, the use of aliquot parts in arithmetic is usually confined to those mixed numbers which are themselves difficult to handle, but which when regarded as aliquot parts of 10, 100, or 1000 are easy to handle, such as $12\frac{1}{2}$, $16\frac{2}{3}$.

EXAMPLE. What is the cost of 184 gallons of oil at $12\frac{1}{2}¢$ per gal.?

$12\frac{1}{2}¢ = \frac{1}{8}$ of \$1.00.
184 gal. at \$1 cost \$184.
\$184 ÷ 8 = \$23. *Ans.*

Since $12\frac{1}{2}¢$ is $\frac{1}{8}$ of \$1.00, 184 gallons at $12\frac{1}{2}¢$ cost $\frac{1}{8}$ of what they would cost at \$1.00, or $\frac{1}{8}$ of \$184, which is \$23.

> Find the cost at \$1.00, and divide by the number of times that \$1.00 contains the price per article.

Aliquot Parts of 100

Memorize the following important aliquot parts of 100:

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{12}$	$\frac{1}{16}$
50	$33\frac{1}{3}$	25	20	$16\frac{2}{3}$	$12\frac{1}{2}$	10	$8\frac{1}{3}$	$6\frac{1}{4}$

Construct tables similar to the above, showing the important aliquot parts of 10 and 1000.

Halves, Fourths, Eighths, and Sixteenths

50¢ is what part of $1.00? When articles are purchased at 50¢ each, how does the number of dollars of cost compare with the number of articles?

Answer the same questions, changing the amount under consideration to 25¢; $12\frac{1}{2}$¢; $6\frac{1}{4}$¢.

ORAL. Find the cost of each of the following:

1. 20 lb. @ 50¢.	**10.** 18 ft. @ $12\frac{1}{2}$¢.	**19.** 60 yd. @ $6\frac{1}{4}$¢
2. 32 lb. @ 25¢.	**11.** 34 ft. @ $12\frac{1}{2}$¢.	**20.** 146 hr. @ $12\frac{1}{2}$¢.
3. 37 lb. @ 50¢.	**12.** 66 ft. @ $12\frac{1}{2}$¢.	**21.** 292 hr. @ $6\frac{1}{4}$¢.
4. 49 lb. @ 25¢.	**13.** 84 ft. @ $12\frac{1}{2}$¢.	**22.** 198 hr. @ $12\frac{1}{2}$¢.
5. 27 lb. @ 50¢.	**14.** 56 ft. @ $12\frac{1}{2}$¢.	**23.** 354 hr. @ $12\frac{1}{2}$¢.
6. 31 yd. @ 25¢.	**15.** 38 yd. @ $12\frac{1}{2}$¢.	**24.** 187 hr. @ 25¢.
7. 246 yd. @ 50¢.	**16.** 64 yd. @ $6\frac{1}{4}$¢.	**25.** 138 oz. @ $12\frac{1}{2}$¢.
8. 307 yd. @ 25¢.	**17.** 72 yd. @ $6\frac{1}{4}$¢.	**26.** 260 oz. @ $6\frac{1}{4}$¢.
9. 283 yd. @ 25¢.	**18.** 48 yd. @ $6\frac{1}{4}$¢.	**27.** 392 oz. @ 50¢.

Interchangeability of Price and Quantity

Price and quantity may be interchanged without affecting the result. Thus if a problem be stated "Find the cost of 17 yd. at 50¢" it may be solved as though it read, "Find the cost of 50 yd. at 17¢," for the answer would be the same in either case; *viz.* $8.50.

This agrees with the general principle of multiplication that changing the order of the factors does not affect the result.

ORAL. Interchanging price and quantity, find the cost of each of the following:

1. 50 yd. @ 19¢.	**9.** 25 bbl. @ $4.27.	**17.** $6\frac{1}{4}$ yd. @ 68¢.
2. 50 yd. @ 31¢.	**10.** 50 bbl. @ $3.84.	**18.** $12\frac{1}{2}$ ft. @ 74¢.
3. 25 yd. @ 37¢.	**11.** 25 bbl. @ $2.76.	**19.** $6\frac{1}{4}$ ft. @ 72¢.
4. 25 yd. @ 83¢.	**12.** 25 bbl. @ $9.83.	**20.** $12\frac{1}{2}$ ft. @ 86¢.
5. 25 lb. @ $1.10.	**13.** $12\frac{1}{2}$ yd. @ 14¢.	**21.** $6\frac{1}{4}$ ft. @ 92¢.
6. 25 lb. @ $1.42.	**14.** $12\frac{1}{2}$ yd. @ 28¢.	**22.** $6\frac{1}{4}$ gr. @ $1.12.
7. 50 lb. @ $1.60.	**15.** $12\frac{1}{2}$ yd. @ 24¢.	**23.** $6\frac{1}{4}$ gr. @ $1.60.
8. 50 lb. @ $2.37.	**16.** $6\frac{1}{4}$ yd. @ 36¢.	**24.** $12\frac{1}{4}$ gr. @ $2.80.

Thirds, Sixths, and Twelfths

$33\frac{1}{3}¢$ is what part of \$1.00? When articles are purchased at $33\frac{1}{3}¢$ each, how does the number of dollars of cost compare with the number of articles?

Answer the same questions, changing the amount under consideration to $16\frac{2}{3}¢$; $8\frac{1}{3}¢$.

ORAL. Find the cost of the following:

1. 375 yd. @ $33\frac{1}{3}¢$
2. 459 yd. @ $33\frac{1}{3}¢$
3. 738 yd. @ $33\frac{1}{3}¢$
4. 522 ft. @ $16\frac{2}{3}¢$
5. 744 ft. @ $16\frac{2}{3}¢$
6. 306 ft. @ $16\frac{2}{3}¢$
7. 336 gal. @ $8\frac{1}{3}¢$
8. 864 gal. @ $8\frac{1}{3}¢$
9. 768 gal. @ $8\frac{1}{3}¢$
10. 261 yd. @ $33\frac{1}{3}¢$
11. 756 ft. @ $16\frac{2}{3}¢$
12. 432 gal. @ $8\frac{1}{3}¢$
13. 639 yd. @ $33\frac{1}{3}¢$
14. 474 ft. @ $16\frac{2}{3}¢$
15. 448 gal. @ $8\frac{1}{3}¢$
16. 384 yd. @ $33\frac{1}{3}¢$
17. 576 ft. @ $16\frac{2}{3}¢$
18. 784 gal. @ $8\frac{1}{3}¢$

ORAL. Interchanging price and quantity when desirable, find the cost of each of the following:

1. $33\frac{1}{3}$ yd. @ 45¢
2. $33\frac{1}{3}$ yd. @ 27¢
3. 54 yd. @ $16\frac{2}{3}¢$
4. 56 yd. @ $8\frac{1}{3}¢$
5. $16\frac{2}{3}$ yd. @ 96¢
6. $33\frac{1}{3}$ yd. @ 54¢
7. $16\frac{2}{3}$ bu. @ 84¢
8. $16\frac{2}{3}$ bu. @ \$1.12
9. 54 bu. @ $8\frac{1}{3}¢$
10. 711 bu. @ $33\frac{1}{3}¢$
11. 50 bu. @ \$1.21
12. 891 bu. @ 25¢
13. $8\frac{1}{3}$ gal. @ \$1.08
14. $8\frac{1}{3}$ gal. @ 27¢
15. 345 gal. @ $33\frac{1}{3}¢$
16. 696 gal. @ $16\frac{2}{3}¢$
17. $12\frac{1}{2}$ gal. @ 58¢
18. 184 gal. @ $6\frac{1}{4}¢$
19. $6\frac{1}{4}$ ft. @ 48¢
20. $12\frac{1}{2}$ gr. @ 94¢
21. 25 gr. @ \$1.47
22. $12\frac{1}{2}$ yd. @ \$1.18
23. 50 yd. @ \$1.63
24. $12\frac{1}{2}$ yd. @ \$2.80

WRITTEN. Using aliquot parts of 100, perform the following multiplications:

1. 50×53791
2. $33\frac{1}{3}\times4623$
3. 25×1209
4. $16\frac{2}{3}\times3864$
5. $8\frac{1}{3}\times2688$
6. $6\frac{1}{4}\times3732$
7. $33\frac{1}{3}\times234$
8. 25×753
9. $6\frac{1}{4}\times324$
10. $16\frac{2}{3}\times468$
11. 25×509
12. 50×72935
13. $16\frac{2}{3}\times936$
14. $252\times16\frac{2}{3}$
15. 5709×25
16. $4653\times33\frac{1}{3}$
17. 5916×50
18. $12\frac{1}{2}\times3468$
19. $5427\times6\frac{1}{4}$
20. $33\frac{1}{3}\times3648$
21. $6\frac{1}{4}\times2019$
22. $3427\times12\frac{1}{2}$
23. $5372\times16\frac{2}{3}$
24. $12\frac{1}{2}\times8493$

WRITTEN. Using aliquot parts of 10, 100, or 1000, perform the following multiplications:

1. $2\frac{1}{2}\times484$
2. $3\frac{1}{3}\times375$
3. $16\frac{2}{3}\times936$
4. $12\frac{1}{2}\times472$
5. $333\frac{1}{3}\times9$
6. 125×824
7. $166\frac{2}{3}\times312$
8. $16\frac{2}{3}\times3120$
9. $753\times3\frac{1}{3}$
10. $456\times16\frac{2}{3}$
11. $382\times2\frac{1}{2}$
12. $1868\times6\frac{1}{4}$
13. $12\frac{1}{2}\times742$
14. $33\frac{1}{3}\times3129$
15. $9138\times16\frac{2}{3}$
16. 4979×50
17. $934\times12\frac{1}{2}$
18. $8\frac{1}{3}\times6096$
19. $228\times166\frac{2}{3}$
20. 500×379
21. $12\frac{1}{2}\times476$
22. 57×250
23. $83\frac{1}{3}\times76$
24. 125×341

To Multiply by a Multiple of an Aliquot Part

Note the following multiplication tables. Memorize the tables of 8ths and 6ths.

Multiplication Tables

Halves		4ths		8ths		16ths	
						1	$6\frac{1}{4}$
				1	$12\frac{1}{2}$	2	$12\frac{1}{2}$
						3	$18\frac{3}{4}$
		1	25	2	25	4	25
						5	$31\frac{1}{4}$
				3	$37\frac{1}{2}$	6	$37\frac{1}{2}$
						7	$43\frac{3}{4}$
1	50	2	50	4	50	8	50
						9	$56\frac{1}{4}$
				5	$62\frac{1}{2}$	10	$62\frac{1}{2}$
						11	$68\frac{3}{4}$
		3	75	6	75	12	75
						13	$81\frac{1}{4}$
				7	$87\frac{1}{2}$	14	$87\frac{1}{2}$
						15	$93\frac{3}{4}$
2	100	4	100	8	100	16	100

Thirds		6ths		12ths	
				1	$8\frac{1}{3}$
		1	$16\frac{2}{3}$	2	$16\frac{2}{3}$
				3	25
1	$33\frac{1}{3}$	2	$33\frac{1}{3}$	4	$33\frac{1}{3}$
				5	$41\frac{2}{3}$
		3	50	6	50
				7	$58\frac{1}{3}$
2	$66\frac{2}{3}$	4	$66\frac{2}{3}$	8	$66\frac{2}{3}$
				9	75
		5	$83\frac{1}{3}$	10	$83\frac{1}{3}$
				11	$91\frac{2}{3}$
3	100	6	100	12	100

Construct comparative tables similar to these, showing the aliquot parts of 10 and 1000.

The following example illustrates a short method of multiplication which involves the use of the aliquot parts multiplication tables.

EXAMPLE. $153 \times 66\frac{2}{3} = ?$

$153 \times 100 = 15300$
$66\frac{2}{3} = \frac{2}{3}$ of 100
$\frac{2}{3}$ of $15300 = 10200$

By the tables it is seen that $66\frac{2}{3}$ is $\frac{2}{3}$ of 100. To find $66\frac{2}{3}$ times any number, add two ciphers, which is equivalent to multiplying the number by 100, and take $\frac{2}{3}$ of the resulting amount.

> Find the cost at \$1.00, divide by the number of times that \$1.00 contains the aliquot unit, and multiply by the number of times that the price contains the aliquot unit.

Construct tables similar to the foregoing, showing the fractional parts of 10 and 1000 based upon the principal aliquot parts.

ORAL

1. What fractional part of 100 is 75? $37\frac{1}{2}$? $62\frac{1}{2}$? $87\frac{1}{2}$?

2. What fractional part of 100 is $18\frac{3}{4}$? $31\frac{1}{4}$? $43\frac{3}{4}$? $56\frac{1}{4}$? $68\frac{3}{4}$? $81\frac{1}{4}$? $93\frac{3}{4}$?

3. Count by 250 to 1000; by 25 to 100; by $2\frac{1}{2}$ to 10; by $3\frac{1}{3}$ to100; by $333\frac{1}{3}$ to 1000.

4. Count to 100 by $8\frac{1}{3}$; by $16\frac{2}{3}$; by $12\frac{1}{2}$; by $6\frac{1}{4}$.

5. 12×75	12. $48 \times 68\frac{3}{4}$	19. $30 \times 37\frac{1}{2}$	26. $44 \times 62\frac{1}{2}$
6. $64 \times 37\frac{1}{2}$	13. $32 \times 81\frac{1}{4}$	20. $40 \times 18\frac{3}{4}$	27. $40 \times 93\frac{3}{4}$
7. $50 \times 62\frac{1}{2}$	14. $64 \times 93\frac{3}{4}$	21. 30×75	28. $33 \times 83\frac{1}{3}$
8. $60 \times 87\frac{1}{2}$	15. $96 \times 66\frac{2}{3}$	22. $21 \times 66\frac{2}{3}$	29. $28 \times 87\frac{1}{2}$
9. $48 \times 18\frac{3}{4}$	16. $42 \times 83\frac{1}{3}$	23. $42 \times 41\frac{2}{3}$	30. $24 \times 43\frac{3}{4}$
10. $32 \times 31\frac{1}{4}$	17. $36 \times 41\frac{2}{3}$	24. $40 \times 31\frac{1}{4}$	31. $56 \times 68\frac{3}{4}$
11. $16 \times 43\frac{3}{4}$	18. $24 \times 58\frac{1}{3}$	25. $56 \times 56\frac{1}{4}$	32. $36 \times 91\frac{2}{3}$

WRITTEN

1. 423 yd. at 75¢	5. 710 yd. at $18\frac{3}{4}$¢	9. 192 yd. at $68\frac{3}{4}$¢
2. 562 yd. at $37\frac{1}{2}$¢	6. 518 yd. at $31\frac{1}{4}$¢	10. 320 yd. at $81\frac{1}{4}$¢
3. 784 yd. at $62\frac{1}{2}$¢	7. 810 yd. at $43\frac{3}{4}$¢	11. 547 bu. at $66\frac{2}{3}$¢
4. 609 yd. at $87\frac{1}{2}$¢	8. 239 yd. at $56\frac{1}{4}$¢	12. 508 bu. at $83\frac{1}{3}$¢

13. 169 bu. at $41\frac{2}{3}$¢
14. 267 bu. at $58\frac{1}{3}$¢
15. 384 bu. at $91\frac{2}{3}$¢
16. 153 bu. at $62\frac{1}{2}$¢
17. 249 bu. at $43\frac{3}{4}$¢
18. 367 bu. at $68\frac{3}{4}$¢
19. 450 bu. at $66\frac{2}{3}$¢
20. 878 bu. at 75¢
21. 786 gal. at $41\frac{2}{3}$¢
22. 962 gal. at $87\frac{1}{2}$¢
23. 191 gal. at $37\frac{1}{2}$¢
24. 856 gal. at $18\frac{3}{4}$¢
25. 960 gal. at $58\frac{1}{3}$¢
26. 325 gal. at $56\frac{1}{4}$¢
27. 274 gal. at $81\frac{1}{4}$¢
28. 432 gal. at $37\frac{1}{2}$¢
29. 503 gal. at $83\frac{1}{3}$¢
30. 747 gal. at $31\frac{1}{4}$¢

WRITTEN. Using aliquot parts of 10, 100, and 1000, perform the following multiplications:

1. $327\times2\frac{1}{2}$
2. $535\times3\frac{1}{3}$
3. $192\times33\frac{1}{3}$
4. $251\times12\frac{1}{2}$
5. 474×125
6. $640\times333\frac{1}{3}$
7. $183\times87\frac{1}{2}$
8. 706×625
9. $918\times83\frac{1}{3}$
10. 829×375
11. $492\times7\frac{1}{2}$
12. 236×250
13. $384\times6\frac{2}{3}$
14. $518\times62\frac{1}{2}$
15. 863×875
16. $649\times666\frac{2}{3}$
17. 372×625
18. $471\times8\frac{1}{3}$
19. $595\times6\frac{2}{3}$
20. $660\times166\frac{2}{3}$
21. $707\times6\frac{1}{4}$
22. $253\times333\frac{1}{3}$
23. 915×750
24. $824\times3\frac{1}{3}$

Compounding Aliquot Parts

Many calculations are shortened by compounding aliquot parts, that is, adding two or more simple aliquot parts instead of using the aliquot parts multiplication table. Multiply 438 by $37\frac{1}{2}$.

(*a*)	(*b*)
$\frac{1}{4}$ of 43800 = 10950	$\frac{1}{4}$ of 43800 = 10950
$\frac{1}{8}$ of 43800 = 5475	$\frac{1}{2}$ of 10950 = 5475
$\frac{3}{8}$ of 43800 = 16425	16425

Instead of treating $37\frac{1}{2}$ as $\frac{3}{8}$ of 100, it is treated as $\frac{1}{4}$ of 100 plus $\frac{1}{8}$ of 100 (Method *a*) or as $\frac{1}{4}$ of 100 plus $\frac{1}{2}$ of $\frac{1}{4}$ of 100 (Method *b*).

Sometimes it will be easier to subtract than to add.

EXAMPLE. Multiply 438 by $87\frac{1}{2}$.

43800
5475
38325

In this case $87\frac{1}{2}$ is treated as $100-12\frac{1}{2}$ instead of $\frac{7}{8}$ of 100 or $50+25+12\frac{1}{2}$. From 100×438 subtract $\frac{1}{8}$ of itself.

An inspection of the multiplication tables of aliquot parts will show that halves, fourths, eighths, and sixteenths are readily compounded with each other. It will also show that thirds, sixths, and twelfths are readily compounded with each other.

ORAL.

1. What two aliquot parts of 100, added, make 75? $37\frac{1}{2}$? $62\frac{1}{2}$? $18\frac{3}{4}$? $31\frac{1}{4}$? $56\frac{1}{4}$?

2. What three aliquot parts of 100, added, make $87\frac{1}{2}$? $43\frac{3}{4}$? $68\frac{3}{4}$?

3. What four aliquot parts of 100, added, make $93\frac{3}{4}$?

4. What two aliquot parts of 100, added, make $66\frac{2}{3}$? $83\frac{1}{3}$? $41\frac{2}{3}$? $58\frac{1}{3}$?

5. What aliquot part of 100, plus what aliquot part of itself, equals 75? (Answer either "$\frac{1}{2}$ of 100, plus $\frac{1}{2}$ of itself," or "$50+\frac{1}{2}$ of 50").

6. What aliquot part of 100, plus what aliquot part of itself, equals $37\frac{1}{2}$? $62\frac{1}{2}$? $18\frac{3}{4}$? $31\frac{1}{4}$? $56\frac{1}{4}$? $58\frac{1}{3}$?

7. What aliquot part of 100 must be subtracted from 100 to give $87\frac{1}{2}$? $66\frac{2}{3}$? $93\frac{3}{4}$? $91\frac{2}{3}$?

8. What aliquot part of 100, minus what aliquot part of itself, gives $18\frac{3}{4}$? $37\frac{1}{2}$? $41\frac{2}{3}$? $43\frac{3}{4}$?

9. What, in your judgment, is the best way to treat 75 as a multiplier? (In answering, say either "$\frac{3}{4}$ of 100," "$\frac{1}{2}$ of 100 plus $\frac{1}{4}$ of 100," or "$\frac{1}{2}$ of 100 plus $\frac{1}{2}$ of itself," whichever appears to you to be best.)

10. What is the best way to treat as a multiplier:

$37\frac{1}{2}$	$87\frac{1}{2}$	$18\frac{3}{4}$	$43\frac{3}{4}$	$63\frac{3}{4}$	$93\frac{3}{4}$	$83\frac{1}{3}$	$58\frac{1}{3}$
$62\frac{1}{2}$	75	$31\frac{1}{4}$	$56\frac{1}{4}$	$81\frac{1}{4}$	$66\frac{2}{3}$	$41\frac{2}{3}$	$91\frac{2}{3}$

WRITTEN. Using the suggestions above, find the total cost of the following:

1. 29 doz. @ $37\frac{1}{2}¢$ ($25¢+12\frac{1}{2}¢$)
2. 47 doz. @ $62\frac{1}{2}¢$ ($50¢+12\frac{1}{2}¢$)
3. 63 doz. @ $87\frac{1}{2}¢$ ($\$1.00-12\frac{1}{2}¢$)
4. 54 doz. @ 75¢ (50¢+25¢)
5. 67 doz. @ $\$1.87\frac{1}{2}$ ($\$2.00-12\frac{1}{2}¢$)
6. 39 doz. @ $\$1.12\frac{1}{2}$
7. 46 doz. @ $\$1.37\frac{1}{2}$
8. 71 doz. @ $\$1.62\frac{1}{2}$
9. 95 doz. @ $1.25
10. 78 doz. @ $1.50

11. 114 gross @ 50¢
12. 139 gross @ $62\frac{1}{2}$¢
13. 500 gross @ $37\frac{1}{2}$¢
14. 117 gross @ $87\frac{1}{2}$¢
15. 137 gross @ $\$1.12\frac{1}{2}$
16. 153 gross @ $\$1.62\frac{1}{2}$
17. 161 gross @ $1.75
18. 285 gross @ $\$1.87\frac{1}{2}$
19. 346 gross @ $1.25
20. 187 gross @ $\$1.37\frac{1}{2}$

Interchanging price and quantity, find the total cost:

1. 75 ft. @ 42¢
2. 50 ft. @ 38¢
3. $62\frac{1}{2}$ ft. @ 29¢
4. $12\frac{1}{2}$ ft. @ 60¢
5. $37\frac{1}{2}$ ft. @ 73¢
6. $87\frac{1}{2}$ ft. @ 47¢
7. 25 yd. @ $1.28
8. $37\frac{1}{2}$ yd. @ 94¢
9. $87\frac{1}{2}$ yd. @ $1.46
10. $62\frac{1}{2}$ yd. @ $1.08
11. $112\frac{1}{2}$ yd. @ $1.12
12. $137\frac{1}{2}$ yd. @ $1.45

WRITTEN. Find the cost of the following groups to the nearest cent on each item. If the fraction in any product is $\frac{1}{2}$¢ or more, figure it as 1¢.

1. 54 lb. @ $16\frac{2}{3}$¢	2. 45 doz. @ $83\frac{1}{3}$¢	3. 58 gal. @ 75¢
60 lb. @ $8\frac{1}{3}$¢	84 doz. @ $58\frac{1}{3}$¢	48 gal. @ $37\frac{1}{2}$¢
75 lb. @ $33\frac{1}{3}$¢	57 doz. @ $66\frac{2}{3}$¢	64 gal. @ $62\frac{1}{2}$¢
30 lb. @ $66\frac{2}{3}$¢	56 doz. @ $16\frac{2}{3}$¢	56 gal. @ $87\frac{1}{2}$¢
4. 54 yd. @ $13\frac{1}{3}$¢	5. 52 yd. @ $58\frac{1}{3}$¢	6. 47 bu. @ $18\frac{3}{4}$¢
90 yd. @ $41\frac{2}{3}$¢	68 yd. @ $87\frac{1}{2}$¢	101 bu. @ $87\frac{1}{2}$¢
36 yd. @ $91\frac{2}{3}$¢	122 yd. @ 75¢	56 bu. @ $56\frac{1}{4}$¢
64 yd. @ $83\frac{1}{3}$¢	95 yd. @ 55¢	92 bu. @ $66\frac{2}{3}$¢
7. 182 yd. @ $18\frac{3}{4}$¢	8. 32 bu. @ $68\frac{3}{4}$¢	9. 126 qt. @ $83\frac{1}{3}$¢
76 yd. @ $31\frac{1}{4}$¢	96 bu. @ $81\frac{1}{4}$¢	96 qt. @ $41\frac{2}{3}$¢
84 yd. @ $43\frac{3}{4}$¢	48 bu. @ $93\frac{3}{4}$¢	58 qt. @ $58\frac{1}{3}$¢
38 yd. @ $56\frac{1}{4}$¢	54 bu. @ $66\frac{2}{3}$¢	75 qt. @ $91\frac{2}{3}$¢
10. 83 pt. @ $31\frac{1}{4}$¢	11. 63 ft. @ $43\frac{3}{4}$¢	12. 36 oz. @ $62\frac{1}{3}$¢
54 pt. @ 75¢	74 ft. @ $81\frac{1}{4}$¢	91 oz. @ $93\frac{3}{4}$¢
65 pt. @ $68\frac{3}{4}$¢	92 ft. @ $37\frac{1}{2}$¢	83 oz. @ $58\frac{1}{3}$¢
72 pt. @ $83\frac{1}{3}$¢	45 ft. @ $41\frac{2}{3}$¢	27 oz. @ $91\frac{2}{3}$¢

To Divide by an Aliquot Part

EXAMPLE. Divide 81625 by $12\frac{1}{2}$.

$81625 \div 100 = 816.25$
$8 \times 816.25 = 6530$

$12\frac{1}{2}$ is $\frac{1}{8}$ of 100. The result of dividing by $12\frac{1}{2}$ is therefore 8 times as great as the result of dividing by 100.

> To divide by a number which is an aliquot part of 100, point off two places and multiply by the number which represents the number of times the divisor is contained in 100.

The rule for dividing by an aliquot part of 10, or of 1000, or any other number which consists of 1 followed by ciphers, is similar to the above, but the number of places pointed off differs.

ORAL

1. $125 \div 12\frac{1}{2}$
2. $275 \div 25$
3. $850 \div 50$
4. $400 \div 16\frac{2}{3}$
5. $200 \div 33\frac{1}{3}$
6. $250 \div 6\frac{1}{4}$
7. $350 \div 8\frac{1}{3}$
8. $425 \div 25$
9. $275 \div 12\frac{1}{2}$
10. $625 \div 50$
11. $550 \div 16\frac{2}{3}$
12. $650 \div 25$
13. $325 \div 6\frac{1}{4}$
14. $250 \div 8\frac{1}{3}$
15. $325 \div 12\frac{1}{2}$
16. $450 \div 16\frac{2}{3}$
17. $700 \div 33\frac{1}{3}$
18. $375 \div 6\frac{1}{4}$

WRITTEN

1. $1375 \div 50$
2. $1425 \div 12\frac{1}{2}$
3. $2700 \div 33\frac{1}{3}$
4. $3450 \div 6\frac{1}{4}$
5. $1825 \div 25$
6. $2750 \div 16\frac{2}{3}$
7. $3233\frac{1}{3} \div 33\frac{1}{3}$
8. $2750 \div 25$
9. $1725 \div 6\frac{1}{4}$
10. $1775 \div 50$
11. $1866\frac{2}{3} \div 16\frac{2}{3}$
12. $2575 \div 12\frac{1}{2}$
13. $1450 \div 8\frac{1}{3}$
14. $3925 \div 6\frac{1}{4}$
15. $2750 \div 8\frac{1}{3}$
16. $3225 \div 12\frac{1}{2}$
17. $1775 \div 6\frac{1}{4}$
18. $4366\frac{2}{3} \div 33\frac{1}{3}$
19. $370 \div 2\frac{1}{2}$
20. $495 \div 3\frac{1}{3}$
21. $1350 \div 33\frac{1}{3}$
22. $775 \div 12\frac{1}{2}$
23. $535 \div 2\frac{1}{2}$
24. $2475 \div 333\frac{1}{3}$
25. $275 \div 16\frac{2}{3}$
26. $425 \div 12\frac{1}{2}$
27. $351 \div 3\frac{1}{3}$
28. $865 \div 16\frac{2}{3}$
29. $1375 \div 8\frac{1}{3}$
30. $1625 \div 12\frac{1}{2}$
31. $3950 \div 333\frac{1}{3}$
32. $850 \div 16\frac{2}{3}$
33. $375 \div 6\frac{1}{4}$

34. How many yards can be bought in each of the following cases, the amount to be spent and the price per yard being given? Express results as mixed numbers.

Amount	Price	Amount	Price
\$87.50	$6\frac{1}{4}¢$	\$27.75	$8\frac{1}{3}¢$
36.50	$8\frac{1}{3}¢$	39.60	$33\frac{1}{3}¢$
49.25	$6\frac{1}{4}¢$	6.50	$16\frac{2}{3}¢$
83.50	$16\frac{2}{3}¢$	22.25	$8\frac{1}{3}¢$
24.00	$33\frac{1}{3}¢$	7.50	$6\frac{1}{4}¢$

35. How many whole articles can be bought, and what will be the remainder in each case, amounts of money available and prices being as shown:

Amount	Price	Amount	Price
\$27.64	$6\frac{1}{4}¢$	\$12.34	25¢
35.89	$8\frac{1}{3}¢$	56.78	$8\frac{1}{3}¢$
16.07	$16\frac{2}{3}¢$	9.05	$6\frac{1}{4}¢$
17.52	25¢	21.48	$33\frac{1}{3}¢$
26.43	$33\frac{1}{3}¢$	76.54	$16\frac{2}{3}¢$

To Divide by a Multiple of an Aliquot Part

Example. Divide 81675 by $37\frac{1}{2}$.

$81675 \div 100 = 816.75$
$816.75 \times 8 = 6534$
$6534 \div 3 = 2178$. *Ans.*

First divide by $12\frac{1}{2}$, following the method previously shown. Since $37\frac{1}{2}$ is $3 \times 12\frac{1}{2}$, it is contained in a given number $\frac{1}{3}$ as many times as $12\frac{1}{2}$. Divide by 3.

Oral

1. $1200 \div 75$
2. $2700 \div 37\frac{1}{2}$
3. $1500 \div 62\frac{1}{2}$
4. $2100 \div 87\frac{1}{2}$
5. $600 \div 18\frac{3}{4}$
6. $2500 \div 31\frac{1}{4}$
7. $4900 \div 43\frac{3}{4}$
8. $8100 \div 56\frac{1}{4}$
9. $2200 \div 68\frac{3}{4}$
10. $3900 \div 81\frac{1}{4}$
11. $7500 \div 93\frac{3}{4}$
12. $900 \div 66\frac{2}{3}$
13. $3550 \div 83\frac{1}{3}$
14. $2500 \div 41\frac{2}{3}$
15. $4200 \div 58\frac{1}{3}$
16. $1210 \div 91\frac{2}{3}$
17. $3500 \div 62\frac{1}{2}$
18. $2828 \div 87\frac{1}{2}$

WRITTEN

1. $3167 \div 37\frac{1}{2}$	7. $3146 \div 43\frac{3}{4}$	13. $6235 \div 83\frac{1}{3}$
2. $2485 \div 56\frac{1}{4}$	8. $8257 \div 75$	14. $3285 \div 91\frac{2}{3}$
3. $5093 \div 18\frac{3}{4}$	9. $2901 \div 93\frac{3}{4}$	15. $9127 \div 41\frac{2}{3}$
4. $4168 \div 68\frac{3}{4}$	10. $3468 \div 87\frac{1}{2}$	16. $6854 \div 62\frac{1}{2}$
5. $7529 \div 62\frac{1}{2}$	11. $3914 \div 31\frac{1}{4}$	17. $7205 \div 87\frac{1}{2}$
6. $1323 \div 81\frac{1}{4}$	12. $5674 \div 66\frac{2}{3}$	18. $5915 \div 58\frac{1}{3}$

Applications of Aliquot Parts to Commercial Transactions

Commodities sold by the ton (2000 lb.) or by the thousand.

EXAMPLE. Find the cost of 1750 ft. of hemlock sheathing at $21.50 per M.

ORDINARY METHOD	SHORT METHOD
$21.50	$21.50 (cost 1 M)
1.750	5.375 (cost $\frac{1}{4}$ M)
107500	16.125 (cost $\frac{3}{4}$ M)
15050	$37.625 (cost $1\frac{3}{4}$ M)
2150	
$37.62500	

EXPLANATION OF THE SHORT METHOD. The advantage of the short method is that much can be done mentally without writing. It is at once apparent that $1750 = 1\frac{3}{4}$ M. The cost of $\frac{1}{4}$ M ($5.375) is written just below $21.50 and subtracted, giving $16.125 as the cost of $\frac{3}{4}$ M. To this $21.50 is added without rewriting it, giving a total of $37.63 for the cost of $1\frac{3}{4}$ M. Note that the number of figures written by the short method is only 18, as against 30 by the ordinary method.

A still shorter solution, but one which would not apply to every problem so easily as it does to this one, is as follows:

$43	It is at once apparent that 1750 ft. = 2M ft. less $\frac{1}{8}$ of itself.
5.37	Multiply $21.50 by 2 mentally and write the result, $43.
$37.63	Then deduct from $43 one-eighth of itself.

NOTE. It is impossible to give arbitrary rules to cover all the various possible applications of aliquot parts in time-saving methods. The alert-minded student will be able to make use of many devices, of which the illustrations here shown are only suggestive. Two students, equally alert mentally, might solve such a problem as the above in different ways, both good.

EXAMPLE. Find the cost of 2500 lb. anthracite coal at $7.25 per ton.

$$\begin{array}{r} \$7.25 \\ 1.81 \\ \hline \$9.06 \end{array}$$

It is at once evident that 500 is $\frac{1}{4}$ of one ton. It is only necessary to add to the price of 1 ton one-fourth of itself.

WRITTEN. Find the cost of:

1. 3500 lb. @ $7.50 per T.
2. 2750 ft. @ $14.40 per M.
3. 2250 lb. @ $8.50 per T.
4. 3250 ft. @ $17.50 per M.
5. 3750 lb. @ $4.60 per T.
6. 2500 ft. @ $22.60 per M.
7. 2250 ft. @ $17.60 per M.
8. 4500 lb. @ $3.80 per T.
9. 3750 ft. @ $18.80 per M.
10. 3250 lb. @ $6.80 per T.
11. 2750 ft. @ $17.50 per M.
12. 2750 lb. @ $7.20 per T.

13.

2250 ft. siding @ $38.50 per M.
2250 shingles @ 4.25 per M.
2950 ft. flooring @ 62.50 per M.
2750 plaster lath @ 4.36 per M.

14.

2875 lb. coal @ $8.25 per T.
2740 lb. plaster @ 8.00 per T.
3750 lb. hay @ 19.25 per T.
2250 lb. feed @ 21.50 per T.

15.

750 ft. siding @ $38.40 per M.
1250 shingles @ 4.50 per M.
1500 ft. flooring @ 44.60 per M.
1750 ft. sheathing @ 29.60 per M.

16.

4250 lb. straw @ $6.40 per T.
1750 lb. feed @ 21.75 per T.
3750 lb. hay @ 18.50 per T.
3500 lb. coal @ 5.40 per T.

17.

750 lb. at $1.35 per C *
625 lb. at 1.26 per C.
375 lb. at .72 per C.
112$\frac{1}{2}$ lb. at 2.16 per C.

18.

1275 lb. at 63¢ per C.
1137$\frac{1}{2}$ lb. at 72¢ per C.
1350 lb. at 85¢ per C.
2262$\frac{1}{2}$ lb. at 96¢ per C.

19.

875 cards at 2.12\frac{1}{2}$ per C.
350 cards at 1.75 per C.
425 cards at 1.28 per C.
575 cards at 1.32 per C.

20.

750 folders at $3.25 per C.
375 folders at 2.60 per C.
425 folders at 1.48 per C.
650 folders at 2.75 per C.

*When the cost unit is 100, the method is so similar to the method employed when the cost unit is 1000 that no separate explanation is necessary.

Miscellaneous Aliquot Parts

Aliquot parts may be based upon other numbers than 10, 100, or 1000. For instance, when the price unit is a dozen or a gross, short methods of multiplication may be used which involve aliquot parts of 12 or 144. Similarly when the unit of measurement of distance is 320 rods (1 mile), or the unit of measurement of time is 60 days, short methods of multiplication may be used involving the use of aliquot parts of 320 and of 60.

There are many cases in which aliquot parts can be used besides those in which the unit of cost or of measurement is 10, 100, 1000, 12, 144, 320, or 60. These seven cases, however, sufficiently indicate the general principle.

ORAL

1. What part of 12 is 6? 9? 3? 1? 7? 10? 2? 4? 5? 8? 11?

2. What part of 1 gross is 1 doz.? 3 doz.? 6 doz.? 9 doz.? 2 doz.? 10 doz.? 8 doz.? 4 doz.? 5 doz.? 7 doz.? 11 doz.?

3. What part of 144 is 72? 36? 108? 12? 84? 120? 24? 48? 60? 96? 132?

4. What part of 320 rods is 160 rods? 40 rods? 80 rods? 200 rods? 120 rods? 240 rods?

5. What part of 60 days is 30 days? 15 days? 10 days? 6 days? 12 days? 45 days? 50 days?

6. What part of 6 days is 3 days? 2 days? 1 day? 4 days?

7. What two aliquot parts of 60 days, added, make 45 days? 40 days? 21 days? 36 days?

8. What aliquot part of 60 days, plus what aliquot part of itself, makes 33 days? 22 days? 45 days? 24 days?

9. How many times 60 days is 180 days? 240 days? 90 days? 210 days? 75 days?

10. What aliquot part of 60 must be deducted from 60 to leave 50? 45? 54? 48? 40?

EXAMPLE. Find the cost of 108 articles at $17.60 a gross.

(*a*)	(*b*)
$17.60	$8.80
4.40	4.40
$13.20	$13.20

It is at once apparent that $108 = 9$ doz. or $\frac{3}{4}$ gro. $\frac{3}{4}$ can be considered either as $1 - \frac{1}{4}$ or as $\frac{1}{2} + \frac{1}{4}$, so that either of the two solutions shown will result.

EXAMPLE. If the use of a certain amount of money costs $4.60 every 60 days, what will it cost in 105 days?

$4.60	105 = 60 + 30 + 15
2.30	The cost for 60 days = $4.60.
1.15	The cost for 30 days = $\frac{1}{2}$ of $4.60 = $2.30
$8.05	The cost for 15 days = $\frac{1}{2}$ of $2.30 (or $\frac{1}{4}$ of $4.60) = $1.15.
	$4.60 + $2.30 + $1.15 = $8.05.

WRITTEN

1. Express the following in miles, decimally to 3 places, and add: 240 rods, 120 rods, 160 rods, 80 rods, 60 rods, 100 rods, 180 rods, 140 rods, 300 rods, 220 rods, 260 rods, 280 rods.

2.	5 doz.	@ $9.35 per gro.	**3.**	11 doz.	@ $32.50 per gro.
	7 doz.	@ 10.40 per gro.		6 doz.	@ 27.00 per gro.
	3 doz.	@ 12.65 per gro.		9 doz.	@ 31.25 per gro.
	8 doz.	@ 11.70 per gro.		4 doz.	@ 32.60 per gro.
	10 doz.	@ 9.50 per gro.		2 doz.	@ 30.00 per gro.
4.	72 art.	@ $8.40 per gro.	**5.**	24 art.	@ $26.50 per gro.
	36 art.	@ 7.25 per gro.		60 art.	@ 27.50 per gro.
	108 art.	@ 9.50 per gro.		48 art.	@ 31.60 per gro.
	84 art.	@ 12.50 per gro.		96 art.	@ 25.00 per gro.
	120 art.	@ 11.20 per gro.		132 art.	@ 28.80 per gro.

6. If the use of money costs $5.40 for 60 days, what does it cost

(*a*) 90 days	(*d*) 75 days	(*g*) 12 days
(*b*) 30 days	(*e*) 105 days	(*h*) 18 days
(*c*) 45 days	(*f*) 6 days	(*i*) 24 days

7. If the use of money costs $2.34 for 60 days, what does it cost

(*a*) 20 days	(*d*) 120 days	(*g*) 48 days
(*b*) 40 days	(*e*) 50 days	(*h*) 72 days
(*c*) 80 days	(*f*) 70 days	(*i*) 100 days

8. If the use of money costs $3.50 for 60 days, what does it cost

(*a*) 120 days	(*d*) 150 days	(*g*) 186 days
(*b*) 180 days	(*e*) 192 days	(*h*) 189 days
(*c*) 240 days	(*f*) 255 days	(*i*) 190 days

Find the cost of the loan in each of the following problems, the cost for 60 days being as shown:

No. of Days	Interest for 60 Days	No. of Days	Interest for 60 Days
9. 30	$2.50	19. 75	$1.75
10. 90	3.24	20. 63	2.14
11. 135	4.40	21. 72	3.56
12. 6	3.70	22. 62	4.50
13. 9	4.80	23. 58	1.29
14. 8	3.50	24. 99	54.32
15. 15	3.00	25. 80	40.00
16. 20	2.75	26. 600	15.00
17. 40	3.25	27. 666	2.90
18. 66	1.50	28. 91	7.50

Aliquot Parts in Billing

Find the amount of each of the following bills:

1.

30 bbl. apples at	4.12\frac{1}{2}$
43 bunches bananas at	1.87$\frac{1}{2}$
24 bbl. cabbages at	1.16$\frac{2}{3}$
15 doz. cauliflower at	.65

2.

24 bx. oranges at	$3.75
36 bx. lemons at	4.20
27 crates grape fruit at	4.25
25 bbl. sweet potatoes at	3.60

3.

840 bu. oats at	$0.51
920 bu. corn at	.82
178 bu. timothy seed at	2.50
940 bu. wheat at	1.12$\frac{1}{2}$

4.

360 lb. coffee at	0.16\frac{2}{3}$
140 bbl. flour at	5.55
124 T. timothy hay at	15.25
60 T. rye straw at	7.75

5.

14 bu. hickory nuts at	$3.25
18 bu. black walnuts at	1.37$\frac{1}{2}$
65 bu. potatoes at	.66$\frac{2}{3}$
73 bu. beans at	2.10

6.

35 bbl. mess pork at	$27.50
648 lb. smoked hams at	.16$\frac{2}{3}$
478 lb. bacon at	.18$\frac{3}{4}$
500 lb. lard at	.12$\frac{1}{2}$

7.

7 shirts at	$13.50 per doz.
8 shirts at	15.60 per doz.
9 shirts at	12.00 per doz.
12 shirts at	12.$\frac{1}{2}$ per doz.

8.

9 doz. games at	$42.00 per gro.
7 doz. games at	40.00 per gro.
8 doz. games at	33.00 per gro.
11 doz. games at	36.00 per gro.

CHAPTER VIII

VOCATIONAL PROBLEMS

Pay Rolls

In figuring the wages of workmen paid by the hour it is useful to be familiar not only with the aliquot parts of $1.00 but with the aliquot parts of the day's pay.

Suppose that a workman is paid on the basis of $5.00 a day, for a day of 8 hours. This is at the rate of $62\frac{1}{2}¢$ an hour. The pay roll clerk is probably familiar with the table of $62\frac{1}{2}¢$, up to $5.00.

If the workman's time for the week is 52 hours, and his rate of pay $62\frac{1}{2}¢$, the pay roll clerk probably figures it as $6\frac{1}{2}$ days at $5.00.

EXAMPLE. Find the total hours and total amount due each workman listed on the following time sheet:

The Superior Mfg. Company

Time Sheet for Week Ending June 8 19____

No.	NAME	Occupation	M	Tu	W	Th	F	S	Rate	Total hours	Amount		Remarks
1	James Adams	Fdry	8	7½	8¼	8	8	5	50	44¾	22	38	
2	Geo D. Black	Yds	8½	9	8½	8	8	6	66⅔	48	32		Dischgd.
3	Wm. Carson	Fdry	5½	8	8	8	8	4	87½	41½	36	32	Sick
4	N. B. Dunn	Yds	8	9	8½	8	7½	5	75	46	34	50	Quit

In figuring the wages due Wm. Carson, the pay roll clerk probably figured as follows:

$87\frac{1}{2}¢$ per hour is equal to $7.00 per day.

5 days at $7.00	$35.00
1 hour	.875
½ hour	.4375
	$36.32

In the other three cases the time was probably figured either by the same plan or by a plan involving aliquot parts of $1.00.

WRITTEN. Find the total hours and the total amount due each workman listed on the following time sheet, and the total amount of the payroll.

THE CITY GAS COMPANY

Time sheet for week ending June 15, 1928

No.	NAME	OCCUPATION	M	TU	W	TH	F	S	TOTAL HRS.	RATE	AMT.	REMARKS
1	Ahern, J.H.	Fitter	8	8	8	8	8	5		75		
2	Babb, Jas.	Fitter	8	7½	8	7¾	8	5		50		
3	Cowan, Geo.	Laborer	6½	8	7¾	9	8	4		55		
4	Drury, W.L.	Laborer	4	8	8	8	8	5		1 00		
5	Ewing, J.G.	Laborer	8	7½	7½	8	8	5		62½		
6	Franks, Levi	S. H.*	8	8	8	8	8	5		50		Disch
7	Goode, Paul	Laborer	7	7	7	7	7½	5		75		
8	Harms, Will	Fitter	7¾	8	7½	8¼	7	5		1 00		
9	Isaacs, B.J.	S. H.	7	8	8	8	7½	5		66⅔		
10	Jacobs, P.	Mach.	7¾	8½	7¾	8¼	8	5		50		
11	King, Jos.	Foreman	8	8	8	8	8	5		83⅓		Quit
12	Lewis, W.M.	Mach.	8	8	8	8	8	5		1 00		
13	Mather, C.M.	S. H.	7	8	7¾	8	7	5		75		
14	Nolan, Pat	Fitter	8	8½	8	7½	8	5		1 00		
15	Owen, H.M.	Laborer	8	8	7¾	8	8	5		1 33⅓		
16	Peters, Geo.	S. H.	8	8	8	8½	8	5		1 00		
17	Quayle, C.	S. H.	8	7¾	8½	8	8	5		75		
18	Roberts, Wm.	Laborer	7	8	9	8	8	5		50		

*Service helper.

WRITTEN. **1.** Copy the above form and data. Make the extensions and foot the amount column.

The amount of the pay roll having been determined, a pay roll check is drawn upon the bank for the amount, payable in currency and coin. In order to have just the right change for making up the pay envelopes, the pay roll clerk usually makes an analysis of the pay roll, thus:

CHANGE REQUIRED FOR PAY ROLL, JUNE 15, 1928

No.	Amt.	$10	$5	$2	$1	50¢	25¢	10¢	5¢	1¢
1	$16.88	1	1		1	1	1	1		3
2	17.26	1	1	1			1			1
3	15.00	1	1							

The form is self-explanatory. When it has been completed, the clerk foots the various columns, and tests the accuracy of the totals by determining whether the total change required equals the total amount of the payroll as shown by the footing of the amount column.

THE PEOPLES TRUST AND SAVINGS BANK
OF CHICAGO

MEMORANDUM OF PAY ROLL

WANTED BY The City Gas Co

Date June 15, 191_

		DOLLARS	CENTS
FOR CURRENCY OR GOLD			
Twenties	5		
Tens	11	325	
Fives	23		
Twos	87	174	
FOR SILVER			
Dollars		93	
Halves	105	52	50
Quarters	137	34	25
Dimes	249	24	90
Nickels	203	10	15
Pennies	468	4	68
Total		718	48

He then prepares a "Memorandum of Pay Roll" on a printed form as shown herewith, which memorandum is sent to the bank along with the pay roll check. The bank teller, in cashing the check, supplies the company with the change in the required denominations.

2. Prepare an analysis of change required for The City Gas Company's pay roll June 15, 1918, calling for the smallest possible number of pieces, and prepare a memorandum to accompany the pay roll check to the bank.

3. Analyze the following pay roll to determine the change required to fill the pay envelopes:

Name	Amount Due	Name	Amount Due	Name	Amount Due
Willis Hill	$24.62	Jno. F. Hewitt	$18.94	C. Harrison	$19.72
H. M. Keal	21.49	Chas. Daly	23.56	M. Fish	18.61
Tom Brown	17.58	Wm. Howell	19.38	C. O. Miller	22.50
H. F. Sanford	22.35	C. J. Kirby	20.83	E. C. Stotts	29.78

4. Analyze the following pay roll to determine the change required to fill the pay envelopes:

Number	Amount Due	Number	Amount Due	Number	Amount Due
106	$25.47	126	$23.38	143	$17.19
109	22.83	140	20.46	186	21.30
110	24.73	141	19.38	187	18.45
111	24.17	142	21.56	192	21.93

5. Prepare a pay roll from the following data. Then analyze the items to determine the change required and write a Memorandum of Pay Roll:

	M.	Tu.	W.	Th.	F.	S.	Rate
James White.....	8	8	8	8	8	5	75
Dan Hart.....	8	$7\frac{1}{2}$	8	$8\frac{1}{2}$	9	4	$66\frac{2}{3}$
James Corbin....	8	$7\frac{3}{4}$	8	$7\frac{3}{4}$	8	5	55
Wm. Callan.....	8	8	9	9	$7\frac{1}{2}$	5	$66\frac{2}{3}$
F. W. Hawkins...	8	$7\frac{3}{4}$	8	$7\frac{3}{4}$	8	5	$63\frac{1}{2}$
Ed. March.....	8	7	$8\frac{1}{2}$	8	9	$4\frac{1}{2}$	$66\frac{2}{3}$
Tom Morrison...	7	$8\frac{1}{4}$	$7\frac{1}{2}$	$8\frac{1}{4}$	9	5	75
F. B. Clark.....	$8\frac{1}{2}$	$7\frac{3}{4}$	$8\frac{3}{4}$	9	$8\frac{1}{4}$	$5\frac{1}{2}$	$66\frac{2}{3}$
J. W. Durkin....	8	8	8	7	8	5	$83\frac{1}{3}$
L. H. Larcom....	8	9	$8\frac{1}{4}$	$7\frac{1}{2}$	$8\frac{1}{2}$	5	$87\frac{1}{2}$
N. C. Lee.....	$7\frac{3}{4}$	$8\frac{1}{2}$	9	$8\frac{3}{4}$	7	$5\frac{1}{4}$	$62\frac{1}{2}$
Joe Black.....	8	7	8	7	8	5	75
E. E. Crabbe....	4	0	8	8	5	5	$66\frac{2}{3}$
Roger Hill.....	0	$8\frac{1}{2}$	$8\frac{1}{4}$	9	$7\frac{3}{4}$	5	$83\frac{1}{3}$
Chas. Dick.....	8	$7\frac{3}{4}$	8	8	$7\frac{3}{4}$	5	$87\frac{1}{2}$
L. Y. Sherman...	8	8	7	9	8	5	1 00
R. L. Scully.....	$7\frac{3}{4}$	$8\frac{1}{2}$	$7\frac{3}{4}$	$8\frac{1}{4}$	$7\frac{3}{4}$	$5\frac{1}{4}$	75

6. Prepare a pay roll from the following data. Then analyze the items to determine the change required and write a Memorandum of Pay Roll:

No.	M.	Tu.	W.	Th.	F.	S.	Rate	No.	M.	Tu.	W.	Th.	F.	S.	Rate
1	9	9	9	9	9	5	$66\frac{2}{3}$	10	9	$8\frac{3}{4}$	9	$8\frac{3}{4}$	9	5	100
2	9	9	8	$7\frac{1}{2}$	9	5	$61\frac{1}{9}$	11	9	0	$8\frac{3}{4}$	6	9	5	$66\frac{2}{3}$
3	9	$8\frac{1}{4}$	9	$8\frac{3}{4}$	$7\frac{3}{4}$	5	$66\frac{2}{3}$	12	$8\frac{1}{2}$	$8\frac{3}{4}$	$9\frac{1}{4}$	$8\frac{1}{2}$	$8\frac{3}{4}$	5	50
4	9	8	9	9	9	5	50	13	0	8	$8\frac{1}{2}$	$8\frac{3}{4}$	9	5	$66\frac{2}{3}$
5	9	$8\frac{3}{4}$	9	$8\frac{3}{4}$	9	5	1 00	14	9	9	9	9	9	5	$44\frac{4}{9}$
6	9	9	9	9	9	5	$66\frac{2}{3}$	15	8	$8\frac{3}{4}$	9	$9\frac{1}{2}$	$8\frac{3}{4}$	5	$66\frac{2}{3}$
7	9	9	9	9	9	5	$72\frac{2}{9}$	16	9	8	9	8	9	5	$61\frac{2}{3}$
8	9	$8\frac{3}{4}$	$8\frac{3}{4}$	$8\frac{3}{4}$	9	5	$66\frac{2}{3}$	17	9	$8\frac{1}{2}$	9	0	$6\frac{3}{4}$	$5\frac{1}{2}$	$66\frac{2}{3}$
9	9	9	9	9	$8\frac{1}{2}$	5	$61\frac{2}{3}$	18	9	8	$8\frac{1}{2}$	9	$9\frac{1}{2}$	$4\frac{3}{4}$	$44\frac{4}{9}$

7. If a man works six 10-hour days and $7\frac{1}{2}$ hours over, what are his services worth at 56¢ an hour?

8. If a man works $60\frac{3}{4}$ hours in a week, and 54 hours is a week's work, the balance being over-time at time and one-half, what is due him if the regular hour rate is 60¢?

9. If a man has been getting $49\frac{1}{2}$¢ an hour, working 10 hours a day, how much an hour must he get to receive the same daily wage if the working day is reduced to nine hours? To eight hours?

10. A contractor pays his men on what is called the "differential bonus" plan. A laborer is paid for shoveling dirt at $3.00 a day. For every yard he shovels over 12, he receives a bonus of 6¢, and for every yard over 18 he gets an additional 5¢.

Find the pay of the following laborers:

No. 29.....15 yd.
No. 30.....16 yd.
No. 31.....18 yd.
No. 32.....21 yd.
No. 33.....20 yd.
No. 34.....22 yd.
No. 35.....19 yd.
No. 36.....17 yd.

11. What is the total amount of the following payroll?

THE ACME MFG. CO., PEORIA, ILL.

Pay Roll for the Week Ending May 28, 1927.

No.	Name	Hours						Total Hrs.	Rate	Total Wages		Re-m'rks
		M	Tu	W	Th	F	S					
1	Andrews, J. A.	8	8	8	8	8	8	48	1.00	48		
2	Busch, W. W.	8	7	8	$7\frac{1}{2}$	7	8		90¢			
3	Canfield, James	8	0	7	6	8	8		75¢			
4	Downer, H. E.	8	$7\frac{1}{2}$	$7\frac{1}{2}$	7	8	7		75¢			
5	Mandel, B. W.	8	8	8	8	8	8		75¢			
6	Noonan, Patrick	8	7	8	7	8	7		80¢			
7	Peters, Paul J	$7\frac{1}{2}$	8	7	8	7	8		90¢			
8	Ryerson, M. C.	0	6	8	$7\frac{1}{2}$	6	8		75¢			
9	Saunders, James	7	0	8	0	0	0		85¢			
10	Thompson, Will H.	8	8	8	7	7	7		1.00			
11	Vance, George	8	8	8	8	$7\frac{1}{2}$	7		80¢			
12	White, J. G.	7	$7\frac{1}{2}$	8	8	8	8		90¢			
	Total											

12. $900 a year is equal to how much per week? How much per month? Counting 4 weeks to a month and using the weekly rate, will the pay for a month equal the monthly rate? One month's pay equals the pay for how many weeks?

13. You have been employed by a certain company and received your pay at the last of each month at the rate of $95 per mo. The company has decided to change to a weekly pay roll. How much per week should you receive?

14. In shops observing an eight-hour day, it is customary to allow time and one-half for overtime each day. Find the amount of the following pay roll. Also complete the listing of the exact coins and currency needed to pay off the men.

ALLISON & BROWN, BURLINGTON, IOWA.

Pay Roll for Week Ending May 21, 1927.

No.	Hours						Total Hours	Rate	Total Wages	Currency and Coin								
	M	Tu	W	Th	F	S				$10	$5	$2	$1	50c	25c	10c	5c	1c
1	8	9	10	8	$9\frac{1}{2}$	8		90¢										
2	7	$7\frac{1}{2}$	$8\frac{1}{2}$	$8\frac{1}{2}$	$9\frac{3}{4}$	9		75¢										
3	8	8	9	7	9	9		1.00										
4	7	8	7	8	9	8		1.00										
5	7	9	7	9	7	9		95¢										
6	8	8	8	7	4	0		75¢										
7	8	$8\frac{1}{2}$	8	8	8	8		90¢										
8	7	$7\frac{1}{2}$	$8\frac{1}{2}$	$7\frac{1}{2}$	7	7		85¢										
9	9	9	9	9	9	9		90¢										
10	7	$8\frac{1}{2}$	$7\frac{1}{2}$	9	8	8		95¢										
11	7	8	9	7	8	9		1.25										
12	8	8	8	7	9	8		1.10										
13	8	8	6	7	8	9		1.00										
14	8	8	8	$7\frac{1}{2}$	7	4		95¢										
15	7	8	$6\frac{1}{2}$	8	7	8		$66\frac{2}{3}$¢										
16	8	8	7	$7\frac{1}{2}$	$8\frac{3}{4}$	$8\frac{1}{2}$		95¢										
17	8	7	$8\frac{1}{2}$	9	10	6		1.00										
Total.																		

Problems of the Farm

1. The freight rate on corn is 62.5¢ a hundred from Chicago to New York. The freight rate on dressed hogs is $1.42 a hundred. It requires 30 bushels of corn of 56 lb. each to grow a 300-lb. hog, dressing 225 lb. How much cheaper is it to ship the dressed hog than the corn?

2. When the average price of corn was 87¢ in Nebraska and $1.23 in Massachusetts, how much more was the cost for feed to raise a 300-lb. hog in Massachusetts than in Nebraska?

3. A farmer applies $2.40 worth of fertilizer to a field and the crop increases in value $16.39. On another plot of the same field he uses $20.90 worth of another fertilizer, and this crop increases in value $40.54. Considering only the profits on this year's crops, which is the more profitable fertilizer to use and how much?

4. A farmer bought a car load of twenty steers at $5.75 a hundred, plus a freight charge of 7¢ a hundred, and eighteen months later sold them at $7.65 a hundred. The steers averaged 898 lb. when bought, and the average weight when sold was 1454 lbs. If the cost of keeping the steers was $900, what was the gain?

5. A farmer bought a herd of 149 steers averaging 982 lb. for which he paid at the rate of 5¢ a pound. The average feed for each steer was: 21.6 bu. of corn @ 50¢; $1\frac{1}{3}$ tons of silage at $3.75; 400 lb. mixed hay at $10.00 a ton; 320 lb. cottonseed meal at $30.00 a ton. The herd was then sold at $8.25 per hundred and they averaged 1254 lb. What was the profit per steer, and the total profit?

6. A boy planted an acre of corn. The seed cost him 75¢; plowing, harrowing, discing, etc., cost $4.50; cultivation cost $3.50, husking he considered worth $2.50. His yield was 80 bu. worth 85¢ a bu. What was his profit? What was the cost per bushel?

7. The University of Minnesota has determined that the cost of raising and harvesting an acre of corn in a certain county was $21.77, and of an acre of barley $19.13. The corn yielded 42 bu. an acre and was worth 85¢ per bu. The barley yielded 37 bu. per acre, and was worth 92¢ per bu. What was the net profit per bushel from each crop? Which was the more profitable crop and how much per acre?

8. A crop of 15 bu. of wheat per acre removes from the soil $21\frac{1}{5}$ lb. of nitrogen, $7\frac{1}{10}$ lb. of phosphorus, and $4\frac{1}{10}$ lb. of potassium. Ordinary barnyard manure contains 10 lb. of nitrogen, 6 lb. of phosphorus, and 12 lb. of potassium per ton. How much manure must be applied to replace the nitrogen used? How much to replace the phosphorus?

9. Find the net loss or gain on hogs for 19—. Value of hogs on hand Jan. 1, \$345.60. Apr. 14, bought a pure bred hog for \$38.50. May 6, paid for services of veterinary, \$2.00. June 4, sold 24 pigs at \$3.20 each. Sept. 8, bought $1\frac{1}{4}$ tons bran at \$22. Sept. 20, sold 8 shoats averaging 156 lb. at \$6.90 a hundred pounds. Nov. 10 sold 20 hogs averaging 278 lb., at \$7.20 a hundred pounds. Dec. 8, sold one hog for \$32. Inventory Dec. 31, 32 hogs averaging 306 lb., at 9¢. Man labor charged, 630 hr. at 16¢; horse labor, 46 hr. at 11¢. Cost of feed, \$803.50.

10. A farmer pays 55¢ a rod for digging trenches, and \$23.00 a thousand for tile. What will be the cost of tiling 84 rd., if the survey costs \$4.30 and 16 tile are required for each rod? How much per rod will the tiling cost?

11. It is estimated that a bushel of corn will make an average increase of 10 lb. when fed to a hog. If corn is worth 95¢ per bu. and hogs $14\frac{1}{2}$¢ per lb., is it better to sell the corn or feed it, and how much, in a field of corn of 50 A. averaging 64 bu. per A.?

12. If the annual cost of maintenance of a dairy cow be \$60.00, and she gives 14,275 lb. of milk testing .04 lb. butter fat per lb. what is the cost per pound of butter fat, making no allowance for the value of the skim milk? What is the total amount received if the milk be sold on the basis of 42.3¢ per pound of butter fat? What should be received if the butter fat be made into butter (making $\frac{1}{8}$ more in weight) and sold at 40.7¢ per lb.?

13. If a gallon of milk weighs 8.6 lb., what would have been the returns from selling the above milk at 8¢ per qt.? What would have been the profit on the cow?

14. A Wisconsin farmer shipped to Chicago in one month 208 cans of milk that were sold by a wholesale commission dealer. He paid $27\frac{1}{2}$¢ per can for transportation. The wholesale dealer sold the milk at \$2.35 per can and deducted 6¢ per can for his charges. What were the farmer's net proceeds?

15. If a can of milk holds 8 gallons and the milk mentioned in the above problem was sold to householders at 14¢ per qt., how much did the householders pay? How much more did they pay than the farmer received?

16. If for feeding purposes 4 lb. of cotton seed meal are the equivalent of 5 lb. of corn meal, which is the better to feed, corn meal at 60¢ per bu. of 48 lb. or cotton seed meal at 72¢ per bu. of 42 lb.?

17. A field of barley is 60 rods wide and 80 rods long. It yielded 1230 bu. What was the yield per acre?

18. What will it cost to enclose a field 80 rd. square with a woven wire fence with a single strand of barbed wire on the top; posts to be 1 rd. apart. The posts cost $23\frac{1}{2}$¢ each; the barbed wire costs $3\frac{3}{4}$¢ a rod, and the woven wire $41\frac{1}{4}$¢ a rod. The staples cost $2.75; the digging of the post holes, $15.00; the setting of the posts, $12.50; and the stretching and fixing the wire, $10.50. What does the fence cost per rod completed?

Problems in Business

1. A contractor built a house for Jones, who agreed to allow him for his pay $\frac{1}{20}$ of all amounts paid for labor and materials. After paying the contractor, Jones found that the house had cost him $6489. How much was paid to the contractor?

2. A dealer cut the price of a dining table one fourth. If the reduced price was $27.60, what was the price before reduction? If this table was marked in the first place to produce a profit of half the cost, what profit is realized if it is sold for $27.60?

3. In North Dakota a farm of 160 A. was purchased for $4500. The entire farm was sown in wheat, and produced $22\frac{1}{2}$ bushels per acre. The wheat was sold for 85¢ per bushel. The first year's crop was what fractional part of the cost of the farm?

4. A cardboard carton is $3\frac{1}{2}''$ x $6\frac{1}{2}''$ x $8\frac{1}{2}''$, outside dimensions. These cartons, when filled, are packed 3 doz. in a larger carton. What must be the inside measurements of the larger carton to contain three tiers of 12 each, of the smaller cartons packed flat? (Two answers.)

5. What fractional part of a cubic foot does each of the smaller cartons mentioned in problem 4 occupy? How many cubic feet do 36 cartons occupy? What outside dimensions can you suggest for cartons to hold 36 of the smaller boxes, allowing $\frac{1}{4}$ in. for the thickness of the material?

6. A label is used on the end of the small cartons mentioned in problem 4 leaving a margin of $\frac{1}{4}$ inch around the label. What is the size of the label?

Cost per Unit

1. Thomas Plant, a farm hand, worked 291.5 hours in one month. His wages were \$60.00. His board was estimated at \$20.00. What did he cost his employer per hour? If Plant worked 31.5 hours in plowing one field what did the man labor cost for plowing that field?

2. In a certain high school a teacher gets \$1500 for 10 months of service, each month consisting of 20 school days. He spends 6 hours each day in class work with classes averaging 25 students each. What is the average cost per student per hour for this teacher?

3. What would be the teaching cost per student in a high school accommodating 1275 students, and maintaining a faculty of 52 teachers receiving the same pay and working the same number of hours as the teacher in Problem 2?

4. An office building to cost \$700,000 is to be erected upon a lot 60′ x 100′. The ground was leased for 99 years at an annual rental of \$12,000 for the first 10 years, \$13,000 per year for the succeeding ten years and \$14,000 per year for the remaining 79 years. If the value of the land is 25 times the average value of the lease, what is the value of this property per front foot and per square foot, including the building?

5. A tract of land containing 80 acres was purchased for \$2,000 per acre and then subdivided into 640 lots each having a frontage of 30 ft., and a depth of 160 ft. What is the cost per lot in this sub-division, and the cost per front foot? How much land was used for streets and alleys?

6. A freight locomotive consumes 212 lb. of coal per mile and a passenger locomotive consumes 109 lb. per mile. What will it

cost to run each of these locomotives 90 miles if the coal costs the railroad company on an average of $1.39 per ton? (Allow 2000 lb. per ton.)

7. If a locomotive averages 31,417 miles per year and fuel costs 10.6¢ per mile, how many tons of coal are used, if the coal costs $1.46 per ton of 2000 pounds?

8. In a certain city the street railway company carried 240,940,-100 revenue passengers, and received $11,952,286 as passenger earnings. What was the average receipt per passenger? If the operating expenses of this railway amounted to $8,694,230, what was the cost per passenger?

9. A roll of wrapping paper 24″ wide and 800 ft. long, costs 5¢ per lb. and weighs 36 lb. It is used to wrap parcels in boxes $4\frac{3}{4}$″ high and $5\frac{1}{2}$″ wide. The full width of the paper is used for each parcel. How many parcels may be wrapped from one roll of paper and what is the average cost per parcel if $1\frac{1}{2}$″ is allowed for lapping over on the box? (Carry the result to the fourth decimal place.)

10. A hardware dealer rents a store for $62.50 per month. During March his sales were $2500, and during April they were $3250. The rent was what decimal part of his sales each month? The decimal rate for March was how much greater than the decimal rate for April?

11. A board 1 in. x 6 in. x 16 ft. contains 8 board feet. Using boards of this size, a man made 3200 wheels each 1 in. thick and 6 in. in diameter, in four days of 8 hrs. each. He received $61\frac{1}{4}$¢ per hr. for his labor. If the lumber was purchased at $28.50 per thousand board feet, what was the average cost of each wheel? (Give result to fourth decimal place.)

12. A company manufactured 150,000 of a certain article during 1917. The cost of these articles was $38,500.00 for materials; $33,850.00 for labor. The company paid out $23,000 for general expenses during the year and estimated that its property had depreciated in value $7500. What was the cost per unit of manufacture for materials? for labor? How much must be charged against each unit to cover its share of expense? of depreciation? What was the total cost per unit?

Profit and Loss

1. A coal dealer buys a car load of coal weighing 82,880 lb. at \$5.90 per ton of 2240 lb. He sells the coal at \$8 per short ton retail after paying 80¢ per long ton for freight. If the cost of delivery averages 32¢ per short ton what will be his profit on the car load?

2. A coal mining Company shipped to a dealer 78,640 lb. of coal. The freight was 60¢ per long ton and the cost of the coal at the mines was \$2.35 per ton. If the cost of delivery was 25¢ per short ton, at what price must the coal be sold per short ton to gain 80¢ per long ton?

3. If coal has been retailing at \$7.50 per short ton what should be the equivalent price per long ton? If the retail price per short ton is increased 50¢ what should be the increase per long ton?

4. A man renting a farm on half shares hired an extra man and team for 10 days at \$6.24 per day to plow corn. By so doing he obtained an increased yield of 4 bu. per acre on 52 acres. If corn is worth 81¢ per bushel, what was the renter's profit on the hired man's labor?

5. It costs \$22.85 to raise an acre of potatoes, exclusive of rent. What yield, in bushels per acre, must a tenant secure in order to make a profit of \$6 per acre, and pay \$5 per acre land rental, the price of potatoes being 65¢ per bushel?

6. One man and a team can cover $1\frac{3}{4}$ acres of land with manure, in a day of 10 hr. If the labor of the man costs 45¢ per hour and the labor of the team costs 40¢ per hour, how many bushels per acre must the yield of corn be increased to meet this additional expense, if corn is worth 87¢ per bu., not counting the value of the manure?

7. According to the statistics of a street railway company, it carried 224,033,000 revenue passengers during the year at an average cost of \$.0388 per passenger. The average receipts per passenger were \$.0441. At this rate what were the profits of the company? What would be the yearly profit on one car that would carry 185 passengers during each trip if it made 4 trips per day every day except Sunday?

8. A dealer bought 400 sheep for \$1175.50. He sold $\frac{1}{4}$ of them at \$9.75 per head, and $\frac{2}{3}$ of the remainder at \$9.25 per head. What

then remained he sold at $9 per head. Find the profit realized and the average profit per head realized.

9. A dealer bought 200 bbl. of apples at $6.25 per bbl. He sold $\frac{2}{5}$ of them at $7.10 per bbl. 10 bbl. were frozen and were a complete loss. At what price must the remainder be sold to realize a net profit on the transaction of $\frac{1}{4}$ of the cost of all the apples?

10. A butcher bought a steer that weighed 1460 lb., at $9.85 per cwt. If this steer dressed 56 lb. to the cwt., how many pounds of dressed meat will the butcher get? What will be the value of the dressed meat if it averages $24\frac{3}{4}¢$ per lb.? Disregarding labor and other costs not mentioned, what will be the butcher's profit?

11. A second hand paper dealer bought 150 tons of old papers at $\frac{1}{2}¢$ a pound. The expense of getting this stock was $760.50. At a further expense of $104.50 he separated it into three lots of 50 tons each, which he sold at $20, $26.50, and $37.00 per ton respectively. What was his profit?

12. A coal dealer bought a car containing 27 tons of coal at $3.36 a long ton of 2240 lb. Freight charges were 78¢ a long ton. Unloading and delivering cost 43¢ a ton. If he retailed it at $6.25 a short ton, what was his profit?

13. If 40 acres of land yield 30,000 lb. seed cotton, and 1500 lb. of seed cotton make a bale of lint, what is the gross return from the crop if the lint sells for $12\frac{1}{2}¢$ a pound and the residue, which is seed, sells for $22.00 a ton? (Assume 1 bale = 500 lb.)

14. If the cost of raising the crop mentioned in the preceding problem is $12.25 an acre, and it costs 90¢ a hundred pounds for picking, 40¢ per 100 lb. of lint for ginning, and 90¢ a bale for bagging and ties, what is the profit on the crop?

15. I was offered $3.50 per hundred for a load of seed cotton weighing 1580 lb. I refused this offer and had the cotton ginned, paying 40¢ per 100 lb. lint, and $1.20 for bagging and ties. The seed cotton ginned 450 lb. lint. I sold the lint for $9\frac{1}{4}¢$ a pound, and the seed at $1.55 per 100 lb. How much more did I receive for my cotton than if I had accepted the offer first stated?

16. A dairyman shipped to a dealer 198 cans of milk. He paid freight charges of $17\frac{1}{2}¢$ a can. The dealer sold the milk at $1.33 a can and deducted 3¢ a can for commission. What were the farmers' net returns?

Investments

1. Brown began business January 1, 1927, with $7500.00. His gains during the year were $6473.89. His losses were $4129.63. What was he worth at the end of the year?

2. On January 1, 1927, Jones' net investment was $12,263.54. At the end of the year his assets amounted to $21,346.05, and his liabilities amounted to $12,564.59. What was his net investment at the end of the year? What was the amount of his gain or loss for the year?

3. Smith's assets on January 1, 1927, were $13,456.27. His liabilities were $3429.68. During the year his gains were $10,564.59 and his losses were $5789.67. His liabilities at the end of the year were $4862.36. What were his assets at the end of the year?

4. On January 1, 1927, Green had on hand cash $1236.59, real estate valued at $8500, furniture and fixtures valued at $560.50, and merchandise $2350.00. There was due him from others on account $3269.57, and he held notes signed by others amounting to $1500. He owed creditors on account $1969.25, and there was a mortgage of $1500 against his property. During the year his net profits were $6427.24. What was he worth at the end of the year?

5. On January 1, 1927, White had $129.73 on hand in cash and had a bank balance of $1263.39. He owned bonds valued at $9250 on which interest had accrued amounting to $127.25. He owned the store in which his business was carried on. The store was valued at $12,500 but there was a mortgage of $3750 against it. He had a stock of goods inventoried at $12,576.48 against which there were three notes outstanding, one for $2000, one for $1000, and one for $256.75. He also owed creditors on account for merchandise $1276.87. He held other persons' notes amounting to $850.00. What was he worth?

6. In the preceding problem it was noted that White had on hand merchandise valued at $12,576.48 on January 1, 1927. During the year he bought merchandise amounting to $18,769.27. At the end of the year he found that he had on hand merchandise valued at $9274.87. What was the cost of the merchandise sold by him during the year? If his sales during the year were

$31,489.47, expenses of running the business $3768.29 and there were no other losses or gains, what was he worth December 31, 1927?

7. On January 1, 1928, W. B. Owen had on hand cash $3903.50, furniture and fixtures valued at $475.00, merchandise inventoried at $3100, notes receivable $1200, accounts against debtors $181.58. He owed creditors $979.00 on account and owed a note for $1425.00. How much was he worth?

8. On January 1, 1927, W. B. Owen had a net investment of $5460.56. During the year his furniture and fixtures depreciated in value $15.00. His profits from the sales of merchandise were $1260.00. His gains through taking cash discounts were $33.45. His losses on interest were $8.11. His expenses were $275.00. What was he worth at the end of the year?

9. On January 1, 1927, the treasurer of The Harvard Club reported that there was cash on hand $95.68. During the course of the year his receipts from initiation fees, membership dues, and special assessments amounted to $349.83. Receipts from other sources were $5750.69. During the year he paid out for various items $5650.29. How much cash should he have had on hand at the end of the year?

10. On January 1, 1927, I bought a house and lot for $5000.00. During the year I built an annex which cost $3,567.27 and a heating plant which cost $963.04. I paid taxes amounting to $25.00, spent $450.00 for repairs and spent $55.60 for grading the yard. My income from rents during the year was $1775.00. At the end of the year I sold the house for $9650.00, paying the agent a commission of $193.00 for selling. What did I lose or gain upon the entire investment?

CHAPTER IX

AVERAGES

An **average** is a medial amount or quantity, determined by dividing the total of several unequal items by the number of such items.

Thus, the average of 10, 12, and 20 is found by adding 10, 12, and 20 and dividing the sum by 3.

Written

1. An automobile manufacturer paid back $15,410,650.00 to 308,213 buyers on a guarantee to refund a certain amount provided 300,000 sales were made during the year. What was the average amount refunded per person?

2. A squad of Boy Scouts hiked $10\frac{1}{2}$ miles on Monday, $8\frac{3}{4}$ miles on Tuesday, 9 miles on Wednesday, none on Thursday, $10\frac{3}{4}$ miles on Friday, and $6\frac{1}{2}$ miles on Saturday. What was the average distance per day?

3. A farmer had four $40\frac{1}{2}$-acre fields planted in corn. On these he raised respectively 1125, 1020, 986, and 1005 bu. What was the average yield per acre in each field and what was the average for the four fields? Carry to one decimal place.

4. Following are the receipts of one department of a store for one week. Monday, $375.20; Tuesday, $438.50; Wednesday, $512.60; Thursday, $439.75; Friday, $487.38; Saturday, $572.21. What were the average daily receipts?

5. The maximum temperatures for seven successive days at a certain place were 63.2°, 58.1°, 67.4°, 71.2°, 65.5°, 73.2°, and 72.6°. What was the mean maximum temperature?

6. A man's net income per month for seven months was $260.25, and for five months it was $241.05. What was his average income per month?

7. At a certain point in Idaho the rainfall by months was as follows: 1.6, 1.2, 1.4, 1.6, 1.0, .5, .4, .7, 1.0, 1.2, 1.3, and 1.3 inches. What was the average monthly rainfall?

8. On May 1, Browning, King & Co. had on deposit in the First National Bank $11,457.20. Their deposits and withdrawals for the first six days of the month were as follows:

May 1	Dep. $983.40, withd. $642.70	May 4	Dep. $511.28, withd. $100.00
2	Dep. 726.81, withd. 263.40	5	Dep. 463.74, withd. 208.26
3	Dep. 620.15, withd. 197.64	6	Dep. 972.65, withd. 311.23

What was their average balance in the bank for the six days?

9. A certain bank makes a charge of $1.00 against any customer whenever he fails to maintain an average daily balance of $250 throughout any month. The following were the deposits and withdrawals of one customer during April, 1918; April 1, deposited $400; April 11, withdrew $300; April 18, deposited $450; April 25, withdrew $400. Should the customer be charged $1.00? What was his average balance for April? What was his lowest balance at any time? What was his balance May 1?

10. Find the total of the following pay roll, and the average cost per hour (to four places), if each employee works 44 hours per week.

CARRUTHERS & CO.

PIECE-WORK PAY-ROLL

WEEK ENDING, Aug. 3, 19

	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Total Pieces	Rate per 100 Pieces	Amount Due
A. S. H.	425	500	550	500	450	225		$66\frac{2}{3}$	
M. A. M.	375	425	400	400	450	175		$66\frac{2}{3}$	
W. L. R.	560	480	525	500	600	250		50	
J. A. L.	225	250	275	200	300	125		1.10	
J. E. O.	425	500	450	475	500	275		55	
M. B. C.	320	285	365	245	350	170		$1.33\frac{1}{3}$	
L. L. H.	550	625	575	600	625	400		75	
E. D. W.	825	930	850	900	925	475		$33\frac{1}{3}$	
J. W. C.	340	520	450	325	650	375		45	
G. C. S.	400	500	450	425	475	325		75	

11. The average yields of potatoes per acre for 40 years were:

From 1867 to 1875.....90.0 bu. From 1889 to 1896.....75.0 bu.
From 1876 to 1888.....81.4 bu. From 1897 to 1906.....85.5 bu.

What was the average for 40 years?

12. In 1912 the electoral vote by states was as follows:

Ala.....12	Ill.....29	Minn.....12	N. C.....12	Tex.....20
Ariz..... 3	Ind.....15	Miss.....10	N. D..... 5	Utah..... 4
Ark..... 9	Ia.....13	Mo.....18	Ohio.....24	Vt..... 4
Calif.....13	Kans.....10	Mont.... 4	Okla.....10	Va.....12
Colo..... 6	Ky.....13	Nebr..... 8	Ore..... 5	Wash..... 7
Conn..... 7	La.....10	Nev..... 3	Pa.....38	W. Va..... 8
Del..... 4	Me..... 6	N. H..... 4	R. I..... 5	Wis.....13
Fla..... 6	Md..... 8	N. J.....14	S. C..... 9	Wyo..... 3
Ga.....14	Mass.....18	N. M.....3	S. D..... 5	
Ida..... 3	Mich.....15	N. Y.....45	Tenn.....12	

It was necessary for the successful candidate for the presidency to have a majority of the electoral votes. Wilson received 435 votes, Taft received 8 votes, and Roosevelt received the remainder. What was Wilson's majority? His plurality?

13. In a certain year Iowa had 497,260 depositors in savings banks. Their total deposits were $168,063,934.80. What was the average deposit?

14. According to the census of a certain year there were 6,361,502 farms, and 878,798,325 acres in these farms, in the U. S. What was the size of the average farm? The value of these farm lands was $34,801,125,697. What was the average worth per acre?

15. Indiana had 33,873 depositors in savings banks in one year, and their total deposits were $12,356,715.57. What was the average deposit?

16. In the United States in 1911 there were $3,224,100,000 of all kinds of money. If the population was 93,792,509, what was the amount per capita?

17. One train runs from Chicago to New York, a distance of 954 miles, in 18 hr. By another road it is 918 miles, and a train on this road travels the distance in 18 hr. At what average speed does each train travel?

18. The Pennsylvania Railroad has an annual income of $164,191,846. Its taxes are $7,021,031; its other expenses are $118,098,295. What is its net profit? average net income daily?

CHAPTER X

PERCENTAGE

Percentage is an application of decimal fractions in which one one-hundredth is the unit used in computations.

A common fraction may have any denominator; that is, the unit in computations may be any part of a thing, as $\frac{1}{7}$. A decimal fraction must have for its understood denominator 10 or some power of 10, as in .001. But in percentage the unit is always equivalent to $\frac{1}{100}$. Thus 1 per cent $= \frac{1}{100}$.

Per cent means *for each hundred.* One one-hundredth, in percentage, is called one *per cent.* The sign for *per cent* is "%." The per cent taken is the *rate.*

Six per cent (6%), which is equivalent to $\frac{6}{100}$, or .06, means *six hundredths,* or *six for each hundred,* or *six to each hundred.*

Common fractions, decimal fractions, and quantities expressed as per cents are only different forms of expression. Any per cent can be expressed in either of the other forms.

EQUIVALENT EXPRESSIONS

Per Cent	Decimal Fraction	Common Fraction
6%	.06	$\frac{6}{100}$ or $\frac{3}{50}$
$5\frac{1}{2}\%$	$.05\frac{1}{2}$ or .055	$\frac{5\frac{1}{2}}{100}$ or $\frac{11}{200}$
$\frac{3}{4}\%$ or .75%	$.00\frac{3}{4}$ or .0075	$\frac{\frac{3}{4}}{100}$ or $\frac{3}{400}$

Any one of the above forms expresses a value which may be expressed in either of the other forms.

WRITTEN. Prepare a table with six columns headed respectively: Common Fraction, Decimal Fraction, Per Cent, Cents on

the Dollar, and Dollars. Fill in with the following, each on a separate line and each under the proper heading: $\frac{2}{3}$, .6, 75%, $\frac{7}{8}$, $62\frac{1}{2}$¢, \$.50, 35¢, 80¢, $37\frac{1}{2}$%, 5%, 10%, .45, .70, .06, .08, $\frac{1}{200}$, $\frac{\frac{1}{4}}{100}$, $\frac{.2}{100}$. Then fill in all blank spaces on each line with equivalents of the value expressed in one space on that line.

To find a given per cent of a number multiply the number by the rate. You will point off two decimal places in the answer, of course, as the rate is always expressed in hundredths.

EXAMPLE. Find 16% of 250.

(*a*)	(*b*)
250	250
.16	16%
1500	1500
250	250
40.00	40.00

EXAMPLE. Find 25% of 468.

$4)\overline{468}$
$\quad 117$

In this case 25% is recognized at once as $\frac{1}{4}$, and the problem is solved by the aliquot parts method.

In solving the following problems, use the aliquot parts method when desirable. Interchange the factors whenever this will make the work easier.

ORAL

1. 16% of 200
2. 25% of 300
3. 20% of 450
4. $33\frac{1}{3}$% of 150
5. $16\frac{2}{3}$% of 300
6. $12\frac{1}{2}$% of 400
7. 50% of 78
8. $37\frac{1}{2}$% of 120
9. 15% of 178
10. 24% of 125
11. 39% of $33\frac{1}{3}$
12. 16% of 75
13. 8% of 926
14. 6% of 31.5
15. 88% of $112\frac{1}{2}$
16. $66\frac{2}{3}$% of 180
17. $62\frac{1}{2}$% of 324
18. 46% of 25
19. 24% of 12.5
20. 48% of 37.5
21. 120% of 625
22. 56% of 87.5
23. 180% of 750
24. $66\frac{2}{3}$% of 2.7

Fractional parts of one per cent

EXAMPLE. Find $\frac{1}{2}\%$ of 148.

$\frac{1}{2}\%$ is also written .5%, and is equivalent to .005.

(*a*)	(*b*)
148.	2)1.48
.005	.74
.740	

In solution (*b*) point off two decimal places first, to find 1 one-hundredth. Then multiply by the number of hundredths in the rate. Dividing by 2 is equivalent to multiplying by $\frac{1}{2}$.

ORAL

1. $\frac{1}{4}$ of 500	5. 1.2% of 500	9. $\frac{3}{4}\%$ of 2400
2. $\frac{1}{8}\%$ of 10,000	6. 1.5% of 400	10. $\frac{4}{5}\%$ of 525
3. $\frac{3}{5}\%$ of 1800	7. .25% of 960	11. 1.21% of 800
4. $\frac{1}{5}\%$ of 1250	8. 2.125% of 160	12. $\frac{3}{8}\%$ of 750

Problems of Commerce and Industry

1. The cost of manufacturing a barrel of flour is \$2.5365. The expenses of selling are \$37,250 for 50,000 barrels. If selling expense is distributed in proportion to manufacturing costs, what must be the selling price per barrel to cover selling expense and to yield a profit of 10% of the selling price?

2. Chinese white copper is made of the following metals:

Copper	40.4%	Tin	2.6%
Zinc	25.4%	Nickel	31.6%

What amount of each will be necessary to make two tons of the composition metal?

3. A contractor agreed to do all the mason work on a certain building for \$43,280. His profits were equal to 10% of the contract price and the labor was equal to 53% of the cost of the contract. What were the profits and what was the cost of the labor on this contract?

4. A certain railroad had a gross revenue in one year of \$15,999,000. The operating expenses were 39.5% of the gross revenue, and

the interest charges were 15.8% of the gross revenue. What was the total amount of operating expenses and interest charges?

5. A gas company's gross receipts were \$16,825,000. A sum equal to 8.2% of the gross receipts was charged to expense on account of the depreciation of property used in manufacturing its products. What was the total charge to expense on account of depreciation?

6. A corporation has gross annual returns from a leasehold amounting to \$136,500. Eight per cent of this sum is reserved each year to discharge a debt of \$131,040. How many years will be required for the amount reserved annually to discharge this debt?

7. The gross area of Maryland is 12,327 sq. mi. Of this area, 19% is water. What is the land area?

8. An estate is valued at \$48,570. 35% of this sum is in real estate and 43.7% of it is invested in bonds. The remainder is in live stock. What is the value of the live stock?

9. The capital of a certain manufacturing company is represented by common and preferred stock, bonds, and surplus. Of this total capital, 12.5% is surplus. What is the surplus if the total capital is \$2,475,000?

10. In a certain year a farmer sowed 100 acres of wheat. The land produced 20 bu. per acre and the wheat was sold at \$1.35 per bu. The next year the farmer sowed the same acreage in wheat. The yield was 20% less in bushels, but was sold at 20% more per bu. How much more or less was the income the second year than the first?

11. In the United States 22% of the population attend school. 90% of these attend the public schools. Of those attending the public schools, 90% are in the grades. Of pupils in the grades

$23\frac{1}{2}$% are in the first grade	$11\frac{1}{2}$% are in the fifth grade
15 % are in the second grade	9 % are in the sixth grade
14 % are in the third grade	$7\frac{1}{2}$% are in the seventh grade
13 % are in the fourth grade	$6\frac{1}{2}$% are in the eighth grade

What per cent of the entire population are in each of the eight grades? (Carry to four decimal places.) How many pupils should there be in the fourth grade in a city of 25,000 population?

To find what per cent one number is of another

The relation of one number to another is most readily expressed as a common fraction. Thus the relation of 4 to 6 is most readily expressed as $\frac{4}{6}$. This common fraction can then be changed to a decimal by dividing the numerator by the denominator. If it is desired to express this value in terms of percentage, the decimal should be carried to two places only (the remainder, if any, being written as a common fraction) and the per cent sign used instead of the decimal point.

EXAMPLE. 5 is what per cent of 20?

The relation of 5 to 20 is $\frac{5}{20}$ or $\frac{1}{4}$. Dividing the numerator of $\frac{1}{4}$ by the denominator gives the decimal .25. Expressed in percentage, this is 25%.

ORAL

1. 180 is what % of 720?
2. 600 is what % of 900?
3. 75 is what % of 450?
4. 75 is what % of 600?
5. 125 is what % of 500?
6. 75 is what % of 375?
7. 60 is what % of 400?
8. $62\frac{1}{2}$ is what % of 250?
9. \$15 is what % of \$60?
10. 25 da. is what % of 75 da.?
11. 700 lb. is what % of 1 ton?
12. 15 qt. is what % of 10 gal.?
13. \$98 is what % of \$490?
14. \$255 is what % of \$1275?
15. 60 da. is what % of 360 da.?
16. 1 ft. is what % of 1 yd.?
17. 5 ft. 6 in. is what % of 1 rod?
18. 9 in. is what % of 1 yd.?
19. 2640 ft. is what % of 1 mi.?
20. 1 long ton is what % of 1 short ton?

Problems of Commerce and Industry

1. A young man receives \$18 per week. His expenses average: board, \$5.00; clothing, \$3.00; incidentals, \$4.00; and the remainder he saves. What per cent of his salary does he save?

2. In a certain letter, 300 words were used. The stenographer misspelled 6 words. What per cent of the words did she spell correctly?

3. A certain base ball club has played 77 games and has lost 32 games. What per cent of the games played have they won?

4. The following is a partial report from the United States Consul at Hobart, Tasmania, showing our exports to that country:

Articles	Last Year	This Year	Increase	Decrease
Apparel and Soft Goods	$16,251	$17,582		
Boots and Shoes	1,689	2,297		
Clocks and Watches	4,709	5,124		
Furniture	4,314	4,744		
Agricultural Implements and Machinery	29,624	23,320		
Musical Instruments	2,142	1,980		
Paper	4,898	18,298		
Automobiles and Bicycles	59,700	55,016		

Find the per cent of increase or decrease in the exports of each of the above articles for this year as compared with last year. Find the per cent of increase or decrease in the total.

5. On April 1, 1928, T. J. Murray bought a hotel credit letter, authorizing hotels to cash paper amounting to $50.00 or less a week for one year, for $5.00. During the year his weekly expenses, all of which were paid by drafts honored on the strength of this letter, averaged $42.50. What per cent did Mr. Murray pay for this service?

6. The report of the United States Consul for the year ending December, 1926, shows that our exports to Turkey for 1926 were $2,927,980 and our imports from Turkey the same year were $16,834,463. Our exports to Turkey were less than our imports by what per cent? Our imports from Turkey were greater than our exports to Turkey by what per cent?

7. Find the per cent of increase in the population of each of the following cities for the period of time covered:

	1910	1920
Birmingham, Ala.	132,685	178,806
Berkeley, Calif.	40,434	56,036
Jacksonville, Fla.	57,699	91,558
Macon, Ga.	40,665	52,995

	1910	1920
Fort Wayne, Ind.	63,933	86,549
Portland, Me.	58,571	69,272
Youngstown, Ohio	79,066	132,358
Richmond, Va.	127,628	171,667

8. In 1920 there were 6,448,343 farms in the United States and in 1925, 6,371,617. What was the per cent of decrease in number of farms?

9. The following table shows the traffic on the Suez Canal for ten years as reported by the Daily Consular and Trade reports:

Year	Number of Boats	Tonnage (Thousands)	Passengers
1904	4237	13,401	210,980
1905	4116	13,134	252,691
1906	3975	13,445	353,881
1907	4267	14,728	243,826
1908	3795	13,633	218,967
1909	4239	15,407	213,122
1910	4533	16,581	234,320
1911	4969	18,324	275,259
1912	5373	20,275	266,403
1913	5085	20,033	282,235

Find the increase or decrease for each successive year, to the nearest per cent, in tonnage; in passenger traffic.

10. The following table represents the number of spindles and looms, and the consumption in bales, of the cotton manufacturing industries of the six leading countries according to the consular reports of the United States Government, September 1, 1926.

Country	Spindles (Thousands)	Looms (Thousands)	Bales (Thousands)
Great Britain	57,286	786	3,121
United States	37,586	695	6,456
Germany	10,480	230	2,405
Russia	7,246	213	2,053
France	9,511	108	2,452
India	8,510	89	1,697

What part of the spindles of these six leading countries (to the nearest per cent) was in the United States? Looms? Bales of cotton consumed?

11. In 1884 the number of post offices in the United States was 48,434; in 1913 they numbered 58,729; and in 1926, 50,601. What was the per cent of increase from 1884 to 1913, and the per cent of decrease from 1913 to 1926?

12. In 1897 there were 83 rural mail carriers and in 1913 there were 42,199. What was the per cent of increase?

What logical relations do you find between the result of the 11th problem and the result of the 12th? Learn to think about these problems.

13. There were 19,967 established newspapers published in the United States in 1927, of which 2,250 were published in New York State. What per cent were published in New York State?

14. In 1923, the United States paid $263,012,500 in pensions. In 1924, $229,994,777; in 1925, $217,150,612; in 1926, $207,844,348. What per cent of decrease is found each year by comparing the amount paid with the amount paid in 1923?

15. Below is a table showing the total sales and total expenses of five traveling salesmen for one year. Fill out the blank spaces showing by what per cent each man's sales were more or less than the average. Fill out the column for expense percentages in the same way.

Salesman	Sales for Year	Per Cent Over or Under Average	Expenses for Year	Per Cent Over or Under Average
A	$12,562.50		$2250.35	
B	16,428.75		2580.62	
C	11,248.26		2345.96	
D	14,854.93		2010.40	
E	8,725.60		2684.62	

To find a number or quantity when a part of it is known

If a given number is known to be a given per cent of a whole, divide it by the given per cent to find the whole.

EXAMPLE. 168 is 8% of what number?

$$.08\overline{)168.00}$$
$$2100$$

Reasoning: If 168 is 8 per cent of the number, 1 per cent is $\frac{1}{8}$ of 168, and 100 per cent is 100 times $\frac{1}{8}$ of 168.
Method: Divide 168 by .08.

EXAMPLE. 120 is $12\frac{1}{2}\%$ of what number?

$12\frac{1}{2}\% = \frac{1}{8}$. If 120 is $\frac{1}{8}$ of the number, the number is 8×120, or 960.
Watch for opportunities to apply the aliquot parts method.

A percentage is the result of a multiplication of some number by a rate. To find the number that was multiplied, therefore, the process is reversed and the percentage is divided by the rate.

ORAL

1. 50 is 10% of what number?
2. 15 is 6% of what number?
3. 25 is 5% of what number?
4. 42 is 7% of what number?
5. 84 is 5% of what number?
6. 72 is 9% of what number?
7. 81 is 3% of what number?
8. 92 is 4% of what number?
9. 138 is 12% of what number?
10. 74 is $16\frac{2}{3}\%$ of what number?
11. 75 is 5% of what number?
12. 9 is .3% of what number?
13. 103 is $33\frac{1}{3}\%$ of what number?
14. 8 is $\frac{1}{2}\%$ of what number?
15. 5 is .2% of what number?
16. 75 is $1\frac{1}{2}\%$ of what number?
17. 9 is $37\frac{1}{2}\%$ of what number?
18. 14 is $\frac{7}{8}\%$ of what number?
19. 25 is $62\frac{1}{2}\%$ of what number?
20. 6 is $66\frac{2}{3}\%$ of what number?

Problems of Commerce and Industry

1. A savings bank invested $5\frac{1}{2}\%$ of its deposits in United States Bonds. If the sum invested amounted to $12,960.23, what were the deposits in this bank?

2. A young man purchased a house for $2500. To pay for it required 25% of his salary for 8 years. What salary did he receive?

3. Tasker Bros.' sales on July 1, 1928, amounted to $12,645.80. This was an increase of 20% in their business as compared with the previous year. What were their sales for the previous year?

4. The following is a partial list of the imports from Nice, France, as compiled by the United States consular office:

Article	Last Year	This Year	Rate of Increase
Ladies' Apparel		$2,656	37.4%
Linen		16,943	13.6%
Olive Oil		602,265	12.8%
Wines and Spirits		4,479	463.4%
Perfumery		1,586,739	22.3%

What was the amount of the imports for last year of each article in the above schedule? (Give the result to the nearest dollar).

5. Wilson and Mathers invested $9600 in a business. If Wilson invested 40% more than Mathers, what was the investment of each?

6. A farmer had 52% of his land in corn, 18% in pasture, 21% in wheat, and the remainder, 27 acres, in meadow. What was the area of the farm and the acreage of each crop?

7. The expenses of a certain store were as follows: April, $3270; May, $2762; and June, $2954. The expenses for these three months were 30% of the expenses for the entire year. What did the expenses of the other nine months average?

8. Five cities in Virginia having more than 25,000 inhabitants according to the census of 1920 are:

	1910	1920	Per Cent of Increase
Lynchburg		30,070	17.6
Norfolk		115,777	71.6
Portsmouth		54,387	63.9
Richmond		171,667	34.5
Roanoke		50,842	47.8

Find the population of each of the above cities in 1910.

9. A house rents for $60 per month and the annual expenses are $112.50. If the net income from this property is $9\frac{1}{2}\%$ of the cost, what did it cost?

10. A salesman receives a commission of 4.5% on all goods he sells. His commission for the second quarter was as follows: April, \$210.70; May, \$192.60; June, \$215.82. What were his total sales for the quarter?

11. A builder sold a house for \$4669.20, which was $12\frac{1}{2}\%$ more than it cost him to build it. What did it cost to build the house?

12. A merchant's sales increased $12\frac{1}{2}\%$ the second year, 20% the third year, and 25% the fourth year. If his sales the fourth year amounted to \$26,482.40, what were his sales the first year?

13. The imports made by the Argentine Republic for the first quarter of 1914 decreased 14.2% as compared with the first quarter of 1913. If the decrease amounted to \$13,027,000, what was the amount imported the first quarter of each year?

14. The following table gives approximately, some of the most important imports from Turkey to the United States. Give the rate of increase or decrease for each article, to the nearest per cent.

ARTICLE	1912	1913	PER CENT INCREASE	PER CENT DECREASE
Attar of Roses	\$ 20,000	\$ 60,000		
Bazaar Goods	40,000	140,000		
Brandies	4,000	11,000		
Brass and Copper Work	7,500	9,000		
Carpets and Rugs	1,800,000	1,500,000		
Cheese	24,000	18,000		
Cotton Lace	600,000	270,000		
Chrome Ore	85,000	170,000		
Emery Stone	300,000	260,000		
Fruits (dried):				
Dates	560,000	700,000		
Figs	880,000	814,000		
Raisins	64,000	96,000		
Furs	25,000	5,000		
Gluestock	15,000	7,500		
Gum:				
Mastic	18,000	12,000		
Tragacanth	96,000	160,000		
Licorice Root	1,300,000	650,000		
Mohair	340,000	540,000		
Mother of Pearl	27,000	18,000		

15. The following is a continuation of the table on page 157.

ARTICLE	1912	1913	
		Increase	Decrease
Nuts:			
Almonds	$ 10,000	300%	
Filberts	224,000		25%
Walnuts	50,000	200%	
Pistachias	145,000	40%	
Olive Oil	115,000		60%
Opium	2,100,000		$33\frac{1}{3}\%$
Provisions	225,000		20%
Puer	25,000		20%
Rags	6,000	$66\frac{2}{3}\%$	
Rubber, old	65,000		20%
Seeds:			
Alfalfa	1,600	$6\frac{1}{4}\%$	
Canary	120,000		40%
Millet	13,000		90%
Poppy	27,000		$66\frac{2}{3}\%$
Silk	300	400%	
Slippers	24,000	$33\frac{1}{3}\%$	
Skins	900,000	20%	
Tobacco	10,000,000	7%	

Find the value of the imports for 1913.

16. The population of the State of Kansas is 1,769,257. The school population is 20% of the civil population. Of the school population, 6% are in high school. The grade school enrollment is divided among the different grades as follows: 1st grade, 21.21%; 2d grade, 15.38%; 3d grade, 14.72%; 4th grade, 13.61%; 5th grade, 11.74%; 6th grade, 9.82%; 7th grade, 7.83%; 8th grade, 5.69%. Every pupil in the 4th, 5th, 6th grades must buy a language book and every pupil in the 7th and 8th grades must buy a grammar. How many of each of these two books will be sold?

17. The following table shows the numbers of pupils enrolled in the various grades of a certain city during five consecutive years. Fill the blank spaces showing what per cent of the total is in each

grade, and the average enrollment for the five years. Carry to four places.

Grade	Enrollment					Per Cent					Average
	1923	1924	1925	1926	1927	1923	1924	1925	1926	1927	
First	6838	6679	6200	6297	6215						
Second	5100	5169	5058	4617	4609						
Third	4586	4786	4773	4603	4516						
Fourth	3990	4416	4532	4644	4540						
Fifth	2996	3108	3530	3755	3895						
Sixth	1966	2231	2406	2823	3021						
Seventh	1303	1507	1642	2043	2112						
Eighth	985	1067	1273	1407	1573						

If the school enrollment, including the high school, was one fifth of the civil population, and if the high school enrollment was $6\frac{1}{2}\%$ of the total school enrollment, what was the size of this city in 1927?

18. The following table represents the increase or decrease, and the rate of increase or decrease, on some of our imports from one Consular district of France during 1913, as compared with 1912:

Find the amount of goods imported for both 1912 and 1913, according to this table.

ARTICLE	Increase over 1912	Decrease from 1912	Per Cent Increase	Per Cent Decrease	Imported 1912	Imported 1913
Aniline Dyes		$4,000		$66\frac{2}{3}\%$		
Bagging, Old	$35,000		$87\frac{1}{2}\%$			
Bags, Hemp		1,750		50%		
Cotton Cloth	225,000		4500%			
Cotton Waste	18,000		$66\frac{2}{3}\%$			
Flax Waste	32,000		40%			
Horses		30,000		$41\frac{2}{3}\%$		
Linen		40,000		10%		
Lithophone Powder	1,600		200%			
Machinery		8,000		20%		

19. Before the war in 1914 silver bullion was worth 54 cents an ounce. In Sept., 1917, it was worth $\$1.00\frac{1}{2}$. If a dollar weighs $412\frac{1}{2}$ grains and is 10% alloy, what per cent did the silver dollar increase in intrinsic value in these three years? How much money would have been required in 1914 to purchase bullion for the coinage of 100,000 silver dollars? in 1917?

Oral Review Problems

1. What is 4% of 75?
2. What is .04 of 80%
3. What is 20% of 150?
4. What is .20 of 120?
5. What is $\frac{1}{5}$ of 140?
6. What is 25% of 160?
7. What is .25 of 140?
8. What is $\frac{1}{4}$ of 180?
9. What is $33\frac{1}{3}\%$ of 165?
10. What is $.33\frac{1}{3}\%$ of 156?
11. What is $\frac{1}{3}$ of 195?
12. What is 40% of 90?
13. What is .4 of 90?
14. What is $\frac{2}{5}$ of 90?
15. What comm. frac. = 60%?
16. What % = $\frac{3}{4}$?
17. What decimal = $37\frac{1}{2}\%$?
18. What comm. frac. = $37\frac{1}{2}\%$?
19. $\frac{5}{6}$ = what %?
20. 35% = what decimal?
21. 18 is 3% of what number?
22. 24 is 8% of what number?
23. 15 is 20% of what number?
24. 40 is 5% of what number?
What number is
25. 20% more than 60?
26. 25% more than 80?
27. 40% less than 150?
28. What is $\frac{3}{5}\%$ of \$100?
29. What is $\frac{3}{8}\%$ of \$1200?
30. How many mills on \$1 = $\frac{2}{5}\%$?
31. 120 is $\frac{4}{5}\%$ of what number?
32. $\frac{3}{5}\%$ = how many cts. on \$1?
33. What is the amt. if 6% = \$30?
34. What is the amt. if 5% = \$60?
35. What is the amt. if .12 = \$150?
36. 75 is 5% of what number?
37. 54 = .06 × what number?
38. 120 = 6% of what number?
39. .12 of what number = 180?
40. 175 = .25 × what number?
41. \$15 = $1\frac{1}{2}\%$ of what amount?
42. \$36 = $\frac{3}{5}\%$ of what amount?
43. What is $2\frac{1}{2}\%$ of \$450?
44. What is $3\frac{1}{3}\%$ of \$780?
45. 80 is .16 of what number?

46. $170 \div 850 =$ what per cent?

47. $45 \div 600 =$ what per cent?

48. What $\% = 30 \div 500$?

49. What decimal $= 75 \div 1500$?

50. $\frac{4}{5}\%$ of \$1200 = what?

51. The number of students enrolled in a certain class last year was 50. The number this year is 60. What is the per cent of increase?

52. A lot was purchased for \$300. It increased in value 20% in one year. What was its value the second year?

53. A house was sold at a profit of 20%. If the owner realized \$360 by the sale, what did the house cost?

54. A contractor constructed a building for \$45,375. If his profit was equal to 10% of the contract price, what was the amount of his profit?

55. A lot was purchased in March, 1923, for \$3500 and was sold one year later for \$3850. What was the per cent of profit? What was the lot worth in March, 1926, if its value increased at the same rate?

56. Certain laborers were working 9 hrs. for \$1.80. Owing to financial depression they were compelled to work 10 hrs. for the same money. What was the per cent of decrease in their wages per hour?

57. A creditor receives from a bankrupt \$1600, which is 40% of his claim. What is the amount of his claim?

58. This year my sales were \$12,000, which was an increase of 20% over last year. What were my sales last year?

59. In a storm a herder lost 24 sheep, or 2% of his herd. How many sheep were in the herd?

60. An estate has a net income of \$16,000, which is equal to 4% of the entire estate. What is the value of the estate?

61. A house rents for \$30 a month. The annual expenses are: Taxes, \$40, insurance \$10, water tax \$10, general expenses \$20. If the owner realizes 7% on the cost of the property, what did it cost?

62. A farmer bought a herd of cattle for \$2500. It increased 20% in value by feeding. He then lost 20% of the herd by disease. What is the value of the remainder of the herd?

CHAPTER XI

COMMISSION AND BROKERAGE

Commission is the performance of a service by one party, called an agent, or commission broker, for a principal, which service consists of effecting a sale or a purchase of merchandise or other property.

The compensation paid for this service is also called the *commission*.

The commission or brokerage is generally a certain per cent of the amount of the sale or purchase, but it may be a certain amount for each unit of weight or quantity; as 2¢ per bu., or $\frac{1}{2}$¢ per lb.

A **commission merchant,** or *broker*, is a merchant who buys or sells property as an agent.

A commission merchant usually has possession of the property he sells.

ST. LOUIS, MO., Sept. 5, 19

Account Sales of 150 bbl. BEN DAVIS APPLES

For Account of A. R. DOWNER,

OLD MISSION, MICH.

BY M. A. FROST & CO.

1918							
Aug.	15	50 bbl.	@ $5.50	275			
	20	25 bbl.	@ $5.65	141	25		
	27	75 bbl.	@ $5.40	405			
						821	25
		CHARGES					
	10	Freight and drayage		21	50		
	27	Commission 3%		24	64		
						46	11
	27	Net proceeds by Chgo. dft. herewith				775	14

ACCOUNT SALES

A broker usually does not have possession of the property he sells, but merely acts as an agent in bringing together the buyer and the seller.

A **guaranty** is a charge made by the agent to cover his personal risk in guaranteeing the goods to be of a certain quality.

This charge is generally estimated as a certain per cent of the amount of the sale or purchase.

Collection. The broker sometimes makes a separate charge for collecting the money for the sale. This charge for collection may be a per cent of the amount of the sale, or may be a flat sum.

An **account sales** is a formal statement prepared by the agent, to show the quantity and prices of sales, the expenses that are properly charged to the principal, and the net proceeds of the sales.

The **net proceeds** is the amount due the principal after all expenses have been taken from the amount of the sales or the amount of the collection.

An **account purchase** is a formal statement prepared by the agent to show the amounts of purchases for the principal, other expenditures made for the principal, and the agent's charges. The commission is based in each case upon the amount or quantity of the purchase.

GULFPORT, MISS., Nov. 15, 1916

Purchase for Account of JEVNE & CO.,

CHICAGO

BY COX AND SONS

500 lb. Success pecans,	@ 40¢	200				
2000 lb. seedling pecans,	@ 15¢	300				
750 lb. Pabst paper shell pecans,	@ 55¢	412	50			
				912	50	
CHARGES						
Hauling, packing and loading		17	50			
Commission, 2%		18	25			
Guaranty, 1½%		13	69			
				49	44	
Total charged to your acct.				961	94	

ACCOUNT PURCHASE

ORAL

1. The commission paid for selling a lot was $37.50. If the rate for commission was 5%, what was the sale price of the lot?

2. My attorney charges me 10% for collecting claims. Last month he retained $48.50 as his commission. How much did he collect?

3. The Chicago agent of a miller deducted from the account of the principal $68.74 as his commission for buying grain at a commission charge of $\frac{1}{2}¢$ per bu. How many bushels of grain did he buy?

4. I sold a lot for a client and returned to him $950, which was the amount left after I had deducted 5% for my commission. Find the selling price of the lot.

5. Hannan & Co. sold a house for Mr. James. They deducted 3% for sales commission, and gave Mr. James a check for $4850. What was the amount of the sale?

6. The Bowlus Fruit Co. sold 100 bbl. apples at $6 per bbl., 100 bbl. at $4.50 per bbl., and 30 bbl. at $4 per bbl. What was their charge for commission at 5%?

7. Logan & Bryan bought 1 car, 1000 bu. wheat; 1 car, 900 bu. wheat; 2 cars, 2500 bu. oats; and 1 car, 1200 bu. oats. At $\frac{1}{2}¢$ per bu., what was their charge for commission?

8. A real estate salesman sold the following lots during July, for a commission of 5%: No. 17, $500; No. 67, $650; No. 85, $800; No. 21, $525; What was his total charge for commission?

9. A farmer shipped 500 lb. of butter to his agent in Cincinnati. It was sold at 22¢ per lb. The agent deducted 5% for commission. Other charges amounted to $4.50. What amount did the farmer receive?

10. An agent bought 1500 bu. of corn at 80¢ and charged 2% for commission. If there were other expenses amounting to $14.50, what was the total cost of the corn?

11. An agent offers to sell oats for you and to charge for commission $\frac{1}{2}¢$ per bushel or 1%. Which is the better offer if oats are 45¢ per bushel?

12. An agent charged 2% for commission and 2% for guaranty. If commission and guaranty amount to $60, how many barrels of apples at $2.50 each did he purchase? What was the gross cost?

13. A lumber dealer bought through an agent 50,000 ft. of cypress at $22.00 per M. What was the total cost of the lumber if the agent's charge for commission was 2% and freight amounted to $250?

14. A contractor constructed a building on a percentage basis. He received the cost of the building plus 10% as his profit. He made a statement asking for $6600. What did the building cost, exclusive of his charge for commission?

15. An agent sold 100 bu. cherries at $2.70 per bu. He deducted for commission 6%, and other charges amounted to $11.50. What were the net proceeds of the consignment?

Problems in Sales on Commission

WRITTEN. Find the commission and net proceeds of each of the following:

	SALE	RATE FOR COMMISSION		SALE	RATE FOR COMMISSION	GUARANTY	CHARGES
1.	$3000	2%	11.	$ 637.49	6%	2%	$14.78
2.	4500	4%	12.	1470.36	7%	------	29.37
3.	5670	5%	13.	1142.63	5½%	2%	54.73
4.	6739.50	6%	14.	650.12	8%	------	60.26
5.	7593.80	5½%	15.	971.46	4%	1%	37.53
6.	3650.29	6½%	16.	1653.90	6½%	2%	15.49
7.	3573.80	3½%	17.	2438.56	7%	2%	37.92
8.	2673.90	4½%	18.	2637.83	4%	------	9.37
9.	2167.42	3½%	19.	960.54	2½%	1%	139.61
10.	1957.20	2½%	20.	1643.50	4%	3%	-------

21. An auctioneer sold goods amounting to $3685.40, for a commission of 5%. What was due him for commission?

22. A real estate salesman sold 3 lots at $460 each, 7 lots at $950, and 5 lots at $1175 each. If he was paid for commission at 4½%, what was his total commission?

23. If a bushel of oats sells for 53½¢, which is the better for the broker, a commission of 2% or a commission of 1¼¢ per bu.? How much better, on 864 bu.?

24. A commission merchant received from a farmer 24 crates (32 boxes each) of strawberries. The merchant sold 384 boxes at $9\frac{1}{2}¢$, 224 boxes at $9¢$, and 160 boxes at $9\frac{3}{4}¢$. The freight charges were \$8.76, drayage \$2.40, collection 1%, and commission 7%. Find the net proceeds of the sale, and the net average price the farmer received per crate.

25. An agent wrote a policy of insurance for \$10,000 at an annual premium of \$34.80 per thousand. The agent received for his commission 25% of the premium paid the first year. What was the amount paid him for commission?

26. A salesman receives a salary of \$90 per month. For commission he receives 2% on all sales up to \$2500 and 3% on amounts beyond that sum. His sales for July amount to \$3385.70. What are his total earnings for the month?

27. An agent sold 6000 bu. of oats and after deducting 3% for his commission remitted his principal \$2677.20. At what price per bu. did he sell the oats?

28. A commission merchant received a consignment of 50 crates (bu.), 32 boxes each, of strawberries; 20 crates (bu.), 32 boxes each, of raspberries; 30 crates (bu.), 32 boxes each, of cherries. He sold 40 crates of strawberries at $9\frac{1}{2}¢$ per box, and 10 crates at $8¢$ per box; the raspberries he sold at \$2.40 per bu., and the cherries at \$2.70 per bu. His charges were as follows: For commission 6%, freight \$47.86, and drayage $5¢$ per crate. Prepare an account sales, supplying imaginary dates and names.

29. A live stock dealer made a shipment of 1 car, 20 head, of steers averaging 1350 lb. each, to a commission dealer in Chicago. They were sold at \$12.35 per cwt. The charges were as follows: For commission $2\frac{1}{2}$%, collection $\frac{1}{2}$%, freight $29¢$ per cwt., and feeding \$63.70. Prepare an account sales.

Purchases and Sales on Commission

1. Find the total cost of a purchase of wheat at Duluth, Minn., consisting of 25,000 bu. at $\$1.84\frac{3}{4}$ per bu., the agent's charges for commission being $\frac{1}{2}¢$ per bu. and 1% of purchase price for guaranteeing the quality purchased.

2. Find the total cost of each of the following:

	Prime Cost	Commission	Guaranty	Other Expenses
(*a*)	$ 3,673.82	2%	1%	$126.49
(*b*)	1,723.89	3%	1%	72.68
(*c*)	8,750.00	5%	-----	21.75
(*d*)	12,500.00	4%	-----	----------
(*e*)	3,729.46	6%	2%	149.74
(*f*)	2,638.93	5½%	-----	263.94
(*g*)	1,746.27	7%	1%	84.12
(*h*)	1,217.50	8%	2%	41.37

3. A principal remitted to his agent $477 with which to purchase strawberries. How many crates (1 bu. each) did he purchase at $3.00 per bu., allowing for a commission of 6%?

4. Find the amount to be invested and the commission in each of the following:

	Amount Remitted	Rate for Commission		Amount Remitted	Rate for Commission
(*a*)	$525	5%	(*g*)	$ 682.50	5%
(*b*)	630	5%	(*h*)	1275.66	3%
(*c*)	312	4%	(*i*)	1502.97	2%
(*d*)	918	2%	(*j*)	1847.69	1%
(*e*)	856	7%	(*k*)	1249.89	6%
(*f*)	714	2%	(*l*)	1375.38	8%

5. A car load of potatoes weighing 32,400 lb. was sold by an agent at 97¢ per bu. of 60 lb. The freight was 24¢ per cwt., commission 3%, and other charges $11.80. The principal then purchased 60 head of sheep at $6.75 per head, commission 2% and drew on the agent who sold the potatoes for the amount necessary to pay for the sheep. What balance was still due the principal?

6. Brown sold 100 bales of cotton (500 lb. each) at 19.8¢ per lb., paying an agent 2% commission, freight $97.50, and other expenses $29.85. He used the proceeds to buy tea at 52¢ a pound plus 2% commission. How many half chests of tea (70 lb. each) did he purchase?

Note. Always state the balance still due the principal, if any.

7. How many crates of cherries, 1 bu. per crate, can be purchased with a remittance of $188.76, if the agent charges 4% for commission and cherries are worth $1.50 per bushel?

8. How many barrels of apples can be bought for \$452.95 if the agent's charge for commission is 4% and other expenses amount to \$31.75, with apples at \$3.50 per bbl.?

9. How many crates of eggs, 30 doz. each, at 34¢ per dozen, can be purchased with a remittance of \$897.75 if the agent is allowed 5% for purchase commission?

10. A savings bank bought bonds for \$50,200. The par value of the bonds was \$50,000, and $\frac{1}{8}$% of the par value was paid the broker for commission. Find the total cost of the bonds.

11. A miller sent to a Chicago grain broker \$7000 for the purchase of wheat. The broker made the following purchases: Sept. 15, 1 car, 900 bu. at \$1.88$\frac{1}{2}$; Sept. 27, 1 car, 1050 bu. at \$1.89$\frac{1}{4}$; Oct. 5, 1 car, 900 bu. at \$1.91. The broker received $\frac{1}{2}$¢ per bu. for commission, and had other expenses amounting to \$41.83. Make a statement of the account on Oct. 5.

12. A grain broker made purchases for his principal as follows: Sept. 1, 1 car wheat, 900 bu. at \$1.89$\frac{1}{2}$; Sept. 10, 1 car wheat, 1000 bu. at \$1.90; Sept. 11, 1 car oats, 1200 bu. at 52$\frac{1}{2}$¢; Sept. 22, 1 car wheat, 860 bu. at \$1.90$\frac{1}{2}$¢ per bu., and 1 car oats, 1200 bu. at 53$\frac{1}{4}$¢. On Sept. 17, the principal remitted \$4000. If the broker charged $\frac{1}{2}$¢ per bu. for commission and other expenses amounted to \$54.78, what was the balance due the broker on Sept. 30?

13. R. B. Gordon & Co., Duluth, are grain brokers. They purchase as follows: For the London Mill Co., 1 car (1000 bu.) wheat at \$1.84$\frac{1}{2}$; 1 car (1100 bu.) wheat at \$1.83$\frac{3}{4}$; 1 car (1400 bu.) oats at 52$\frac{1}{4}$¢; 1 car (1200 bu.) oats at 53¢; 1 car (900 bu.) wheat at \$1.84$\frac{3}{4}$¢; 1 car (1350 bu.) oats at 53$\frac{1}{4}$¢. They receive 2% for commission and have incurred expenses amounting to \$45.83. The mill company, at the time they requested Gordon & Co. to buy for them, enclosed a draft for \$5750. Prepare an account purchase.

14. The gross cost of a carload of wheat containing 1150 bu. was \$2044.44. The agent charged 2% for buying and freight amounted to \$26.88. At what price per bu. was the wheat purchased?

15. An agent charged \$21.60 for buying a car of shelled corn on 2$\frac{1}{2}$% commission. If the freight was \$30.82 and the car contained 800 bu., what was the gross cost per bu. to the principal?

16. Wennemer Bros. & Co., general contractors, agreed on May 15 to construct a building for the Central Realty Co. on a commission basis. The realty company was to pay the cost of the building plus 10% profit on sub-contracts. The masonry work was done by the general contractors for $24,875. They sub-let other parts of the work as follows: Carpentry, $10,375; electrical work, $2,150; plumbing and heating, $4,385.50; plastering, $3250; decorating, $857.50. The Central Realty Co. made the following payments: June 27, $5000; July 20, $8000; Aug. 31, $3500; Sept. 25, $10,000; Oct. 29, $12,500. Make a statement showing the amount due the general contractors on Nov. 30, $2500 to be withheld for 6 mo. as a guaranty.

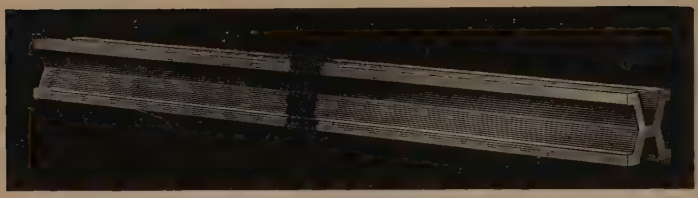
17. An agent agreed to sell 120 city lots for a commission of 15% and pay all incidental expenses. The first 30 lots were to be sold for $650 each, the next 50 lots at $575 each, and the last 40 lots at $500 each. The agent employed two sub-agents and paid them for commission 5% on all sales they made. One sub-agent sold 15 lots at $650, 20 lots at $575, and 10 lots at $500; the other sold 10 lots at $650, 15 lots at $575, and 20 lots at $500; the agent sold the remainder of the lots. The expenses were as follows: Advertising, $385.45; automobile expenses, $486.93; other expenses, $653.20. What profit did the agent realize?

18. The executor of an estate was allowed for fees as follows: 5% on the first $1000 paid out, $2\frac{1}{2}$% on the next $4000 paid out, and 1% on all payments over $5000. The payments for the first month were $6390.75. What was due him for fees?

19. In a certain state an executor is allowed for fees on personal property 5% for the first $1000, $2\frac{1}{2}$% for the next $4000, and 1% for all sums beyond the first amounts. If the total charges for commissions allowed the executor amounted to $200, what was the value of the estate?

CHAPTER XII

PRACTICAL MEASUREMENTS



Section of U. S. Standard Meter

Fundamental Units of Measurement. The fundamental units of measurement are units of numerical quantity, length, weight, and time. Other units, such as units of value, surface, volume, and capacity, are derived from these fundamental units. Direction is an essential factor in some calculations.

The fundamental unit of numerical quantity is the figure 1. All other numerical quantities, integral or fractional, are based upon it.

U. S. Standard Kilogram

The fundamental unit of length is the meter. Our foot of 12 inches is .3068 meter. Our linear, square, and cubic measures, are all based upon this fundamental unit.

The fundamental unit of weight is the kilogram. Our pound (Troy) is equivalent to .373267 kilogram. Our various tables of weight are all based upon this fundamental unit.

The fundamental unit of time is the second.

The solution of the problems of this chapter requires the use of the tables of measurement found in the appendix. All these tables are based directly or indirectly on one of the fundamental units mentioned above.

Note that the tables of numerical quantity have a uniform decimal progression, since they are based upon our decimal system of numbers, but that the tables based upon the units of length, weight, and time (except the metric tables) have an irregular progression.

In calculations involving the tables of measurement, the principles of addition, subtraction, multiplication, and division are applied in the usual way, except that the values of units "borrowed from" or "carried to" the next higher order, instead of being regularly 10, will vary, depending upon the table applicable.

Changing denominate numbers from larger units to smaller, or from smaller units to larger, is such a simple matter that no special form of solution will be given for it. The pupil can solve such problems in his own way, avoiding long numbers and fractions as much as possible.

WRITTEN

1. Express 3 miles in feet.
2. Express 840 rods in miles.
3. Express 160 rd., 54 yd., 1 ft., 8 in. in inches.
4. Express 1 long ton in ounces, avoirdupois.
5. Express $\frac{3}{4}$ rd. in inches.
6. Express $\frac{1}{2}$ ton, $\frac{3}{4}$ cwt., $\frac{7}{8}$ lb., in pounds.
7. Express $2\frac{1}{2}$ A. in sq. ft.
8. Express 1 A., 80 sq. rd. in sq. yd.
9. Express $3\frac{1}{9}$ cu. yd. in cu. in.
10. Express 1 cu. yd., 4 cu. ft., 8 cu. in., in cubic inches.
11. Express 1 bu., 3 pk., 4 qt., in pints.
12. Express 2 bu. 1 pk., 1 pt., in quarts.
13. Express $2\frac{1}{2}$ gal. in pints.
14. Express 3 gal., 1 pt., 2 gills, in pints.
15. Express 2.48 lb. gold (Troy weight) in ounces.
16. Express $\frac{3}{4}$ of a quadrant in degrees and minutes.
17. Express 1 wk., 3 days, 30 min., in hours.
18. Express 2 gross, $1\frac{5}{6}$ doz., in units.

Express in the largest possible units (Thus: 21 pt. = 2 gal. 2 qt. 1 pt.):

19. 1111 pt. **22.** 12,526 cu. in. **25.** 9849 sec. (circular)

20. 1317 oz.(AV.) **23.** 5729 sq. in. **26.** 9238 ft.

21. 9726 in. **24.** 4835 sec. (time) **27.** 6529 A.

28. Express 9 in. decimally in rods.

29. Express 1580 feet decimally in miles.

30. Express $\frac{7}{8}$ cu. ft. in cu. yd., common fraction form.

31. 13.5 qt. are what part of a barrel containing 31.5 gal.?

32. 20″ are what part of a circle?

33. 7 hr., 25 min. are what part of a month?

34. Express 60 days as a fractional part of a year.

35. How many times 60 days are:

180 days	90 days	1 mo. 12 da.
120 days	2 mo. 6 da.	2 mo. 20 da.
5 months	96 days	6 mo. 24 da.
15 days	75 days	4 mo. 15 da.
96 days	7 months	10 mo. 18 da.
135 days	11 months	5 mo. 20 da.

Addition

EXAMPLE. A milk dealer made deliveries to a restaurant during the week ending Jan. 8, 1916, as shown. What was the total quantity of milk delivered during the week?

	gal.	qt.	pt.
Mon.	3	2	1
Tues.	4	0	1
Wed.	3	1	0
Thurs.	3	3	1
Fri.	4	2	1
Sat.	3	3	1
	23	1	1

First add the pints column. The total is 5 pints. $5 \div 2 = 2$, the number of quarts, with 1 pt. over. Write the 1 under the *pints* column, and add 2 to the *quarts* column.

Second, add the *quarts* column, including the 2 quarts carried from the *pints* column. The total is 13 quarts, which equals 3 gal. 1 qt. Write 1 under the *quarts* column.

The total of the *gallons* column, including the 3 gallons carried from the *quarts* column, is 23. The total quantity is therefore 23 gal., 1 qt., 1 pt. The foregoing method can be applied to the addition of all kinds of denominate numbers. If the student is not familiar with all the principal tables of denominate numbers, he should be assigned one or two each day to be memorized.

WRITTEN

1. What is the total acreage of four fields as follows: 24 A., 76 sq. rd.; 41 A., 84 sq. rd.; 33 A., 55 sq. rd.; 29 A., 114 sq. rd.?

2. What must be the length of a fence around a field having five sides that measure as follows: 14 rd., 12 ft.; 21 rd., 8 ft.; 15 rd., 6 ft.; 18 rd., 9 ft.; 10 rd., 14 ft.?

3. The glass in the windows and doors of a certain room consisted of pieces of the following sizes: 4 sq. ft., 112 sq. in.; 9 sq. ft. 84 sq. in.; 5 sq. ft. 90 sq. in.; 7 sq. ft. 36 sq. in.; 6 sq. ft. 84 sq. in.; 4 sq. ft. 120 sq. in. What was the total amount of glazing for the room?

4. A room measures 24 ft. 2 in. in width and 27 ft. 9 in. in length. What is the distance around it?

Subtraction

EXAMPLE. Find the time between Nov. 23, 1908 and Mar. 4, 1919.

yr.	mo.	da.
1919	3	4
1908	11	23
10	3	11

4 days + 30 days (1 mo.) = 34 days. 34 days − 23 days = 11 days. 2 mo. + 12 mo. (1 yr.) = 14 mo. 14 mo. − 11 mo. = 3 mo. 1918 yr. − 1908 yr. = 10 yr. The time is therefore 10 yr. 3 mo. 11 da.

The method of subtraction illustrated above can be applied to all kinds of denominate numbers. Do you know all the tables given in the appendix?

WRITTEN

1. Find the time from July 12, 1920 to Feb. 8, 1926.

2. Find the time from Oct. 25, 1926 to Mar. 13, 1928.

3. A street is 3 mi. 140 rd. long. Of this, 80 rd. 12 ft. are paved with Belgian block; 200 rd. 10 ft. are paved with brick; 180 rd. 9 ft. are paved with asphalt. The remainder is unpaved. What is the length of the unpaved portion?

4. From a vat containing 212 gal. 3 qt. 1 pt. of vinegar, enough was drawn to fill six barrels containing respectively: 31 gal. 2 qt.; 32 gal. 3 qt.; 30 gal. 2 qt. 1 pt.; 32 gal. 1 pt.; 33 gal. 1 qt. 1 pt.; 32 gal. 3 qt. How much remained?

5. Nathan Shepherd sold the following quantities of hay: 3 T. 590 lb.; 5 T. 840 lb.; 1 T. 1070 lb. If his total crop was 36 T., how much had he left for his own use?

6. Jones gave Smith his note for 3 months, 15 days. The note was dated Feb. 2, 1924, and if unpaid at maturity was to bear interest from its due date until date of payment. Jones paid the note and interest on Jan. 20, 1928. For what period of time did he pay?

7. For how long did each of the following bear interest?

Date	Time to Run	Interest from When	Paid When
May 7, 1927	3 mo.	From date of note	July 15, 1928
Mar. 10, 1926	6 mo.	After maturity	June 10, 1928
Dec. 20, 1927	60 days*	After maturity	Sept. 25, 1928
Sept. 9, 1927	90 days	From date of note	At maturity

*To find the due date, when the time of the note is stated in days, count forward actual days, thus: There are 11 days left in December and 31 in January. $11+31=42$, leaving 18 days to run after January. Due date February 18.

Multiplication

Example. A can of peaches weighs 2 lb. 14 oz. What will a quarter gross of such cans weigh?

lb.	oz.
2	14
	36
103	8

Write the problem in the form shown. Then multiply 14 oz. by 36. The result is 504 oz. $504 \div 16 = 31$, the number of lb., with 8 oz. over. Write 8 in the *ounces* column. Then multiply 2 lb. by 36 and add the 31 lb. carried from the *ounces* column. 36×2 lb. $= 72$ lb. 72 lb. $+ 31$ lb. $= 103$ lb. The completed result is 103 lb. 8 oz. or $103\frac{1}{2}$ lb.

Example. What is the cost of 14 barrels of vinegar, each containing 30 gal., 3 qt., 1 pt., at $12\frac{1}{2}$¢ a gallon?

gal.	qt.	pt.
30	3	1
		14
432	1	0

Multiplying by the process shown above, it is found that the total quantity is 432 gal. 1 qt. or $432\frac{1}{4}$ gal., which at $12\frac{1}{2}$¢ is \$432.25 ÷ 8 or \$54.03 (aliquot parts method).

Had the price been given per quart it would have been necessary either to change the total quantity to quarts ($432 \times 4 = 1728$. $1728 + 1 = 1729$, the number of quarts), or to express the contents of one barrel in quarts and multiply by 14.

Written

1. Cans of tomatoes of a certain kind weigh 1 lb. 10 oz. each. They are packed 3 doz. cans to each case. If the packing case weighs $4\frac{1}{2}$ lb. what will be the weight of the case and its contents?

2. What would be the weight of 400 such cases (problem 1) when filled and ready for shipment? What would be the total freight charge at 27¢ per 100 pounds?

3. A factory employed 50 persons on an average of 8 hr. 10 min. a day for 26 days. If the average wage was 42¢ an hour, what was the total pay for the month?

4. A farmer wishes to measure a rectangular field.. He does not possess a surveyor's tape but instead uses a wire 3 rd. 10 ft. in length. What is the distance around the field if its length is 18 lengths of this wire and its width 10 lengths of the wire?

5. How many bushels of clover seed are there in 200 bags each containing 2 bu. 1 pk.? What is the value of the clover seed at \$6.50 per bu.?

6. During one year the state of Missouri produced 49,100 ounces (Troy weight) of silver, which was worth 54¢ per oz. What was its total value?

7. North Carolina produced in one year 3478 fine ounces of gold. Find the total value of the output, if gold was worth \$20.67 per oz.

8. Silas Wright sold at $9\frac{1}{2}$¢ per lb. 5 steers weighing as follows: 1390 lb., 1420 lb., 1450 lb., 1370 lb., and 1430 lb. How much did he receive for them?

9. A grocer bought a barrel of vinegar containing 31 gal. 3 qt. for \$7.50. He emptied the contents into pint bottles costing 5¢ a dozen, and sold it for 4¢ a pint. What was his profit, allowing 3 pints for leakage?

Division

To divide a compound denominate number by an integer.

EXAMPLE. If 277 gal., 3 qt., 1 pt. of oil be divided equally among 9 barrels, how much will be placed in each of the barrels?

	gal.	qt.	pt.
9)277	3	1	
30	3	1	

Write in form for division as shown. 277 gal. divided among 9 bbl. gives 30 gal. for each bbl., with 7 gal. over. 7 gal. = 28 qt. 28 qt. + 3 qt. = 31 qt. 31 qt. ÷ 9 = 3 qt. with 4 over. 4 qt. = 8 pt. 8 pt. + 1 pt. = 9 pt. 9 pt. ÷ 9 = 1 pt. The completed answer, therefore, is 30 gal., 3 qt., 1 pt.

Another way to divide in denominate numbers is to reduce the entire expression to one denomination and divide.

Thus in the above case the entire expression could have been written as gallons, or as quarts, or as pints, and then divided by 9.

277 gal. 3 qt. 1 pt. = $1111\frac{1}{2}$ qt.

$1111\frac{1}{2}$ qt. ÷ 9 = $123\frac{1}{2}$ qt. or 30 gal. 3 qt. 1 pt.

To divide a compound denominate number by another, express both in terms of the same denomination and divide.

EXAMPLE. A barrel of vinegar containing 32 gal. 2 qt. 1 pt. is emptied into small jugs containing 1 qt. 1 pt. How many jugs will be required?

32 gal. 2 qt. 1 pt. = 261 pt.

1 qt. 1 pt. = 3 pt.

261 ÷ 3 = 87, the number of jugs required.

WRITTEN

1. A car of shelled corn weighs 54,880 lb. How many bushels does it contain if shelled corn weighs 56 lb. per bushel? What is it worth at 70¢ per bushel?

2. Samuel Bowers owns land having 45 rd. 10 ft. 6 in. frontage. How many 35 ft. lots can he get out of this strip of land if an alley 18 ft. wide is to be allowed for?

3. A railroad rail is 33 ft. long and weighs 80 lb. per running yard. How many rails will be required to lay one mile of track and how many tons will the rails weigh?

4. A car is marked "capacity 60,000 lb." If a car can be loaded to 110% of its marked capacity and if oats weigh 32 lb. per bushel, how many bushels of oats may be shipped in this car?

5. A coal dealer buys in one car load 40 long tons (2240 lb.) of anthracite coal at \$5.70 per ton. Find the cost per short ton (2000 lb.).

6. A freight locomotive consumes about 220 lb. of coal to each mile it runs. How many miles can it run on a supply of 10 long tons? (Give nearest integer.)

7. A certain aluminum casting weighs 4 lb. 10 oz. Making no allowance for waste, how many castings can be made from 1 short ton of metal? (Give nearest integer.)

8. Adams loaned Burton the services of 5 helpers for 2 weeks, 4 days, and 3 hours. For how long must Burton loan the services of his three helpers to repay Adams, valuing the services of all helpers at the same rate?

9. A factory occupies one floor of a building 40′ by $62\frac{1}{2}$′, and pays a rental of \$150 per month. Find the annual and daily rental cost of this building per square foot.

10. The office of the above factory occupies a space 20′ by $22\frac{1}{2}$′. What fractional part of the entire space does the office occupy?

11. Johnson bought from a coal dealer seven loads of Pocahontas coal at \$4.10 per ton delivered. The net weights in pounds were as follows: 10,800, 9300, 9500, 9900, 10,500, 10,300, 9900. What was the amount of the bill?

CHAPTER XIII

INTEREST

- KINDS OF INTEREST
  - Simple
  - Compound
  - Simple Interest on Unpaid Interest
- FACTORS OF EVERY PROBLEM
  - (*a*) Counting Time
    - Calendar Months and Days
    - Compound Subtraction
    - Exact Days
  - (*b*) Expressing the time fractionally in terms of a year
  - (*c*) The computation
- METHODS OF SOLUTION
  - Ordinary Method
  - Exact Method
  - Bankers' Method
- PROCESSES OF FIGURING
  - Cancellation Process
  - Sixty Day Process

Interest. As commonly defined, interest is a compensation paid for the use of money.

Strictly speaking, interest is the *use of money*, without regard to what compensation was paid or whether any compensation was paid. The two uses of the word *interest* correspond to the two uses of the word *rent*. Neither interest nor rent is money, yet common speech sanctions the expressions "paid the interest," and "paid the rent," instead of "paid for interest" and "paid for rent."

Principal. The money at use is called the *principal*.

Rate. The *rate* is the number of hundredths of the principal paid for the use of the principal for one year. The rate is usually expressed as a per cent.

Legal Rate. This is the rate fixed by state law to apply to any interest-bearing obligation in the absence of an agreement between debtor and creditor as to the rate.

Maximum Rate. This is the highest rate that contracting parties are permitted by law to agree upon.

NOTE. The legal rate and the maximum rate are two distinct things. Most states prescribe both by statute, though a few states fix no maximum. In a few states the maximum rate and the legal rate are the same per cent. Any rate not greater than the maximum rate may be fixed by contract.

Usury is the charging of a rate of interest in excess of the maximum rate allowed by law.

In the states where usury is forbidden, the law usually prescribes some sort of punishment for those who charge usurious rates of interest. See *Burgess' Commercial Law.*

Kinds of Interest. Interest may be simple or compound.

Simple interest is interest upon principal only.

If $1000 be at interest, drawing 6% simple interest payable annually, $60.00 interest falls due at the end of each year. This $60.00 should be paid annually but even if it is not paid it is not permitted itself to draw any interest.

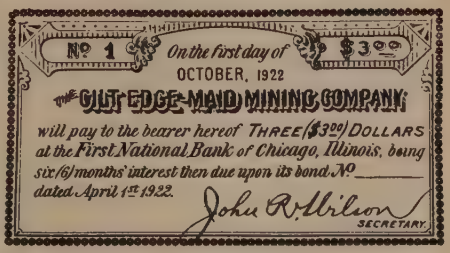
Compound interest is interest upon the original principal plus interest accruals periodically calculated.

If $1000 be at compound interest, at 6% calculated annually, it would draw interest as follows:

1st yr.	Int. on $1000.00 at 6%	=$60.00
2d yr.	Int. on $1060.00 ($1000.00+$60.00)	= 63.60
3d yr.	Int. on $1123.60 ($1000.00+$60.00+$60.00+$3.60)	= 67.42

Note that the first year's interest is upon principal only; the second year's interest is upon principal and interest; and the third year's interest is upon principal, interest, and interest on interest. This would be continued, further and further compounded, until the end of the time for which the obligation runs.

Simple interest on unpaid interest. This is a kind of interest that is sometimes used and which is a compromise between simple and compound. While not permitting interest to be compounded by the method just described, it permits interest items, if unpaid when due, to draw simple interest until paid.


No. 1 On the first day of OCTOBER, 1922 $3.00
THE GILT EDGE MAID MINING COMPANY
will pay to the bearer hereof THREE ($3.00) DOLLARS at the First National Bank of Chicago, Illinois, being six (6) months' interest then due upon its bond No. _____ dated April 1st 1922.
John R. Wilson SECRETARY

A COUPON

An illustration of this is an interest-bearing interest coupon attached to a bond. If not paid when due, it bears simple interest until paid.

Methods of Solution

There are three methods of solution commonly followed: The Ordinary Method, the Exact Method, and the Bankers' Method. A different result is secured by each method. Before you can understand these it will be necessary for you to study carefully the following statements:

Three factors in figuring interest. Every calculation of interest involves three factors:

(*a*) Counting the time
(*b*) Expressing the time fractionally in terms of a year
(*c*) The computation

(*a*) *Counting the time:* The time is sometimes counted by calendar months and days, sometimes by "compound subtraction," and sometimes by exact days.

Months and Days: Count forward the number of whole calendar months indicated (as 9 mo. from Jan. 10 to Oct. 10), then count exactly the days remaining. This method is used on the New York Stock Exchange and other stock exchanges.

Compound Subtraction: This is the method you learned under *denominate numbers*. It gives the same result as the months and days method when the extra days (after whole months have been counted) do not include the 31st of any month. If the extra days do include the 31st of some month, it gives one day less than the month and day method. If the extra days include the end of February, it gives two days more than the month and day method.

Exact Days: This method of counting time requires no explanation, as the term defines itself. If the time exceeds a year, whole years are counted as such.

(*b*) *Expressing the time fractionally in terms of a year.* The time having been counted, it is necessary to know how many days are to be considered a year. Sometimes a year is considered to be 360 days, and sometimes it is considered to be 365 days.

(*c*) *The computation.* Having counted the time, and having determined whether the basis of calculation is to be 365 days or 360 days, the figuring itself presents little difficulty.

RULE: The rule for figuring interest whatever method is used, is

Multiply together the principal, the rate, and the time expressed in years.

Ordinary Method. Count the time by compound subtraction and figure the interest on the basis of 360 days to the year. Consider a year as 12 months of 30 days each. This is the legal method for ordinary business transactions.

EXAMPLE. Find the interest on $1000 at 6% from Oct. 21, 1926 to Apr. 6, 1928.

Year	Month	Day
1928	4	6
1926	10	21
1	5	15

Int. for 1 yr. = $60.00

5 mo. 15 da. = 165 da.

$$\frac{\$1000.}{1} \times \frac{6}{100} \times \frac{165}{360} = \$27.50$$

Total = $87.50

It is stipulated by law in many states that in calculations of interest a year is to be considered as 12 months of 30 days each. This applies to the calculation of interest and also to the counting of the time.

Exact Method. Count time by exact days and treat these days as 365ths of a year.

EXAMPLE. Find the interest on $1000 at 6% from Oct. 21, 1926 to Apr. 6, 1928.

Oct. 21, 1926 to Oct. 21, 1927	= 1 yr.
Days left in October, 1927	= 10
Days in November	= 30
Days in December	= 31
Days in January, 1928	= 31
Days in February	= 29
Days in March	= 31
Days in April	= 6
	168

$$168 \text{ days} = \frac{168}{365} \text{ of a year}$$

$$\frac{\$1000}{1} \times \frac{6}{100} \times \frac{168}{365} = \$27.61$$

Int. for 1 yr. = $60.00

$87.61

Bankers' Method. Count the time by exact days, and figure the interest on the basis of 360 days to the year.

EXAMPLE. Find the interest on $1000 at 6% from Oct. 21, 1926 to Apr. 6, 1928.

NOTE. Do not confuse the expressions "Exact time" and "Exact interest." Counting time by exact days is not all of the process of finding exact interest, nor is this method of counting time confined to the finding of exact interest.

The time by exact days, as computed under the "exact days" method, is 1 yr. 167 days.

$$\frac{\$1000}{1} \times \frac{6}{100} \times \frac{167}{360} = \$27.83$$

$$\text{Int. for 1 yr.} = \underline{\$60.00}$$

$$\$87.83$$

NOTE. The interest figured by the Banker's Method is $87.83 as against $87.45 by the exact method and $87.50 by the so-called ordinary method.

Problems in Counting Time

Find the time between the two dates given in each of the following problems by exact days, by compound subtraction, and by months and days.

When using the months and days method, count from Jan. 28, 29, 30, or 31 to Feb. 28, as one month; and from the 31st of any month to the 30th of any month which has only 30 days, as one month.

1. April 10, 1928, to May 18, 1928.
2. April 18, 1927, to June 24, 1927.
3. May 5, 1927, to Sept. 1, 1927.
4. July 29, 1928, to Dec. 1, 1928.
5. Oct. 15, 1927, to Feb. 27, 1928.
6. Sept. 18, 1926, to Jan. 12, 1927.
7. Aug. 7, 1927, to Jan. 1, 1928.
8. June 18, 1927, to Oct. 13, 1927.
9. May 27, 1927, to Aug. 12, 1927.
10. Sept. 15, 1927, to Feb. 28, 1928.

Find the time given between the two dates given in each of the following problems by compound subtraction and by exact days.

1. May 27, 1926, to Dec. 1, 1927.
2. Nov. 5, 1925, to Feb. 8, 1928.
3. Jan. 18, 1924, to May 3, 1927.
4. Feb. 21, 1926, to June 10, 1928.
5. June 28, 1924, to April 14, 1927.
6. Oct. 9, 1923, to April 14, 1928.
7. Mar. 17, 1923, to June 8, 1928.
8. May 29, 1918, to Sept. 18, 1926.
9. June 8, 1923, to Oct. 1, 1927.
10. May 12, 1920, to Sept. 5, 1927.

Find the time between the two dates given in each of the following problems by compound subtraction.

1. Jan. 5, 1926, to May 29, 1928.
2. Aug. 10, 1924, to June 30, 1928.
3. Oct. 12, 1926, to Feb. 28, 1928.
4. Feb. 28, 1923, to Aug. 25, 1927.
5. June 29, 1924, to Apr. 13, 1927.
6. Mar. 24, 1917, to Sept. 5, 1926.
7. July 16, 1918, to Nov. 4, 1924.
8. Dec. 28, 1923, to Feb. 29, 1927.
9. Sept. 20, 1927, to Jan. 31, 1928.
10. May 31, 1925, to Aug. 31, 1927.

Processes of Figuring Interest

Given a method of counting time, and having determined whether 360 or 365 days is to be considered a year, the result of a problem will not vary, whatever process be used in the actual figuring. There are a great many processes of figuring in use. In this book there will be only two of these processes explained. Learn one of these processes and stick to it.

The Cancellation Process

RULE. Multiply together the principal, the rate, and the time expressed in years. Express all three terms fractionally, and cancel.

EXAMPLE. Find the interest on \$1500.00 from June 10, 1925 to April 6, 1927, at 6%, ordinary method, cancellation process.

Year	Month	Day
1927	4	6
1925	6	10
1	9	26

Int. on \$1500 for 1 yr. at 6% = \$90.00

9 mo. 26 da. = 296 da. = $\frac{296}{360}$ yr.

$$\frac{\$1500}{1} \times \frac{6}{100} \times \frac{296}{360} = \$74. \quad \$74 + \$90 = \$164. \text{ ans.}$$

WRITTEN. Find the interest by the ordinary method, cancellation process, on

1. \$1800 from Jan. 15, 1927 to Aug. 30, 1928, at 6%.
2. \$2160 from Apr. 4, 1924 to Feb. 28, 1927, at 6%.
3. \$1750 from Dec. 21, 1923 to Mar. 31, 1928, at 6%.
4. \$864.50 from July 5, 1925 to Aug. 26, 1927, at 6%.
5. \$720 from Nov. 22, 1923 to May 30, 1927, at 7%.
6. \$1440 from Jan. 25, 1926 to Mar. 5, 1928, at 8%.
7. \$536.25 from May 20, 1924 to Sept. 10, 1927, at 5%.
8. \$461.57 from Aug. 30, 1923 to July 25, 1928, at $4\frac{1}{2}$%.
9. \$632.89 from Feb. 28, 1925 to Apr. 20, 1927, at $6\frac{1}{2}$%.
10. \$1250.50 from June 15, 1922 to Nov. 15, 1927, at 4%.

The Sixty-day Process

This is sometimes called the *60-day process* and sometimes called the *6% process*.

> RULE. Point off two places in the principal, which will give you the interest for 60 days at 6%. Multiply this by the number of times 60 days is contained in the whole time. Increase or decrease the result so secured, according as the rate is more or less than 6%.

Cases involving 6% interest

EXAMPLE. Find the interest on $1500 from June 10, 1926, to April 6, 1928, at 6%, ordinary method, sixty-day process.

Year	Month	Day
1928	4	6
1926	6	10
1	9	26

Preliminary. If the rate for 360 days is 6%, the rate for 60 days is $\frac{1}{6}$ of 6%, or 1%. Pointing off 2 places in the principal gives $15.00 as the interest for 60 days at 6%.

1st Step. The greatest number of times that 60 days (or 2 mo.) is contained in the time (1 yr. 9 mo. 26 da.) is 10. $15.00 × 10 = $150.00 for 1 yr. 8 mo.

2d Step. There still remain 1 mo. 26 da. One month is half of 60 days. $15.00 ÷ 2 = 7.50 for 1 mo.

3d Step. There still remain 26 da. (20+6)
Twenty days are $\frac{1}{3}$ of 60 da. $15.00 ÷ 3 = 5.00 for 20 da.
Six days are $\frac{1}{10}$ of 60 da. $15.00 ÷ 10 = 1.50 for 6 da.

$164.00 for 1 yr. 9 mo. 26 da.

$ 15.00	
150.00 − 1 − 8	
7.50 − 1	
5.00 −	−20
1.50 −	− 6
$164.00 = 1 − 9 − 26	

NOTE. In writing the solution the explanations are omitted. All that need be written are figures, as shown at the left.

WRITTEN. Find the interest, in the following problems, by the ordinary method, sixty-day process:

1. $336.00 from Mar. 12, 1927, to June 22, 1927, at 6%.
2. $420.50 from Sept. 12, 1924 to Oct. 21, 1926, at 6%.
3. $991.25 from May 20, 1928 to Aug. 20, 1928, at 6%.
4. $835.00 from July 31, 1926, to June 17, 1928, at 6%.
5. $768.75 from Nov. 28, 1927 to Apr. 4, 1928, at 6%.
6. $156.50 from Jan. 1, 1927 to Apr. 7, 1927, at 6%.
7. $340.40 from Apr. 15, 1926 to Jan. 30, 1928, at 6%.
8. $372.25 from Aug. 16, 1928 to Oct. 16, 1928, at 6%.
9. $426.60 from Feb. 28, 1927 to Apr. 15, 1928, at 6%.
10. $157.50 from June 25, 1927 to Sept. 7, 1927, at 6%.

When the rate of interest is more or less than 6%, the problem is first solved as for 6%, and to or from the result so secured is added or deducted the proper fractional part. Thus

To find int. at 9%, find int. at 6% and add $\frac{1}{2}$.
To find int. at 8%, find int. at 6% and add $\frac{1}{3}$.
To find int. at $7\frac{1}{2}$%, find int. at 6% and add $\frac{1}{4}$.
To find int. at 7%, find int. at 6% and add $\frac{1}{6}$.
To find int. at 5%, find int. at 6% and deduct $\frac{1}{6}$.
To find int. at $4\frac{1}{2}$%, find int. at 6% and deduct $\frac{1}{4}$.
To find int. at 4%, find int. at 6% and deduct $\frac{1}{3}$.
To find int. at 3%, find int. at 6% and deduct $\frac{1}{2}$.

EXAMPLE. Find the interest on $1500.00 from June 10, 1926 to April 6, 1928 at $4\frac{1}{2}$, ordinary method, sixty-day process.

Year	Month	Day
1928	4	6
1926	6	10
1	9	26

$15.00 = 60 da. at 6%.
$150.00—1—8—0—at 6%.
7.50— 1—0
5.00— —20
1.50— — 6
4)$164.00 = 1—8—26 at 6%.
41.00
$123.00. Ans.

No part of the above process needs explanation except the last step. After the interest for the time has been found to be $164 at 6%, it is necessary to deduct $\frac{1}{4}$ of this amount, or $41, leaving $123 as the result at $4\frac{1}{2}$%.

WRITTEN. Find the interest, in the following problems, by the ordinary method, sixty-day process:

1. \$457.50 from Aug. 20, 1926 to Jan. 1, 1928, at 5%.
2. \$589.75 from Feb. 28, 1925 to Aug. 13, 1927, at 7%.
3. \$636.00 from Sept. 30, 1924 to Feb. 16, 1928, at 6%.
4. \$145.25 from Jan. 15, 1927 to Mar. 15, 1928, at $7\frac{1}{2}$%.
5. \$724.50 from Dec. 20, 1927 to Mar. 12, 1928, at $4\frac{1}{2}$%.
6. \$918.00 from Mar. 18, 1927 to Aug. 4, 1928, at 5%.
7. \$263.75 from July 25, 1927 to Feb. 28, 1928, at 4%.
8. \$872.00 from Oct. 31, 1927 to Apr. 13, 1928, at 7%.
9. \$591.50 from Feb. 16, 1926, to June 10, 1927, at 5%.
10. \$300.30 from Apr. 30, 1921 to Jan. 1, 1928, at 8%.

Time-saving applications of the sixty-day process

It is well to bear in mind that since the interest for 60 days is 1% of the principal, the interest for 6 days ($\frac{1}{10}$ of 60 days) is $\frac{1}{10}$ of 1%, and can be found by pointing off 3 places instead of 2. Note the following:

The interest for 6 days at 6% is found by pointing off 3 places.
The interest for 60 days at 6% is found by pointing off 2 places.
The interest for 600 days at 6% is found by pointing off 1 place.
The interest for 6000 days at 6% equals the principal.

It is also well to bear in mind that factors in multiplication can be interchanged without affecting the result. Thus $3\times4=4\times3$. Applied to solutions of problems in interest, the interest on \$100 for 60 days = the interest on \$60 for 100 days.

Interchanging principal and time, find the interest on the following:

1. \$600 at 6% for 124 days.
2. \$500 at 6% for 174 days.
3. \$400 at 6% for 137 days.
4. \$300 at 6% for 162 days.
5. \$200 at 6% for 185 days.
6. \$750 at 5% for 187 days.
7. \$800 at $4\frac{1}{2}$% for 191 days.
8. \$900 at 3% for 249 days.
9. \$1000 at 4% for 85 days.
10. \$450 at 5% for 119 days

11. \$1500 at 5% for 146 days.
12. \$1800 at 4% for 162 days.
13. \$2100 at 8% for 185 days.
14. \$2400 at 8% for 219 days.
15. \$3000 at $4\frac{1}{2}$% for 197 days.
16. \$3600 at 5% for 237 days.
17. \$4200 at 6% for 242 days.
18. \$1200 at 3% for 261 days.
19. \$1500 at 5% for 276 days.
20. \$1600 at $5\frac{1}{2}$% for 239 days.

NOTE. After completing this group of problems, it is suggested that the student should decide once for all whether he will use the cancellation process or sixty-day process, and use the process decided upon in all future work.

Finding the Rate

To find the rate, the principal, interest, and time being known, find the interest at 1% and divide the given interest by this amount.

EXAMPLE. I had \$620 invested in a business. It earned for me \$9.92 in 96 days. To what rate of interest was this equivalent?

$\frac{\$620}{1} \times \frac{1}{100} \times \frac{96}{360} = \$1.65\frac{1}{3}$, interest on \$620 at 1%.

$\$9.92 \div \$1.65\frac{1}{3} = 6$, the rate %.

NOTE. Be careful not to drop fractions from a number which is to be used as a divisor. Give the result to the nearest $\frac{1}{4}$%

WRITTEN. At what rate of interest would the principal have to be placed, in each of the following cases, to earn the amount shown?

	Principal	Time	Earning		Principal	Time	Earning
1.	\$360	150 days	\$ 9.00	5.	\$612	78 days	\$ 7.96
2.	\$475	109 days	5.75	6.	\$954.38	72 days	11.45
3.	\$942	140 days	10.99	7.	\$1076.90	84 days	17.59
4.	\$875	54 days	6.56	8.	\$1238.50	117 days	24.15

	Principal	Time	Earning
9.	\$920.54	3 mo. 18 da.	\$22.09
10.	\$870.85	5 mo. 27 da.	25.69
11.	\$1143.30	6 mo. 21 da.	47.88
12.	\$1649.75	8 mo. 20 da.	71.49

Finding the Time

To find the time, the other factors of the problem being known, find the interest on the principal at the rate stated, for one year, and divide the total interest by the interest for one year. The result will be the number of years in the time.

EXAMPLE. In what time will \$350 produce \$5.60 interest at 6%?

\$21.00 = the interest at 6% for 1 year.
\$5.60, the given interest ÷ \$21 = .26$\frac{2}{3}$, the number of years.
.26$\frac{2}{3}$ yr. expressed as months and days is 3 mo. 6 da.
Sometimes the solution can be shortened by using one month or one day instead of one year, as the unit of time. Give the result in whole days.

Find the time for which the following amounts must be invested to produce the interest given, at the rate given.

	Principal	Rate	Interest		Principal	Rate	Interest
1.	\$627.50	6%	\$12.55	7.	\$645.80	6%	\$54.79
2.	\$1200	5%	\$42.50	8.	\$895.42	5%	\$62.68
3.	\$3000	4$\frac{1}{2}$%	\$97.50	9.	\$1800	8%	\$210.40
4.	\$1800	4%	\$50.00	10.	\$1132.90	3$\frac{1}{2}$%	\$187.46
5.	\$2500	7%	\$124.93	11.	\$1372.38	9%	\$437.10
6.	\$3500	7$\frac{1}{2}$%	\$165.52	12.	\$1579.25	3%	\$207.80

Finding the Principal

To find the principal, the other factors of the problem being known, find the interest on \$1.00 at the rate stated and for the time stated. Divide the total interest by this amount. The result will be the number of dollars in the principal.

EXAMPLE. What principal will produce \$12.80 at 6% interest in 4 mo. 15 da.?

\$1.00 will yield \$0.0225 interest in 4 mo. 15 da. at 6%.
\$12.80 ÷ \$.0225 = \$568.89, the number of dollars in the principal.

WRITTEN. What sum of money must be invested to produce the interest given below at the rate and for the time given?

1. \$13.60 interest at 6% for 5 mo. 18 da.
2. \$27.30 interest at 5% for 6 mo. 27 da.

3. $31.50 interest at 4% for 8 mo. 15 da.
4. $45.60 interest at 4½% for 2 mo. 12 da.
5. $112.25 interest at 7% for 1 mo. 15 da.
6. $137.80 interest at 8% for 11 mo. 27 da.
7. $94.76 interest at 4% for 1 yr. 3 mo. 15 da.
8. $139.57 interest at 3½% for 2 yr. 5 mo. 18 da.
9. $178.63 interest at 5% for 1 yr. 10 mo. 24 da.
10. $95.37 interest at 9% for 6 mo. 26 da.
11. $382.55 interest at 3% for 234 da.
12. $196.48 interest at 5½% for 219 da.

Finding the Present Value

It is often important to know the present value of an obligation maturing at some future time. How much money should be placed at interest at a given rate on some previous date, in order that principal and interest will together exactly equal the amount of the obligation on the day it is due?

First find what one dollar will amount to, in the given time, at the rate specified. Divide the amount of the obligation by the sum so secured; the result will be the number of dollars required.

EXAMPLE. What is the present value of a debt of $765 due in 4 mo., if interest is worth 6%?

The interest on $1.00 for 4 mo. at 6% is $.02.
$1.00 will therefore amount in 4 mo. at 6% to $1.00+$.02=$1.02.
$765÷$1.02=750, the number of dollars required.

The principal found in the above problem is frequently called *Present Worth* and the interest, *True Discount*.

WRITTEN. What amount of money placed at interest at the rate and for the time specified, will produce the amount due, in each of the following cases?

	Amount Due	Time	Rate
1.	$1150.40	2 mo. 15 da	6%
2.	$2167.20	6 mo. 27 da	5%

	Amount Due	Time	Rate
3.	$3287.40	1 yr. 3 mo. 15 da.	$4\frac{1}{2}\%$
4.	$5262.25	2 yr. 4 mo. 10 da.	6%
5.	$3750.85	4 mo. 24 da.	7%
6.	$483.40	178 da.	8%
7.	$596.32	217 da.	$7\frac{1}{2}\%$
8.	$1346.26	247 da.	5%
9.	$2637.89	1 yr. 5 mo. 26 da.	$6\frac{1}{2}\%$
10.	$867.97	1 yr. 1 mo. 10 da.	9%

INTEREST DRILL PROBLEMS

Find the amounts to be put in the columns that are left blank.

	Principal	Rate	Time	Interest	Total (Prin. and Int.)
1.	$ 500	6%	90 da.		
2.	400	6%	75 da.		
3.	600	6%	80 da.		
4.	800	6%	72 da.		
5.	300	6%		$ 12.00	$ 312.00
6.	500	6%	4 mo.		
7.		4%	2 yr.	48.00	
8.		5%	1 yr. 6 mo.	150.00	
9.	1200	6%		24.00	1224.00
10.		8%	1 yr.		324.00
11.	200	5%		25.00	225.00
12.		4%	2 yr. 6 mo.	14.50	
13.	500		2 yr.	80.00	580.00
14.	300		90 da.	4.50	304.50
15.	1000		6 mo.	15.00	1015.00
16.	1200		2 mo.	8.00	1208.00
17.	250	6%		10.00	
18.	400	8%	6 mo.		
19.		4%	1 yr. 6 mo.		318.00
20.		3%	2 yr. 4 mo.		535.00
21.	360	9%	1 yr. 8 mo.		
22.	300	6%		24.00	324.00
23.		4%	2 yr. 3 mo.	18.45	
24.	600	6%		4.50	604.50

Problems in Cost of Interest

Use the ordinary method in solving the following problems.

1. On Apr. 1, 1927, F. C. Keach gave his note to Myron Duckels for $964.50. The note was due in six months and bore 6% interest. How much was due on the day of maturity?

2. On Jan. 10, 1927, H. C. Schoolcraft received D. C. Brown's 90-day note for $560 at 5%. The note was dated Jan. 1, 1927. How much was due Aug. 21, 1928, no payments having been made?

An interest-bearing note if unpaid at maturity continued to bear interest at the rate stated in it. A non-interest-bearing note bears interest after maturity at the legal rate of the state in which it is to be paid.

3. Brookman & Co. bought from the Yellow Pine Lumber Co. mdse. worth $2256.50, terms 90-day note without interest on day of shipment. The note was payable at Rock Island, Ill. The lumber was shipped Nov. 24, 1927. The note was not paid until June 16, 1928. How much was then paid?

See table of statutory rates of interest at the end of the chapter.

4. A bill of goods was sold on June 27, 1927 amounting to $783.17 on the following terms: 2/10, *n*/30, int. at 6% after maturity. How much would have paid the bill on July 7, 1927? On July 10, 1927? On July 27, 1927? On Feb. 28, 1928?

5. Upon graduation from high school a young man enters an office at a salary of $12.00 a week. What sum of money would have to be invested at 6% to yield the same income? (Simple interest is understood unless compound interest is specified.)

6. A man bought a farm of 160 A. in North Dakota at $35 per A. He paid $1600 cash at the time of purchase and gave four notes for $1000 each, with interest at 6%, payable in 1 yr., 2 yr., 3 yr., and 4 yr. respectively. If all these notes were met at maturity, what was the total cost of the farm, including interest?

7. If a sum of money is loaned at 6% per annum, how long will it take the principal to double itself? at 5%? at 8%? at 7%?

8. What is the interest on $1262.50 at 6% from Aug. 15, 1927 to Mar. 23, 1928, ordinary method? exact method? bankers' method?

9. A sells furniture for cash. B sells on six months' time. If A sells a dresser for $37.50, what will B have to charge for the same

dresser, if the use of money in a trading business is worth 12% per annum?

10. On Sept. 1, 1927, Mr. Thomas bought a cottage for $2500, paying $1000 at time of purchase and $100.00 a month thereafter until paid. Interest was charged upon unpaid balance at 6%. The monthly payment was applied first upon interest due, the balance being applied upon principal. Show how each month's payment was applied. What was the total cost of interest?

11. Two men are negotiating a contract by which one agrees to give the other three notes for $2000 each with interest at 6%, one note to be paid in one year, one in two years, and one in three years. The debtor asks that the interest be paid on each note only when the note is paid. The creditor insists that all interest be paid annually. How much difference is there between them?

12. A house which cost $6500 was purchased by paying $1500 cash at the time of sale and giving a mortgage, bearing interest at 6%, for the balance. The house accommodates two families, each of whom pays $30 per month rent. The annual expenses are: Taxes, 21.27 mills on each dollar of a valuation of $4000; insurance, $15.40; water tax, $12.50 for each family; and repairs, $60. What rate of interest is earned on the money invested by the purchaser?

13. A young man, when he becomes 25 years of age, is to receive, by the will of his father, $10,000. He is now 22 years of age and desires to sell his inheritance. If the use of money is worth 6%, simple interest, what should he receive at present for his inheritance?

Problems of Interest and Profit and Loss

1. A bill of goods amounting to $1247.83 is bought on the terms "2/10, *n*/30." The purchaser is short of cash but wishes to save the discount on the bill. He can get a loan at the Union Trust Co. for 30 da. at 6%, but is undecided. Will it be to his interest to obtain a loan sufficient to pay the bill? If so, how much?

2. An investor bought a house for $3750 and rented it for $30 per month. During the first year he paid $21.74 for taxes, $18.67 for insurance, $12.50 for water tax, and $15.00 for repairs. At the

end of one year he sold the house for $4000. What was his profit on the venture and to what rate of interest on his original investment was it equivalent?

3. A hardware dealer bought certain articles for $5.52 each. Three months after the purchase he sold them for $7.50 each. What was the per cent of profit? To what rate of interest was this equivalent?

4. An apartment house containing six apartments is for sale. Two apartments rent for $32.50 per month each, two for $30 each, and two for $25 each. The annual expenses are: Taxes on $9500 at $20.48 per M; water tax, $8 for each apartment; insurance, $27.50; repairs (estimated), $120; other expenses and decorations (estimated), $200. What price can an investor afford to pay to realize 9% net on the investment?

5. A peddler bought a car load of watermelons containing 1225 melons at $9\frac{1}{2}¢$ per melon, borrowing the purchase money for 30 days at 7%. There were 85 bad and broken melons and the remainder were divided into three grades. There were 240 1st grade, 650 2d grade, and 250 3d grade melons that retailed at 25¢, 20¢, and 15¢ respectively. Eight wagons and eight men were employed to peddle these melons and closed out the car load in two days. If a man and wagon were worth $5 per day was there any profit beyond wages, and if so how much, after paying $22.78 for freight?

6. A country bank, having a surplus of funds, deposited $50,000 with a city bank. The latter agreed to pay interest at the rate of $2\frac{1}{2}\%$ annually, payable at the end of the year. The city bank loaned this money for a year at 6% and received the interest semi-annually. The interest so received was loaned at 6%. Find the profit made by the city bank the first year.

7. A grain dealer bought on Sept. 27, 1927, three car loads of corn, containing respectively 900 bu., 1000 bu., and 950 bu., at $82\frac{1}{2}¢$ per bu. The corn was stored on Sept. 29 at a charge of $\frac{1}{2}¢$ per bu. per month or fraction thereof. On Jan. 17 following, the corn was sold at $97\frac{3}{4}¢$ per bu. There was a shrinkage of 60 bu. and there were expenses amounting to $57.83. The profit realized from the transaction was equivalent to what rate of interest on original investment?

8. Wennemer Bros., Inc., bought of the American Gypsum Co. a bill of material amounting to $11,483.62 on which there was a discount of 5% if paid in 10 da. They were short of ready cash and borrowed sufficient money at a bank at 6% on a note at 60 days to meet the bill. How much was the discount on the bill in excess of the interest on the loan?

9. A builder bought a lot for $800 and erected a house upon it at a cost of $2850. The house and lot rented for $32.50 per mo. The expenses were as follows: Taxes, 19.45 mills per dollar on a valuation of $2500; insurance, 62¢ per $100 for 3 yr. on a valuation of $2000; water tax, $7.50 per year; repairs, $25. What per cent per annum did he receive upon the investment?

10. On Sept. 1, 1927 a young man bought a lot for $600, paid $150 cash at the time of purchase, and agreed to pay interest at 6% and $10 on principal each month. If all payments were made for 10 months what were the total payments? What was his profit if he sold the lot at the end of 10 months for $850?

11. Taking the total amount paid on the lot in the above problem as the investment, the profit was equivalent to what rate of interest?

12. I own 10 building lots each 40 ft. wide. The city lets a contract to pave the street upon which they are located and my assessment amounts to $2.95 per foot of frontage. The city allows me to pay 25% of my assessment 3 mo. after the completion of the paving and 25% at the end of each year after the completion of the paving. No interest is to be charged unless payments are not met at the date of maturity. On the day my first payment is due I offer to pay the present worth of the debt at 6% simple interest. If this offer is accepted, what will I pay?

13. Williams and Clayton bought a bill of goods amounting to $6732.94 dated Oct. 21, 1926. The terms were, "5/10, 2/30, n/4 mo." On Oct. 29 a payment of $2000 was made in accordance with an agreement that they be allowed a proportionate discount upon the amount paid. Nov. 15 they made a further payment of $2500 under the same conditions. When the bill was due they had difficulty in raising cash and did not pay the balance until April 27, 1928. What amount was then due if the use of money was charged for at 6%?

14. Samuel Edwards bought an apartment house containing twelve apartments that rent for $40 each per month and one that rents for $25.00. The purchase price was $34,500. He paid $12,000 cash, gave a first mortgage for $17,500 at 5%, and a second mortgage for $5000 at $6\frac{1}{2}\%$. Expenses are as follows: Taxes, $20.83 per thousand dollars on a valuation of $19,500; insurance, 65¢ per hundred dollars for 3 years on $25,000; coal, 50 tons at $3.75 per ton; janitor service, $30 per month and the use of one apartment worth $25 per month; repairs and general expenses (estimated) including water tax, $475 per year. The net income from this property is what per cent upon the cash investment?

15. In the foregoing problem, the net income would have been what per cent of the cost of the property had the building been paid for in full without borrowing?

16. Herewith is an abstract of J. R. Hadley & Co.'s accounts with Greenfield and Northrup, receivable and payable.

Dr.		J. R. Hadley & Co.					Cr.		
19—					19—				
Jan.	15	To Mdse	950	60	Feb.	13	By Mdse (30 da.)	560	
Feb.	20	To Mdse (30 da.)	1500	00	Mar.	20	By Mdse (30 da.)	789	50
Feb.	25	To Mdse (60 da.)	1175	50	Apr.	5	By Mdse (30 da.)	450	

The above shows an exchange of merchandise, with the balance in favor of Greenfield and Northrup. Interest at 6% is added to each item after due. What is the balance of the account July 1 including interest?

What is the interest on the balance of the above account, not considering interest items, for one day at 6%?

If money is worth 6%, how many days before July 1, or how many days after July 1, should the account be paid, in order that neither Greenfield and Northrup nor J. R. Hadley & Co. would lose any interest?

17. Black bought from White $1260.50 worth of merchandise on July 15, 1927, on 60 days' time, interest at 6% after maturity. How much did Black owe White on Jan. 1, 1928? What was the present worth of the debt on the date of the bill?

Interest Table

ORDINARY INTEREST TABLE FOR $1000.00 AT VARIOUS RATES

TIME	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	6
1 da.	.069	.083	.097	.111	.125	.139	.153	.167
2 da.	.139	.167	.194	.222	.250	.278	.306	.333
3 da.	.208	.250	.292	.333	.375	.417	.458	.500
4 da.	.278	.333	.389	.444	.500	.556	.611	.667
5 da.	.347	.417	.486	.556	.625	.694	.764	.833
6 da.	.417	.500	.583	.667	.750	.833	.917	1.000
7 da.	.486	.583	.681	.778	.875	.972	1.069	1.167
8 da.	.556	.667	.778	.889	1.000	1.111	1.222	1.333
9 da.	.625	.750	.875	1.000	1.125	1.250	1.375	1.500
10 da.	.694	.833	.972	1.111	1.250	1.388	1.528	1.667
11 da.	.764	.917	1.069	1.222	1.375	1.528	1.681	1.833
12 da.	.833	1.000	1.167	1.333	1.500	1.667	1.833	2.000
13 da.	.903	1.083	1.264	1.444	1.625	1.806	1.986	2.167
14 da.	.972	1.167	1.361	1.556	1.750	1.944	2.139	2.333
15 da.	1.042	1.250	1.458	1.667	1.875	2.083	2.292	2.500
1 mo.	2.083	2.500	2.917	3.333	3.750	4.167	4.583	5.000
2 mo.	4.167	5.000	5.833	6.667	7.500	8.333	9.167	10.000
3 mo.	6.250	7.500	8.750	10.000	11.250	12.500	13.750	15.000
4 mo.	8.333	10.000	11.667	13.333	15.000	16.667	18.333	20.000
5 mo.	10.417	12.500	14.583	16.667	18.750	20.833	22.917	25.000
6 mo.	12.500	15.000	17.500	20.000	22.500	25.000	27.500	30.000

To find the interest on more or less than $1000.00, find the interest on $1000.00 for the given time and rate from the table. Divide this by 1000 and multiply the quotient by the number of dollars in the required amount.

The table gives the interest for any number of days from 1 to 15. If the interest be required for 16 days, to the interest for 15 days add the interest for 1 day; and so on for other numbers of days.

WRITTEN: Using the table find the interest on:

1. $1000 for 8 da. at $4\frac{1}{2}\%$
2. $500 for 4 mo. at $3\frac{1}{2}\%$
3. $367 for 3 mo. at $2\frac{1}{2}\%$
4. $482 for 29 da. at 5%
5. $15,650 for 8 mo. 13 da. at $3\frac{1}{2}\%$
6. $1924 for 99 da. at $4\frac{1}{2}\%$
7. $1000 for 9 da. at 9%
8. $750 for 23 da. at 8%
9. $840 for 8 mo. at $7\frac{1}{2}\%$
10. $525 for 3 mo. 27 da. at $3\frac{1}{3}\%$

The following table shows the legal rate of interest and the maximum rate allowed by law in each of the several states.

States and Territories	Legal Rate	Maximum Rate	States and Territories	Legal Rate	Maximum Rate
Ala.	8%	8%	Neb.	7%	10%
Alaska	8%	12%	Nev.	7%	any
Ariz.	6%	12%	N. H.	6%	6%
Ark.	6%	10%	N. J.	6%	6%
Calif.	7%	any	N. Mex.	6%	12%
Colo.	8%	any	N. Y.	6%	6%
Conn.	6%	12%	N. C.	6%	6%
Del.	6%	6%	N. D.	7%	12%
Dist of Col.	6%	6%	Ohio	6%	8%
Fla.	8%	10%	Okla.	6%	10%
Ga.	7%	8%	Ore.	6%	10%
Hawaii	8%	12%	Pa.	6%	6%
Ida.	7%	12%	P. I.	6%	any
Ill.	5%	7%	P. R.	6%	12%
Ind.	6%	8%	R. I.	6%	any
Ia.	6%	8%	S. C.	7%	8%
Kans.	6%	10%	S. D.	7%	12%
Ky.	6%	6%	Tenn.	6%	6%
La.	5%	8%	Tex.	6%	10%
Me.	6%	any	Utah	8%	12%
Md.	6%	6%	Vt.	6%	6%
Mass.	6%	any	Va.	6%	6%
Mich.	5%	7%	Wash	6%	12%
Minn.	6%	10%	W. Va.	6%	6%
Miss.	6%	8%	Wis.	6%	10%
Mo.	6%	8%	Wyo.	8%	12%
Mont.	8%	any			

CHAPTER XIV

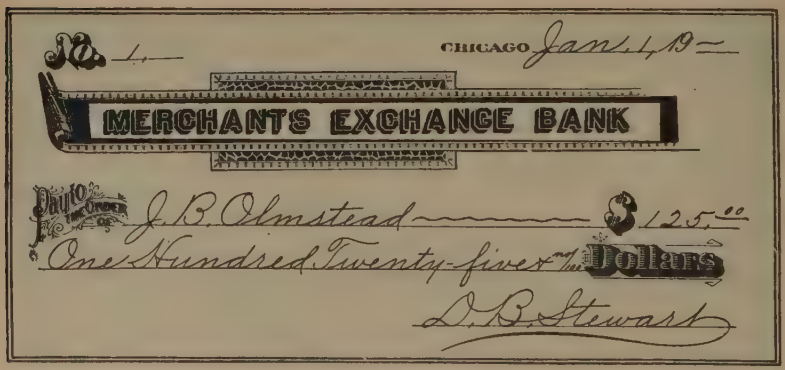
NEGOTIABLE PAPER

A **negotiable paper** is a written promise or order for the payment of money which if properly made out and transferred under proper conditions* passes to the purchaser free of any defenses which may have existed against it in the hands of the original holder.

Checks, notes, drafts, certificates of deposit, and bonds are examples of negotiable paper.

A full discussion of negotiable paper will be found in your commercial law text. It is sufficient for the purposes of this book to note that the intent of the laws concerning commercial paper is to create a medium of exchange that will circulate freely because people are not afraid to take it at its apparent value (face, less discount or plus interest, with a possible deduction for exchange or for collection). Speaking broadly, it can have no concealed defenses.

Checks


No. 1
CHICAGO Jan. 4, 19—
MERCHANTS EXCHANGE BANK
Pay to the order of J. B. Olmstead $125.00
One Hundred Twenty-five and no/100 Dollars
D. B. Stewart

CHECK

A **check** is an order on a bank, payable in cash on demand. To be of value it must be drawn by one having funds in the bank

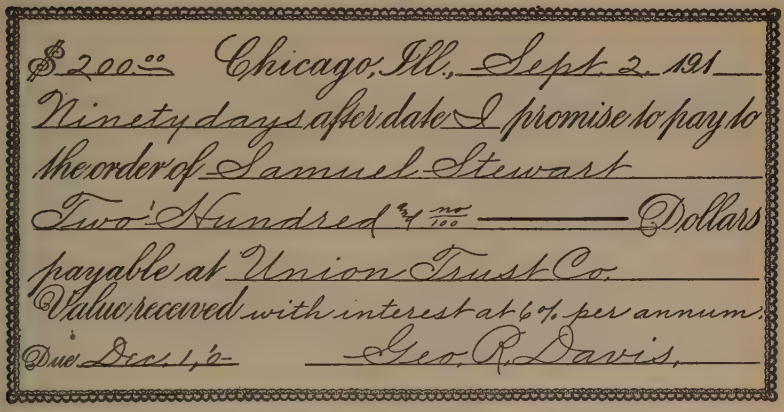
* These conditions relate to the proper endorsement and delivery. The holder, to be protected in his rights, must be an innocent holder, for value, and must have secured the paper in the usual course of business.

subject to check sufficient to cover it. Savings accounts are not subject to check.

The parties to a check are the *drawer*, the *drawee*, and the *payee*. In the model the drawer is D. B. Stewart, the drawee is the Merchants' Exchange Bank, and the payee is J. B. Olmstead.

Checks should be presented for payment promptly. Failure to present a check within a reasonable time makes the holder liable for loss in case of failure of the bank and subjects him to inconvenience and perhaps loss if, when he finally presents the check, there are not sufficient funds to the drawer's credit in the bank to pay it.

Notes


$200.00 Chicago, Ill., Sept. 2, 191_

Ninety days after date I promise to pay to the order of Samuel Stewart

Two Hundred 00/100 ——— Dollars

payable at Union Trust Co.

Value received with interest at 6% per annum.

Due Dec. 1, '_ ——— Geo. R. Davis.

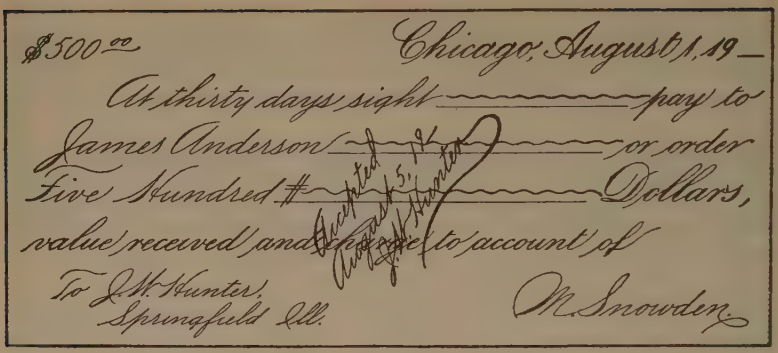
A Promissory Note

A **negotiable promissory note** is an unconditional promise in writing made by one person to another, signed by the maker, engaging to pay on demand or at a fixed or determinable future time, a certain sum of money. It may be payable to order or to bearer.

The parties to a note are the *maker* and the *payee*. In the above note the maker is George R. Davis and the payee is Samuel Stewart.

The note shown is interest-bearing. How much is it worth on Sept. 2? How much on Oct. 2? on Nov. 1? on Jan. 1? How much 4 mo. after maturity, if unpaid?

Drafts


$500.00
Chicago, August 1, 19—
At thirty days sight pay to
James Anderson or order
Five Hundred & Dollars,
value received and charge to account of
To J. W. Hunter,
Springfield Ill.
M. Snowden
Accepted Aug. 5, 19— J. W. Hunter

TIME DRAFT

A **draft,** or bill of exchange, is an unconditional order in writing, addressed by one person to another, signed by the person giving it, requiring the person to whom it is addressed to pay on demand or at a fixed, determinable future time a given sum of money, to some person named therein or to his order or to bearer. If payable on demand, or "at sight," it is called a *sight draft.* If payable a certain number of days, months, or years after sight it is called a *time draft.* If payable on a given future date it is called a time draft.

Sight drafts are used largely for making collections when the debtor lives at a distance from the creditor. They are usually drawn through a bank but not necessarily so. Drafts through a bank constitute a good collecting medium because the debtor may not wish his bank to know that he owes money and is slow to pay it.

There are three parties to a draft, called *drawer, drawee,* and *payee.* In the model shown, the drawer is M. Snowden, the drawee is J. W. Hunter, and the payee is James Anderson. The payee has no claim against Hunter until Hunter indicates his acceptance of the terms of the draft. In the model Hunter has written his *acceptance* across the face of the draft. Until Hunter accepts, Snowden is responsible for the paper, but after the paper is honored by Hunter, Hunter is primarily responsible and Snowden secondarily responsible.

$114.00 Chicago, Ill. Apr. 25 19—

At Sight Pay to

the order of Merchants Exchange Bk

One Hundred Fourteen & no/100 Dollars

Value received and charge the same to account of

To A. F. Harvey, H. T. Raynor

La Grange, Ill.

SIGHT DRAFT

The above paper has value because H. T. Raynor, the drawer, is primarily responsible for it until A. F. Harvey, the drawee, has honored it by payment.

Peoria, Ill. May 24 19—. No. 362

THE FIRST NATIONAL BANK

OF PEORIA, ILL.

Pay to the order of H. T. Raynor $300.00

Three Hundred & no/100 Dollars

To FIRST NATIONAL BANK, CHICAGO.

H. W. Blondel, Cashier

BANK DRAFT

A **bank draft** is a draft drawn by one bank upon another. It is widely used as a medium of remittance.

If A of Chicago wishes to make a remittance to B of St. Louis he will go to some Chicago bank and buy a draft upon some St. Louis bank with which the Chicago bank has connections. A will then send the draft to B, who will cash it at the St. Louis bank upon which it is drawn. For the services the Chicago bank may charge a fee, which is called exchange.

Endorsement

The transfer of negotiable paper is accomplished by endorsement and delivery. The endorsement is the signature of the payee, written usually on the back of the left end of the paper, either by itself or with accompanying words specifying to whom the paper is transferred.

If the payee writes his name only as his endorsement, the effect is to transfer the paper to any bearer. This is called an *endorsement in blank*. If he precedes his signature with the words "Pay to the order of John Doe" or "Pay to John Doe or order" the effect of the endorsement is to transfer the paper only to John Doe or to some other person whom he may designate. This is called an *endorsement in full*. Other forms of endorsement are *restrictive endorsements*, *qualified endorsements*, and *endorsement "without recourse,"* each of which has the effect indicated by its name. (*Endorsement* is also spelled *indorsement*.)

Finding the date of maturity

Demand or sight paper is payable on being presented for payment. Time paper is due a certain number of days, months, or years after sight or after date. If "after date," time begins to run at once. If "after sight," time begins to run after the date of acceptance.

If paper is not accepted when presented for acceptance, or if it is due and payment is refused, it is said to be dishonored. If dishonored, the fact of dishonor is often attested by a notary public, in a form called a *protest*. Endorsers are entitled to a formal notice of protest, and failure to properly protest paper and duly notify endorsers will often relieve endorsers from liability.

In counting time to maturity, omit the day upon which the note is dated; then count forward the number of days, or months, or years in the time as stated; the last day of the period stated is the due date.

Thus to fix the date of maturity for a 60-day note dated March 15, count March 16 to 31 inclusive as 16 days. 16 days left in March plus 30 days in April = 46 days. There are 60—46 or 14 days to be counted in May. The due date is then May 14.

The time when interest begins to run is indicated in the wording of the paper. If the paper is interest-bearing, interest begins to run from its date, or from the date of acceptance in case of a time draft payable a certain number of days after acceptance.

ORAL. What is the due date in each of the following cases:

	Date of Paper	Time to Run	Time to Run		Date of Paper	Time to Run	Time to Run
1.	Mar. 18	2 mo.	60 da.	9.	Jan. 31	1 mo.	30 da.
2.	Mar. 27	3 mo.	90 da.	10.	Jan. 30	1 mo.	30 da.
3.	Apr. 14	4 mo.	120 da.	11.	Sept. 14	3 mo.	90 da.
4.	Nov. 30	3 mo.	90 da.	12.	Oct. 31	4 mo.	120 da.
5.	June 24	2 mo.	60 da.	13.	Feb. 28	2 mo.	60 da.
6.	Aug. 10	6 mo.	180 da.	14.	May 12	4 mo.	120 da.
7.	Oct. 21	3 mo.	90 da.	15.	July 27	1 mo.	30 da.
8.	Dec. 22	3 mo.	90 da.	16.	Aug. 31	1 mo.	30 da.

When the time is stated in months and the due date is ascertained to be some date that does not exist, such as Feb. 30 (Jan. 30+1 mo.) count the last day of the calendar month as the date of maturity.

WRITTEN. Use numerals for the months, as 8/11 instead of Aug. 11.

	Date	Time	Time		Date	Time	Time
1.	6/12	2 mo	60 da.	9.	5/31	1 mo.	30 da.
2.	9/4/19	4 mo.	120 da.	10.	4/30	2 mo.	60 da.
3.	11/26/18	3 mo.	90 da.	11.	3/31	3 mo.	90 da.
4.	8/31	1 mo.	30 da.	12.	2/28	1 mo.	30 da.
5.	1/28/18	1 mo.	30 da.	13.	7/31	2 mo.	60 da.
6.	1/29/18	1 mo.	30 da.	14.	9/30	3 mo.	90 da.
7.	1/30/18	1 mo.	30 da.	15.	11/30/17	2 mo.	60 da.
8.	1/31/18	1 mo.	30 da.	16.	10/31/17	3 mo.	90 da.

17. Find the amount due at maturity on the note shown in the illustration.

18. J. W. Hunter, the drawee named in the time draft in the illustration, did not pay the draft until Dec. 15. The rate of interest on the draft is 6%. How much did Mr. Hunter pay?

CHAPTER XV

BANK DISCOUNT

The holder of paper not yet due may realize cash upon it by selling it. The buyer of such paper usually accepts it at a discount. Banks do a large business in discounting paper before maturity.

For purposes of figuring discount, paper may be divided into two general classes — interest-bearing paper and non-interest-bearing paper.

Non-interest-bearing paper is worth at maturity only its face, and it is upon this amount that discount is figured. Interest-bearing paper is worth at maturity its face plus interest for the time it has run, and it is upon the full amount of its principal and interest that discount is figured. Discount is figured in the same way as interest, but generally at a different rate.

The value of any note or other written obligation, at any time, is the face of the paper plus interest if any, for the full period of the obligation, and minus discount. The discount must be based on the combined principal and interest.

The period of time for which discount is figured is called the term of discount. It is usually counted in exact days.

ORAL. Find the term of discount on the following paper, counting the discount period by exact days.

	Date of Paper	Time	Date Discounted
1.	Mar. 27	3 mo.	Apr. 10
2.	May 9	60 da.	June 1
3.	Oct. 1	2 mo.	Oct. 10
4.	Aug. 15	30 da.	Aug. 15
5.	Sept. 24	2 mo.	Oct. 1
6.	July 17	4 mo.	Aug. 26
7.	Nov. 5	1 mo.	Nov. 23
8.	Jan. 29	1 mo.	Feb. 21
9.	Apr. 30	3 mo.	May 2
10.	Feb. 28	4 mo.	Apr. 16

11. June 20	3 mo.	June 20
12. Dec. 17, 1918	3 mo.	Jan. 4, 1919
13. Apr. 10	3 mo.	Apr. 10
14. Feb. 29	1 mo.	Mar. 1
15. Aug. 31	2 mo.	Aug. 31
16. July 31	2 mo.	Aug. 7
17. Jan. 30	1 mo.	Jan. 30
18. Mar. 17	3 mo.	Mar. 25
19. Nov. 22, 1917	4 mo.	Nov. 30, 1917
20. Apr. 12	6 mo.	Apr. 30

Many banks use a table, of which the following is a copy, for computing the actual days between two dates.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January.....	365	31	59	90	120	151	181	212	243	273	304	334
February.....	334	365	28	59	89	120	150	181	212	242	273	303
March.....	306	337	365	31	61	92	122	153	184	214	245	275
April.....	275	306	334	365	30	61	91	122	153	183	214	244
May.....	245	276	304	335	365	31	61	92	123	153	184	214
June.....	214	245	273	304	334	365	30	61	92	122	153	183
July.....	184	215	243	274	304	335	365	31	62	92	123	153
August.....	153	184	212	243	273	304	334	365	31	61	92	122
September.....	122	153	181	212	242	273	303	334	365	30	61	91
October.....	92	123	151	182	212	243	273	304	335	365	31	61
November.....	61	92	120	151	181	212	242	273	304	334	365	30
December.....	31	62	90	121	151	182	212	243	274	304	335	365

This table gives the exact number of days from any day of any month to the corresponding day of any other month for any period of time not more than one year.

Thus, the number of days from April 27 to July 27 would be 91. This is found by locating *April* in the column to the left and following along the horizontal line to the right until the column headed *July* is reached. The number at the junction of these two columns is 91. This indicates that the time from April 27 to July 27 is 91 days.

The time from Apr. 27 to Aug. 10 would be found as follows: The time from Apr. 27 to July 27 is 91 days. From July 27 to Aug. 10 is 14 days. 91 days + 14 days = 105 days.

Problems in the Use of the Table

ORAL. Find the number of days between the two dates given in each of the following:

1. Jan. 18 to June 18.
2. Feb. 12 to July 5.
3. April 15 to Oct. 9.
4. July 18 to Dec. 1.
5. July 1 to Jan. 1.
6. Oct. 20 to Jan. 12.
7. Aug. 5 to Nov. 27.
8. Jan. 1 to July 1.
9. May 20 to Oct. 8.
10. June 27 to Nov. 10.
11. July 17 to Dec. 5.
12. Oct. 20 to Apr. 4.
13. Sept. 15 to Feb. 1.
14. Mar. 12 to Sept. 5.

For a further use of the table find the time in some group of problems which your teacher will designate.

Non-Interest-Bearing Paper

EXAMPLE. A note for $350, dated Mar. 1, due in 60 days, was discounted Mar. 18 at 6%. Find the proceeds of the note.

Mar. 1 + 60 days = Apr. 30, date of maturity.
From Mar. 18 to Apr. 30 = 43 days, the term of discount.
Interest on $350 for 43 days = $2.51, amount of discount.
$350 − $2.51 = $347.49, proceeds of note.

WRITTEN. Find the proceeds of the following notes:

	Face of Note	Time to Run	Date of Note	Date Discounted	Rate of Discount
1.	$ 600.00	60 da.	Mar. 12, 1928	Mar. 27, 1928	6%
2.	380.00	90 da.	Jan. 22, 1927	Feb. 15, 1927	6%
3.	352.25	3 mo.	Feb. 17, 1928	Apr. 2, 1928	5%
4.	1150.00	30 da.	June 16, 1927	June 18, 1927	6%
5.	875.00	4 mo.	May 12, 1927	July 1, 1927	7%
6.	392.80	2 mo.	July 7, 1928	July 7, 1928	6%
7.	3000.00	6 mo.	Aug. 19, 1927	Aug. 25, 1927	6%

8.	$ 925.40	5 mo.	Sept. 1, 1927	Sept. 12, 1927	5½%
9.	4500.00	4 mo.	Oct. 31, 1928	Nov. 17, 1928	6%
10.	7200.00	3 mo.	Sept. 22, 1927	Oct. 1, 1927	4%
11.	362.75	90 da.	Oct. 2, 1927	Oct. 15, 1927	7%
12.	1281.30	2 mo.	Nov. 17, 1928	Dec. 1, 1928	7½%
13.	860.35	8 mo.	Aug. 31, 1927	Dec. 15, 1927	7%
14.	125.00	1 mo.	Mar. 1, 1927	Mar. 2, 1927	6½%
15.	278.30	30 da.	May 21, 1928	May 25, 1928	6%
16.	387.28	2 mo.	Sept. 26, 1928	Oct. 15, 1928	6%
17.	3500.00	6 mo.	Feb. 11, 1927	Mar. 1, 1927	4½%
18.	712.45	60 da.	Apr. 30, 1928	May 10, 1928	6%
19.	383.47	90 da.	May 31, 1928	May 31, 1928	7%
20.	516.40	2 mo.	Jan. 15, 1928	Jan. 31, 1928	8%
21.	857.16	4 mo.	Oct. 31, 1927	Nov. 5, 1927	6%
22.	1575.81	30 da.	Feb. 24, 1928	Mar. 1, 1928	7½%
23.	4500.00	90 da.	May 27, 1927	May 28, 1927	6%
24.	6200.00	3 mo.	June 30, 1928	July 2, 1928	5%

$2478.50 DENVER, COLO., *May 24, 19*

Sixty days AFTER DATE *we* PROMISE TO PAY TO THE ORDER OF *The Amoskeag Mills*

Twenty-four Hundred Seventy Eight and 50/100 DOLLARS.

VALUE RECEIVED.

John Albright & Co.

25. The above note was discounted on June 6, 1928 at 6%. What were the proceeds?

WRITTEN. Find the proceeds of the following drafts:

	Face	When Payable	Date Drawn	Date of Acceptance	Date of Discount	Rate of Discount
1.	$ 900.00	60 da. after sight	Mar. 19	Mar. 24	Apr. 1	6%
2.	472.50	30 da. after date	May 16	May 19	May 22	6%
3.	1637.20	3 mo. after date	Nov. 14	Nov. 18	Dec. 1	5½%
4.	371.25	90 da. after sight	July 27	July 28	Aug. 10	7%
5.	800.00	2 mo. after sight	Sept. 8	Sept. 15	Sept. 20	7½%
6.	750.37	30 da. after date	Apr. 5	Apr. 10	Apr. 15	8%
7.	978.46	1 mo. after sight	June 11	June 12	June 14	5%
8.	436.18	4 mo. after date	Aug. 20	Aug. 24	Aug. 24	6%
9.	214.70	4 mo. after sight	Oct. 28	Oct. 31	Nov. 4	6½%
10.	1356.45	60 da. after sight	Dec. 18	Dec. 19	Dec. 19	7%
11.	4250.00	90 da. after sight	Feb. 12	Feb. 15	Mar. 1	5%
12.	3783.19	3 mo. after sight	Jan. 24	Jan. 29	Feb. 20	8%

$1264.75 NEW YORK, N. Y., *May 15, 19*

Thirty days AFTER DATE PAY TO THE ORDER OF

The H. B. Claflin Co.

Twelve Hundred Sixty-four and 75/100 DOLLARS.

VALUE RECEIVED AND CHARGE TO THE ACCOUNT OF

To *Armbruster & Co.*,
Columbus, Ohio. *The H. B. Claflin Co.*

13. The above draft was accepted on May 18 and discounted on May 21 at 7%. Find the proceeds.

14. How much profit was realized by the bank which discounted the above draft (see problem 13) if the draft was paid by Armbruster & Co. on July 20, with interest at 7% from date of maturity?

15. If the draft shown at the top of the next page was accepted by the drawee on Mar. 19 and discounted at 6% on Mar. 25, what were the proceeds?

$3170.45 NEW YORK, N. Y., *Mar. 16, 19*

At sixty days sight PAY TO THE ORDER OF

Chas. G. Price

Thirty-one Hundred Seventy and 45/100 DOLLARS.

VALUE RECEIVED AND CHARGE TO ACCOUNT OF

To *Mabley-Carew Co.,*
Cincinnati, Ohio. *Chas. G. Price.*

16. The Mabley-Carew Co. paid the above note on April 15, less discount at 4%. What profit was realized by the bank which handled the paper?

Interest-Bearing Paper

EXAMPLE

$627.50 DAYTON, OHIO, *Mar. 18, 19*

Three months AFTER DATE *I* PROMISE TO PAY TO

Richard W. Wright, OR ORDER,

Six Hundred Twenty-seven and 50/100 DOLLARS.

VALUE RECEIVED, WITH INTEREST AT *6%*.

James Powell.

The above note was discounted at a bank on Mar. 29. What were the proceeds, if the rate of discount was 6%?

Interest on $627.50 for 3 mo. at 6%	$ 9.41
Face of note	627.50
Amount due at maturity June 18, 1918	$636.91
Mar. 29 to June 18 = 2+30+31+18 = 81 days, term of discount.	
Discount on $636.91 for 81 days	8.60
Proceeds of note	$628.31

WRITTEN. Find the proceeds of the following paper:

	Face	Time	Dated	Rate of Interest	When Discounted	Rate of Discount
1.	$ 500.00	3 mo.	May 1	6%	May 21	6%
2.	1250.00	2 mo.	Apr. 10	6%	May 1	6%
3.	1160.45	60 da.	Mar. 15	5%	Mar. 20	7%
4.	1473.18	90 da.	Aug. 12	7%	Sept. 1	6%
5.	274.80	30 da.	Jan. 5	6%	Jan. 8	6%
6.	728.32	4 mo.	Oct. 1	7%	Oct. 10	$6\frac{1}{2}$%
7.	417.35	90 da.	Sept. 22	$5\frac{1}{2}$%	Oct. 1	6%
8.	3500.00	6 mo.	Apr. 1	7%	Apr. 14	6%
9.	1250.00	60 da.	June 1	5%	June 15	7%
10.	1637.80	90 da.	July 17	6%	Aug. 1	$7\frac{1}{2}$%
11.	763.27	4 mo.	2/10/17	$5\frac{1}{2}$%	Feb. 28	7%
12.	471.80	3 mo.	July 30	5%	Aug. 31	6%
13.	267.45	2 mo.	Nov. 18	6%	Dec. 1	8%
14.	1439.60	30 da.	Dec. 1	5%	Dec. 2	6%
15.	1129.70	3 mo.	2/3/18	6%	Mar. 31	7%
16.	4500.00	90 da.	Mar. 11	5%	Mar. 31	7%

17. Draft dated Aug. 6 for $1273.94 payable 60 days after date with interest at 6%, accepted Aug. 8, and discounted Aug. 31 at 8%.

18. Draft dated April 15 for $2185.77 payable 4 mo. after sight with interest at 5%, accepted April 28, and discounted May 1 at $7\frac{1}{2}$%.

19. Draft for $1250.50 dated July 3, payable 3 months after sight, with interest at 5%, accepted July 7, and discounted July 15 at $7\frac{1}{2}$%.

20. Note for $750 at $4\frac{1}{2}$%, dated Oct. 1 and maturing in 4 mo. Discounted Oct. 1 at 6%. 745.64

Collection Charges. An additional charge is frequently made in discounting paper which has to be sent out of the city for collection.

WRITTEN

Read carefully the paragraph in small type on page 204.

1. On Dec. 31 Jones had on hand eight interest-bearing notes as listed below. Each was for one year. What was their total inventory value, discounted at 5%?

Face	Dated	Rate of Interest	Face	Dated	Rate of Interest
$1200.00	Sept. 27	6%	$ 682.70	Oct. 1	5%
1154.37	Nov. 5	5½%	243.18	Nov. 3	5%
397.50	Oct. 17	7%	1178.40	Aug. 12	5½%
575.00	Nov. 20	6%	2739.45	July 28	6%

2. Find the proceeds of the first note in the first problem if it were payable in 6 months and discounted at a bank on Dec. 31 at 6%.

3. Find the proceeds of the first note in the first problem had it been payable in 6 mo. without interest, and discounted at a bank on Dec. 31 at 6%.

4. What would have been the proceeds of the fourth note in the first problem if it had been payable in 3 mo. with interest at 6% and had been discounted at 6%?

5. What would have been the proceeds of the fourth note in the first problem has it been payable in 3 mo. without interest and discounted on Dec. 31 at 6%?

6. A note for $1800 payable in 6 mo. without interest is dated Mar. 1, 1918. What is the cash value of this note on July 1, 1918, if the current rate of discount is 6%?

7. The Bronson Machinery Co. sold $4500 worth of machinery to the Wilson Valve Co., on April 1, 1918, on the following terms: Cash $1000; note for $1500, without interest, payable in 4 mo.; note for $2000, without interest, payable in 6 mo. At what value should these notes have been listed among the assets of the Bronson Machinery Co. on July 1, 1928, discounted at 6%?

8. On May 1 the Farmers' Loan and Trust Co. discounted at 6% for John Powers & Co., a note for $3000 due in 90 days without interest and dated May 1. On July 1 following, the bank prepared a statement of its condition and a statement of its income. At what valuation should the note and interest have been listed on this statement?

Problems in Negotiable Paper

1. The Art Metal Co.'s Notes Receivable book showed the following notes on Sept. 1, 1928, at the time of consolidation with the Wolverine Motor Co.

Principal	Date	Time	Interest
$357.86	June 24, 1928	3 mo.	
146.37	July 18, 1928	60 da.	6%
95.82	July 27, 1928	2 mo.	6%
239.70	Aug. 5, 1928	2 mo.	
75.37	Aug. 15, 1928	3 mo	7%
438.80	Aug. 21, 1928	90 da.	6%

At what value should the above notes be taken into the business, assuming that the maker in each case is financially responsible? Show the value of each note separately, discounted at 6%.

2. The Notes Payable book of The Johnson Tool Co. shows the following, all payable in Indiana:

Dated	Time	Interest Bearing	Principal
April 15, 1928	2 months	Yes. 6%	$318.40
May 25, 1928	3 months	No	562.90
June 30, 1928	3 months	Yes. 5%	427.60
July 17, 1928	90 days	No	272.35

On Nov. 1, 1928, no payments having been made on any of these notes, and all being past due, The Johnson Tool Co. made arrangements to have its non-interest-bearing note discounted at a bank for funds sufficient to pay the above indebtedness. If the note issued in favor of the bank is for 90 days from Nov. 1 and money is worth 6%, what must be its face to yield sufficient funds to pay the other notes?

3. The North End Construction Co. received from the Intervale Realty Co. for the erection of a building four payments, as follows:

June 20	$5000	Paid by 3-mo. note without interest
July 12	$5000	Paid by 60-day note without interest
Sept. 1	$5000	Paid in cash
Oct. 3	$5000	Paid by 4-mo. note without interest

The construction company discounted all notes the day they were received, the first and third at 6%, the second at 7%. How much cash did it realize?

$1456.50 PORTLAND, ORE., *Apr. 1, 19*

Three years AFTER DATE, *I* PROMISE TO PAY

William J. Roberts, OR ORDER,

Fourteen Hundred Fifty-six and 50/100 DOLLARS,

WITH INTEREST AT *8%*. VALUE RECEIVED.

C. J. Smith.

4. The above note was discounted by Mr. Roberts on May 5, 1925, at 6%, no interest having been paid. How much money did he receive?

5. The above note was not collected, and no interest was paid, until Nov. 22, 1928. How much was then due?

6. A non-interest-bearing 90-day note for $150.50 dated May 7, 1926, was not collected until Dec. 5, 1927. Both parties to the note lived in Illinois. How much should have been collected?

CHAPTER XVI

TRADE AND CASH DISCOUNT

Trade discount is a deduction from list or catalog price, made for the purpose of fixing the invoice price to the trade. It is usually expressed as a given per cent of the list price.

This method of fixing price is convenient when the seller wishes to quote different prices to different classes of customers, or wishes to make changes in price from time to time. Many other conveniences and economies of the method will be apparent to the thoughtful student.

EXAMPLE. A merchant who sells to both wholesale and retail trade publishes a catalog with prices for the retail trade. He offers a 25% trade discount to wholesalers. What is his wholesale price on an article listed at $28.00?

List price	$28.00
Trade discount (25%)	7.00
Invoice price	$21.00

Series of Trade Discounts

EXAMPLE. The merchant mentioned in the preceding example offers an additional trade discount of 10% to jobbers; that is, his price to jobbers is "list less 25 and 10%." What is his July price to jobbers on an article listed at $28.00, if as a special inducement during July he offers an additional 5%?

List price	$28.00
1st trade discount (25%)	7.00
	$21.00
2d trade discount (10%)	2.10
	$18.90
3d trade discount (5%)	.94
Invoice price	$17.96

NOTE. The foregoing problem could have been solved as follows: $100\% - 25\% = 75\%$; $75\% - 10\%$ of itself $= 67\frac{1}{2}\%$; $67\frac{1}{2}\% - 5\%$ of itself $= 64\frac{1}{8}\%$; $64\frac{1}{8}\%$ of $\$28 = \$17.96$ or, 95% of 90% of $75\% = 64.125\%$; $.64125 \times \$28 = \$17.96$.

TERMS CASH.—PAYABLE IN CHICAGO OR NEW YORK FUNDS. LOCAL CHECKS MUST INCLUDE COST OF COLLECTION
CLAIMS FOR ALLOWANCES MUST BE MADE UPON RECEIPT OF GOODS. NOT RESPONSIBLE FOR GOODS SENT BY MAIL.

INVOICE NO. 102
SALESMAN Scott
ORDER NO. 0886
YOUR REQ. NO.

The Franklin Company
ESTABLISHED 1861
Engravers-Designers-Electrotypers
Catalogue and Booklet Printers
720-734 South Dearborn Street

TELEPHONES HARRISON 1224

Chicago April 6th, 19

Sold to Lyons & Carnahan,
623 S. Wabash Ave., Chicago.

1	Zinc of Letter	2	76	
1	" "	1	85	
1	" "	2	79	
		7	40	
	Less 20%	1	48	
		5	92	
	Inking Purple Copy		50	
				6 42

Invoice Showing Trade Discount Figured by Percentage

Note. Fractions of a cent sometimes appear in the extensions of the items on a bill. In solving the problems in this book, charge the customer the full cent when the fraction is $\frac{1}{2}¢$ or more in the final extension of any single item. In practice, there is no uniformity in this matter — some merchants always give the half cent, some always take it, and others alternately give and take.

Written

1. What is the invoice price of a stove listed at \$90 less 50, 25, 10, and 5%?

2. What is the invoice price of paper bags listed at \$2.40 per 1000 if the terms to the trade are 50, 20, and $7\frac{1}{2}$% off?

3. A novel catalogued at \$1.50 sells to jobbers at $33\frac{1}{3}$ and 25% off. If a quantity discount of 5% is allowed on orders of 100 or more, what is the invoice price of 125 books to jobbers?

4. A manufacturer bills goods to his western depository at 25 and 15% off. The depository sells at 10% above its cost. What is its profit on goods listed at \$150?

5. A merchant buys mdse. listed at \$962.50. One firm offers discounts of 20, 10, and 5%. Another firm offers 5, 10, and 20% off. Which offer shall he accept? Explain.

NEW YORK, Jan. 26, 19

M Western Grocery House
45 E. Randolph St.,
Chicago, Ill.

Bought of **The American Tobacco Company**

111 Fifth Avenue

Terms Cash: 60 days, net.

We allow a discount of 2% for cash in 10 days from date of bill, payable in New York exchange.
NO INSURANCE ON GOODS UNLESS SO ORDERED.
No Claims allowed unless made within five days after receipt of Goods.
No Payment (Except at this Office) or Advances of any Nature on our Account are Authorized Except on Our Written Order. Goods Sold Upon the Express Condition that Transportation is at Purchaser's Risk.

Shipped from F. O. B. New York- Order of _____

	BRAND	STYLE	Total Quantity	PRICE	AMOUNT		TOTAL	
15	M Cremo Cigars	1/20's		30.			450	00
10	M Francis Wilson Cigars	1/40's		65.	650	00		
		Less $5 per M			50	00	600	00
							1050	00
			Frt. Paid				6	60
							1056	60
3¾	M Cremo Cigars	Gratis						
2½	M Francis Wilson Cigars	"						

TRADE DISCOUNTS IN MONEY AND MERCHANDISE

6. What single discount is equivalent to each of the following series?

(*a*) 25 and 20%
(*b*) 25 and $12\frac{1}{2}$%
(*c*) 20 and 15%
(*d*) 20 and $2\frac{1}{2}$%
(*e*) 40 and 10%
(*f*) 40 and $7\frac{1}{2}$%
(*g*) $33\frac{1}{3}$ and 25%
(*h*) 25 and $33\frac{1}{3}$%
(*i*) $33\frac{1}{3}$ and $12\frac{1}{2}$%
(*j*) 50 and $33\frac{1}{3}$%
(*k*) 40 and $12\frac{1}{2}$%
(*l*) $33\frac{1}{3}$ and 20%
(*m*) 20, 10, and 10%
(*n*) 10, 10, and 5%
(*o*) 20, 10, and 5%
(*p*) 25, 10, and 5%
(*q*) 50, $33\frac{1}{3}$, and 25%
(*r*) 60, 40, and 25%
(*s*) 80, 20, and $2\frac{1}{2}$%
(*t*) 25, $16\frac{2}{3}$, and 10%
(*u*) 20, 20, and 20%
(*v*) 50, 50, and 10%
(*w*) 70, 25, and 3%
(*x*) 10, 10, and 10%

7. On the invoice at the top of this page the rebates of money and cigars are together equivalent to what trade discount, on the Francis Wilson cigars?

Cash discount is a deduction from invoice price made in consideration of payment within a specified time.

The offer of a discount for cash is usually made in the terms of sale, printed or written as a part of the heading of an invoice. Thus "2/10 *n*/30 (2% 10 days, net 30 days)" means that the customer may deduct 2% if he pays cash in ten days, but if he fails to do this the bill is due without discount in 30 days.

Sometimes the terms of sale contain the additional specification that the charge draws interest if not paid at maturity.

Taking a discount for cash in advance of maturity is usually regarded as a purely financial transaction, not a trading transaction. Hence cash discount is not deducted, usually, in figuring costs. Some authorities favor the practice, however.

WRITTEN. Find the net cash due on each of the following bills if discounted, and the latest date on which discount will be allowed.

1.

CENTRAL PAINT & GLASS CO.

DETROIT, MICH., Oct. 2, 19__

SOLD TO Pickard Hdw. Co.,
Sheboygan, Wis.

Terms: 2% Cash Nov. 12, 1920.

			GROSS	NET	EXTENSION
		RED SHIELD COLORS			
	5#	5's American white @ .07			
		ACME QUALITY PAINT AND VARNISH REMOVER			
	½	Gal. ¼'s @ $2.00			
	¼	Gal. ¼'s @ $2.15			
		ACME QUALITY OIL WOOD STAIN			
	2	Hf. Pt. Dark Oak @ $ .18			
		Less 40%			
		ACME QUALITY ART WOOD FINISH			
	1	Hf. Pt. 1/32 @ $ .25			
		Less 40%			
		RED SHIELD COLORS			
	6	lb. Ivory Drop Black @ $ .32			
		Less 50% and 5%			

"5's" means packed 5 lb. to the can. "$\frac{1}{4}$'s" means $\frac{1}{4}$ gal. to can.

2.

Schroeder Paint and Glass Co.
Detroit, Mich.

August 19 19

Sold to STEWART & STEWART,
346 Cass Avenue, City.

Terms: 2% 10 days; net 30 days.

1	Box 12x32 S. A.	$37.00
2	" 12x36 " "	37.00
1	" 18x30 " "	36.00
3	" 10x12 " "	37.00
4	" 24x26 " "	39.00
1	" 20x30 " "	37.00
	Less 90%	

3.

Our invoices are all made at net cash prices and are payable in New York City Bankable Funds. Money sent by mail at the risk of those thus remitting.

SARGENT

No deduction allowed unless claim is made as soon as goods are received. All goods at risk of purchaser after shipment.

TERMS
30 Days or two per cent cash in 10 days.

Sargent & Company
Manufacturers of Hardware
220 North Michigan Ave.
Chicago

NO GOODS TAKEN BACK OR EXCHANGED

March 7, 19

Make remittance to Sargent & Company New Haven, Conn.

SOLD TO Stebbins Hdw. Co.

ADDRESS 15 W. Van Buren St., Chicago, Ill.

	THIS SHIPMENT FROM FACTORY AT NEW HAVEN, CONN.		GROSS	NET	TOTAL
1/2	doz Cow Bell 4	at $5.00			
		50-3/10			
10	gro Sc Hook 412-¾	at $4.50			
		80%			
3	only Checks 1625	at $4.00			
		50-20-10-10			
1	doz Pulley 25-3	$3.00			
		50-10-10-10			
1	doz C D Fasts AB2168P	$14.60			
		33 1/3-20-10-10			
1/2	doz Parts 11 for Spring 1653	$9.00			
1/2	doz Parts 11 for Spring 1655	$12.00			
		33 1/3-20-2/10			
2	doz Clamps 3	$.65			
		10-10			
10	gro Sc eyes 112	$1.70			
5	gro Hks and Eyes 40-1	$6.50			
		87 1/2-10			
2	doz Paper Files 34	$1.40			
		25-20-10-10			

"50—3/10" means less 50 and three 10's; or 50, 10, 10. and 10 off

4.

STANDART BROTHERS

DETROIT, MICH., Sept. 30, 19___

SOLD TO J. M. & W. Westwater,

529 Main St., City.

			GROSS		NET		EXTENSION	
1/4	doz.	3/8 #6 Wood Chisels	@	$14.00				
1/4	"	1/2 " "	@	14.00				
1/4	"	5/8 " "	@	15.50				
1/4	"	3/4 " "	@	16.00				
1/4	"	7/8 " "	@	18.00				
1/6	" 1	1/2 " "	@	22.00				
1/6	" 1	3/4 " "	@	24.00				
		Less 80, 10, and 10%						
1/6	"	3/8 #5 Wood Chisels	@	12.50				
1/6	"	1/2 " " "	@	12.50				
1/6	"	3/4 " " "	@	14.50				
1/6	"	1 " " "	@	16.50				
		Less 80, 10, and 10%						
1/4	"	67 R. R. Picks	@	15.00				
		Less 80%						

5.

THE VODREY POTTERY CO.

EAST LIVERPOOL, OHIO, Jan. 3, 19___

SOLD TO Messrs. Willis & Fenn,

Detroit, Mich.

TERMS: 1% 15 days; net 30 days.

5	doz.	Risco Teas, Handled	@	$1.60			
5	doz.	" " Cups, only	@	.93			
2	doz.	" Fruits, 4 in.	@	.60			
5	doz.	" Plates, 7 in.	@	1.30			
3	doz.	" " 8 in.	@	1.50			
2	doz.	" Bakers, 3 in.	@	1.40			
5	doz.	Bristol Teas, Handled	@	1.60			
5	doz.	" " Cups, only	@	.93			
2	doz.	" Fruits, 4 in.	@	.60			
5	doz.	" Plates, 7 in.	@	1.30			
3	doz.	" " 8 in.	@	1.50			
		Less 60 and 10%					

6.

S. A. WELLER & CO.

ZANESVILLE, OHIO, July 11, 19__

SOLD TO The Racket Store,

Cleveland, Ohio

TERMS: 1% 15 days; net 30 days

1	doz.	Risco Bone Dishes	@	$1.20
3	"	" Ind. Butters	@	.40
1	"	" Creams	@	2.40
1/2	"	" Dishes 6"	@	1.40
1/2	"	" 10"	@	3.60
1/2	"	" 12"	@	6.00
6	"	" Fruits 4"	@	.60
6	"	" 4½"	@	.65
3	"	" Nappies 7"	@	2.40
2	"	" 8"	@	3.60
1	"	" 9"	@	4.80
3	"	" Plates 4"	@	.70
3	"	" 5"	@	.90
3	"	" 6"	@	1.10
1	"	" 7"	@	1.30
1	"	" 8"	@	1.50
1/4	"	" Sugars	@	5.40
5	"	" Handled Teas	@	1.60

Less 60, 10, and 2 1/2%

7.

PARK & McKAY

Detroit, Mich., Oct. 14, 19__

Sold to Fitzpatrick & Hoepfner,

Columbus, Ohio

Terms: 2% 10 days; net 30 days.

300'	4"	Standard Single Hub Soil pipe	@	$1.70
200'	4"	" Double " " "	@	1.80
	Less 70, 5 and 5%			
200'	4"	Extra Heavy Single Hub Soil pipe	@	$2.80
150'	4"	" " Double " " "	@	2.95
100'	3"	" " Single " " "	@	2.50
50'	3"	" " Double " " "	@	2.60
	Less 75, 5, and 5%			

8.

PARK & McKAY

Detroit, Mich. Oct. 14, 19__

Sold to Fitzpatrick & Heopfner,

Columbus, Ohio

Terms: 2% 10 days; net 30 days.

18	4" Standard Ys	@	$3.65
12	4" " 1/4 Bends	@	2.65
12	4" " Sanitary Ts	@	3.65
24	4" " 1/8 Bends	@	2.65
6	3" x 2" Standard Ys	@	3.70
24	4" Extra Heavy 1/8 Bends	@	3.80
12	4" x 3" Extra Heavy Ys	@	4.20
12	3" Extra Heavy 1/8 Bends	@	3.15
6	3" " " 1/4 Bends	@	2.60
6	3" " " Ys	@	4.20
3	4" " " Xs	@	4.20
	Less 80 and 5%		
1	Coil 5/8 strong lead Pipe 184#	@	$ .27
1	Coil waste pipe 174#	@	.30
6	4" x 6" lead Ferrule	@	.80
	Less 55 and 5%		
3	20" x 30" P 6800 sinks	@	21.50
	Less 30%		
6	4" Extra Heavy 1/4 Bends	@	3.90
	Less 80, 5, and 5%		

ORAL

1. What single discount is equal to 25 and 20%?

2. Which is the better offer, a discount series of 20 and 20%, or a single discount of 35%?

3. Which is the better offer, a series of 25, 20, and 10%, or 40 and 10%?

4. What is the selling price of an article listed at \$60 less $33\frac{1}{3}$, 25, and 10%?

5. An article is listed at \$150 by two different firms. One offers discounts of $33\frac{1}{3}$, 20, and 10%, and the other offers discounts of 40, 10, and 10%. Which offer is the better?

6. What single discount is equivalent to 20 and 10%?

7. An article is listed at \$50 less 80 and 10%. What is the selling price?

8. Which is the better offer, discounts of 40 and 20% or discounts of 20, 20, and 20%?

9. What single discount is equal to discounts of 80 and 20%?

10. Which is the better offer, a series of 40 and 10% or a single discount of 50%?

11. Wood chisels of a certain kind are listed at \$14 per dozen less 90%. What is the net cost per chisel?

12. Plates were bought at \$1.50 per dozen less 60 and 10%. They were sold at 20% from the list price. What was the profit per plate?

Plumbers' Problems — Written

1. Two manufacturers of plumbing supplies each list a bathroom outfit for \$265. One offers discounts of 20, 20, and 10% and the other discounts of 25, 20, and 5%. Which offer is the cheaper and how much?

2. A plumber is offered a certain quantity of radiators by one firm for \$975.40 less 30, 20, and 10%. Another firm offers the same quantity for \$847.50 less 40 and 25%. The first firm offers a further discount of 3% if cash is paid within 10 days, while the second firm will not allow a discount for cash. Which is the better offer and how much, if cash is to be paid?

3. A plumber buys certain fixtures at \$27 per dozen less 60 and 20%. He sells these fixtures at list price less 20 and 10%. How much profit does he make on 40 dozen? What per cent, based on cost?

4. One dealer offers a plumber certain goods for \$258.75 less 60, 20, and 10%, with an additional 5% for cash in 10 days. Another dealer offers the same goods for \$296.37 less 75, 10, and 2%, net cash f.o.b. dealer's city. The first dealer also allows one-half the freight, the freight charge being for 1275 lb. at 21¢. The freight rate from the second dealer is 32¢. Which is the better offer and how much?

5. A plumber bought a coil of lead pipe weighing 186 lb. at $5\frac{1}{2}$¢ per lb. less 50 and 10%. He was offered a cash discount of 3% for payment of the bill within 10 days. If he accepted the offer and paid cash within 10 days, what was the net cash price per lb.?

WRITTEN

1. A dealer advertised an ice chest for \$14.56. These chests had been selling for \$20. The advertisement read, "To each person who presents this ad, a further discount of \$1.00 will be allowed." Brown presented the advertisement and bought an ice chest. At what rate of discount did he buy it?

2. A canvasser sold stereoscopic views at $12\frac{1}{2}¢$ each or \$1.50 a dozen. As an inducement for large orders, he made it a rule to give two pictures extra with each \$3.00 purchase. To what rate of discount was that equivalent?

3. A merchant's sales amounted to \$30,000.00 the first year. His regular terms allowed a discount of 2% for cash. If every customer took advantage of the cash discount, what was the amount of his net cash receipts from sales for the year?

4. The merchant in the foregoing problem adopted for his second year the plan of giving customers \$2.00 worth of goods extra for every \$100 worth of goods purchased. His cash receipts amounted to \$30,000 for the year. How much did he increase his sales the second year? Was the rate of discount more or less than the preceding year? How much?

5. I purchased goods listed at \$2685.30, at $33\frac{1}{3}$, 20, and 10% off. I sold these goods at the list price less 40 and $12\frac{1}{2}$%. What was my rate of profit over cost? My profit was what per cent of the sale price?

6. A certain department store advertised that to all customers living within 100 miles who would come to the store and buy \$25 worth of goods, the store would refund the amount of car fare one way. One woman living 75 miles from this store bought \$32.58 worth of goods. The car fare refunded was at the rate of 2¢ per mile. To what discount was this equivalent?

7. Another woman whose car fare was the same as that of the woman mentioned in the preceding problem bought \$65 worth of goods. What rate of discount did she secure?

8. H. O. Sharp sold H. E. Smith a bill of goods amounting to \$500.00. At the conclusion of the sale, Mr. Smith asked for a cash discount of 2%. Mr. Sharp replied that his house did not give cash discounts, but that he would throw in a suit of clothes

worth 2% of what Mr. Smith had spent. To what rate of discount was this equivalent?

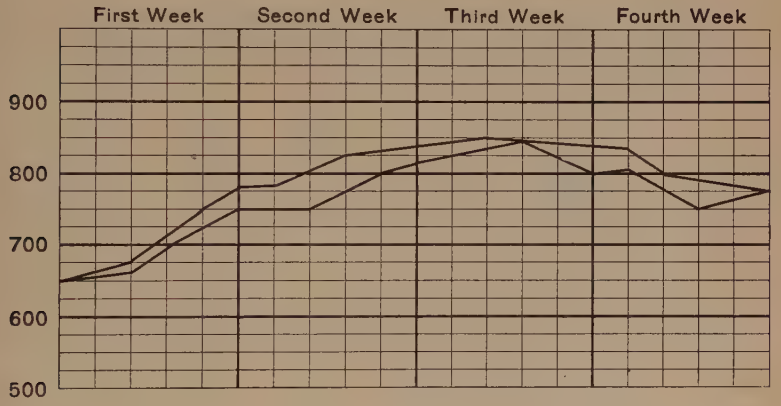
9. Greenebaum & Co. make a practice of allowing customers a cash discount of 5% on partial payments of invoices. Wolfe & Co., who owed them a large sum of money, made them a payment of $500.00. What was the amount of discount to which they were entitled because of this payment?

CHAPTER XVII

GRAPHIC ILLUSTRATIONS

A **graph** is a diagram by which some relationship is illustrated, usually by means of straight or curved lines.

The following graph illustrates the variation between enrollment and attendance at a certain school for each day for one month.



The upper charted line represents the enrollment of the school, and the lower charted line, the attendance.

What does each space between the light vertical lines represent? Between the heavy vertical lines?

What does each space between the light horizontal lines represent? Between the heavy horizontal lines?

What does the space between the two charted lines represent? When are the lines farthest apart and what does this indicate?

Graphs may be used to illustrate temperature, rain fall, earnings and expenses, receipts and expenditures, fluctuations in prices, to compare one year's sales with another, or for a variety of other purposes.

1. The net profits and dividend payments of the International Harvester Co. for ten years were as follows:

Year	Net Profits	Dividends Paid	Year	Net Profits	Dividends Paid
1903.....	$5,640,000	$3,600,000	1908....	$8,890,000	$4,200,000
1904.....	5,660,000	4,800,000	1909....	14,890,000	4,200,000
1905.....	7,480,000	4,800,000	1910....	16,080,000	7,400,000
1906.....	7,850,000	4,800,000	1911....	15,520,000	8,200,000
1907.....	7,080,000	4,200,000	1912....	16,390,000	8,200,000

Illustrate graphically how the net earnings and the dividends paid by the company compare.

2. The following represents the production, in barrels, of the Universal Portland Cement Company's plants during ten consecutive years:

Year	Barrels	Year	Barrels	Year	Barrels
1906.....	2,076,000	1910.....	7,001,500	1913.....	11,197,000
1907.....	2,129,000	1911.....	7,737,500	1914.....	9,116,000
1908.....	4,535,000	1912.....	10,114,500	1915.....	7,645,000
1909.....	5,786,000				

Illustrate graphically the increase in the company's production.

3. The following schedule represents the gross income and total expenses of the Chicago Stock Yards Co. for seven years:

Year	Gross Income	Expenses	Year	Gross Income	Expenses
1906.....	$5,472,000	$3,432,000	1910.....	$5,291,000	$3,146,000
1907.....	5,903,000	3,677,000	1911.....	5,578,000	3,476,000
1908.....	5,203,000	3,013,000	1912.....	5,601,000	3,455,000
1909.....	5,058,000	2,910,000			

Compare gross earnings with expenses by means of a graph.

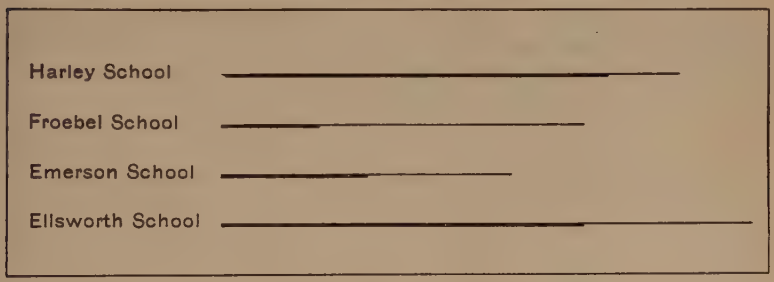
4. By means of a graph compare, on the blackboard, the daily attendance in your class this month with the attendance last month.

5. The receipts of an insurance company for one month amounted to $125,000. The expenses for the month amounted to $45,000, and claims paid amounted to $70,000, the remainder being held in the cash balance. Illustrate how the receipts, disbursements, and cash balance compare, by means of a circular graph.

6. Rule vertical lines $\frac{1}{4}$ inch apart and let their length represent graphically the average values of the principal crops of the United States during ten years, from the following figures. Let $\frac{1}{4}$ inch represent $50,000,000.

Corn	$869,575,310	Hay	$489,912,828
Wheat	431,717,233	Potatoes	124,812,869
Oats	246,936,311	Cotton	468,843,688
Barley	46,158,110		

7. The following graph shows the comparative enrollments of four different schools. The thickened part of each line represents the enrollment of the same school ten years ago. Each inch represents 100 enrollments. What is the present enrollment of each of the four schools? What was the enrollment of each school ten years ago?



8. Draw a graph illustrating the average prices of munition stocks in the United States compared with the average prices of railroad and industrial stocks, from June 21, 1916, to Nov. 22, 1916, from the following data. Let each horizontal line represent $20.

1916	Munitions	R. R. and Ind.	1916	Munitions	R. R. and Ind.
June 21	$314	$85	Sept. 20	$428	$92
July 5	343	84	Oct. 4	417	95
July 19	369	82	Oct. 18	421	97
Aug. 9	390	83	Nov. 1	534	99
Aug. 23	368	89	Nov. 15	472	98
Sept. 6	426	90	Nov. 22	458	97

ANNUAL INCOME

PROTRACTOR

9. The circle herewith represents an annual income of $1000. The parts into which it is divided are marked to show what the money was spent for. Construct a protractor similar to the illustration, slightly smaller than the circle, figure the angle of each part of the circle, in degrees, and compute the amounts spent on each item and what per cent each was of the whole.

CHAPTER XVIII

INVOLUTION AND EVOLUTION

Involution

A **power** of a number is the product obtained by using the number two or more times as a constant factor.

Thus, 4, 8, and 16 are powers of 2. 4 is the second power of 2 because 2 is used twice as a factor to give 4. 8 is the third power of 2 because 2 is used three times as a factor to give 8.

The second power of a number is called the **square** of the number. The third power of a number is called the **cube** of the number.

Consideration of the rules for finding the area of a square and the volume of a cube will make apparent the meanings of the above terms.

Involution is the process of raising a number to one of its powers.

The power to which a number is to be raised is often indicated by writing a small figure to the right and slightly above the number to be raised. This small number is called the **exponent.**

Thus, 5^2 means that 5 is to be squared; 6^3 means that the cube of 6 is to be found; 2^4 means that 2 is to be raised to the 4th power; etc.

WRITTEN

1. Find the square of all numbers from 1 to 20 inclusive.
2. Find the cube of all numbers from 1 to 20 inclusive.
3. Find the fourth power of all numbers from 1 to 20 inclusive.
4. Find the square of $\frac{1}{4}$, $\frac{3}{8}$, $\frac{7}{12}$, $\frac{5}{6}$, $\frac{3}{7}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{1}{2}$.
5. Find the cube of $\frac{3}{5}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{12}$, $\frac{5}{6}$, $\frac{1}{3}$, and $\frac{1}{2}$.
6. Find the fourth power of $\frac{3}{4}$, $\frac{1}{2}$, $\frac{3}{8}$, $\frac{4}{5}$, $\frac{1}{4}$, $\frac{5}{6}$, $\frac{5}{12}$, and $\frac{3}{5}$.
7. Find the square of .25, .75, .46, .55, .85, .16, .15, and .18.
8. Find the cube and fourth power of .12, .15, .18, .20, .25, .45, .27, and .32.
9. Make, and memorize, a table of the squares of the nine digits; also a table of the cubes of the nine digits.
10. How many acres in a field 80 rods square?
11. What is the value, at $2.10 per acre, of the land in a field 160 rods square?

Evolution

The **root** of a number is that number which was used two or more times as a constant factor to give the number.

Thus 2 is the square root of 4; 2 is the cube root of 8; 2 is the 4th root of 16; 5 is the square root of 25.

Evolution is the process of finding what number was used as a constant factor to produce a certain power. Evolution is the opposite of involution.

The radical sign is the character ($\sqrt{\ }$) placed above a number, considered as a power, to indicate that the root is to be extracted. When no number is used within the radical, square root is indicated.

Thus: $\sqrt{256}$ indicates that the square root is to be found.
$\sqrt[3]{1728}$ indicates that the cube root is to be found.
$\sqrt[4]{891}$ indicates that the fourth root is to be found.

A **perfect power** is a number whose root can be extracted leaving no remainder.

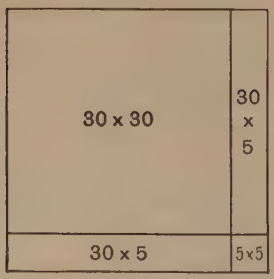
25 is a perfect square whose root is 5.

An improper power is a number whose root cannot be found exactly.

10 is an improper power, as its square root is 3.1622+, and no exact cube root or other root can be found.

To find the square root of a number.

EXAMPLE. What is one side of a square the area of which is 1225 sq. in?



	12'25.	30
	9 00	5
Trial div., $30\times2=$ 60	3 25	35
5		
300		
$5^2=$ 25		
325	3 25	

1. Point off the number into periods of two places, beginning at the decimal point.

2. Determine the largest perfect square which may be subtracted from the left hand period (9). Write the root of this square, followed by a cipher or ciphers, as a part of the result (30), and subtract the square of this partial result from the number.

3. For a trial divisor, multiply by 2 the part of the result already formed (giving 60). Try this (60) into the remainder and add the largest integral number that can be so obtained (5), to the partial result already formed. Multiply the trial divisor (60) by this number (5) and to the result add the square of this number ($5^2 = 25$).

4. Deduct from the remainder the total so secured. If the total is larger than the remainder, a smaller number will have to be selected for the second part of the result, and step 3 performed over again.

If there are more periods, continue the solution by the same process.

The method of solution shown above is a formula. See if you can reason the process out by consulting this diagram as you follow the steps of the formula.

When the number of which the square root is to be extracted contains decimals, be sure there are an even number of decimal places, adding a cipher if necessary. Solve as before, pointing off as many decimal places in the root as there are periods in the decimal part of the number.

Thus the square root of 9′.92′25 will have two decimal places. The square root of .00′05′76 will have three decimal places.

To find the square root of a common fraction find the square root of its numerator and of its denominator and use these roots as the numerator and denominator, respectively, of the result.

WRITTEN. Find the square root of each of the following:

1. 324	7. 2809	13. 4.7089	19. .576
2. 441	8. 4624	14. 12.378*	20. .0075
3. 529	9. 5625	15. 115.006	21. $\frac{25}{49}$
4. 729	10. 7225	16. 17.154	22. $\frac{2}{3}$
5. 1296	11. 8649	17. 31.0075	23. .1
6. 1764	12. 9604	18. 591.4624	24. $\frac{1}{4}$

*In extracting the root of an imperfect power, carry the result to three decimal places.

CHAPTER XIX

PRACTICAL MEASUREMENTS — Continued

Areas of Surfaces

Preliminary Definitions and Rules

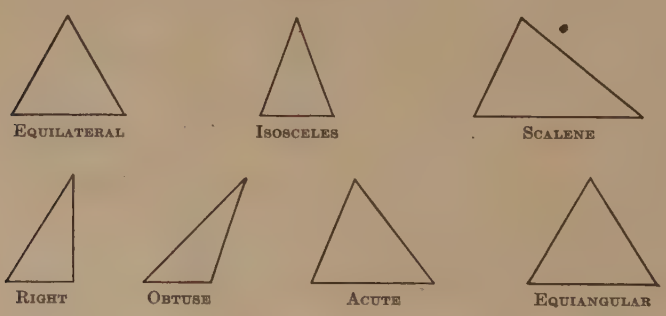
It is assumed that the student is familiar with most of the terms used in measurements, and that he knows the elementary rules. No attempt is made here to give a complete or connected list. A few definitions that the student may have forgotten, are given. These are accompanied and followed by some rules that will be needed in the work in practical measurements which follows. For tables, the student is referred to the appendix.

A line is **perpendicular** to another when the two meet at right angles, thus forming two angles of 90° each.

Thus a vertical and horizontal line are perpendicular to each other. Do not confuse *vertical* and *perpendicular*. A line may be perpendicular to another without being vertical.

The **hypotenuse** of a right triangle is the line which is opposite the right angle.

TRIANGLES



The **perimeter** of a surface is the combined length of its sides.

The **area** of a rectangle is the product of its length and width.

The **area** of a triangle is half the product of its base and altitude.

The **area of a triangle the altitude of which is unknown** is found as follows: From half the perimeter subtract each side separately;

QUADRILATERALS

TRAPEZEUM TRAPEZOID PARALLELOGRAM

PARALLELOGRAMS

RECTANGLE SQUARE RHOMBOID RHOMBUS

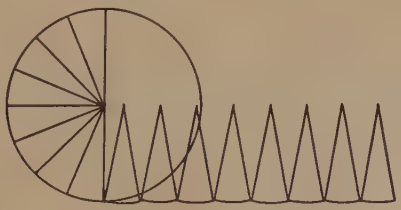
multiply the product of these three results by half the perimeter; extract the square root of the result.

The **square of the hypotenuse** of a right triangle is equal to the sum of the squares of the other two sides.

From this we can determine the length of any side of a right triangle if the other two sides are known.

The **circumference** of a circle is 3.1416 times the diameter.

For approximate measurements $3\frac{1}{7}$ is sometimes used instead of 3.1416. Use $3\frac{1}{7}$ only when instructed to do so.



The **area of a circle** is

3.1416 (approx. $3\frac{1}{7}$) $\times$ the square of the radius, or
.7854 (3.1416 $\div$ 4) $\times$ the square of the diameter, or
the circumference $\times$ one fourth the diameter (or $\frac{1}{2}$ of radius)

The square of the diameter is four times the square of the radius. Therefore if 3.1416 $\times$ square of radius = area, then .7854 (3.1416 $\div$ 4) $\times$ square of diameter = area. Again, if 3.1416 $\div$ 4 $\times D^2$ = area, then 3.1416 $\times D \times D \div 4$ = area, then (3.1416 $\times D$) $\times$ ($D \div 4$) = area, and since (3.1416 $\times D$) is the circumference, then circumference $\times$ one fourth of diameter = area.

Knowing the relations of radius, diameter, circumference, and area, each to the others, any one of them can be found if any other one is known.

WRITTEN

1. Draw a diagram of a rectangular floor 12 ft. x 15 ft. on a scale of $\frac{1}{4}$ in. to 1 ft. (If drawn on a blackboard, make the scale 1 in. to 1 ft.) How many sq. ft. in this floor? How many sq. yd.? (Draw heavier lines to show the square yards.)

2. A room is 10 ft. wide, 12′ 6″ long, and the walls are 9′ high. Make a flat diagram of the walls and ceiling, on a scale of $\frac{1}{4}$ in. to each foot.

3. Find the area of each of the above walls, and the total area of walls and ceiling to the nearest square yard.

4. John Clayton owns a strip of land on Main St., at the corner of Euclid, containing one acre. It is 160 ft. wide. Find its length.

5. The above land is to be divided into lots 30 ft. wide, adding any remainder over the 30 ft. to the corner lot. Make a plat of this property on a scale of $\frac{1}{32}$ in. to each foot.

6. What is the length of one side of a square field that contains 10 A.?

7. Find the area and perimeter of right triangles as follows:

(*a*) Base, 12 ft.; alt., 18 ft.
(*b*) Base, 21 yd.; alt., 30 yd.
(*c*) Base, 40 yd.; alt., 70 yd.
(*d*) Base, 45 rd.; alt., 65 rd.
(*e*) Base, 60.4 rd.; alt., 130.7 rd.
(*f*) Base, 12.8 ch.; alt., 18.3 ch.*

8. A field, triangular in shape, is 40 rd. long and $24\frac{1}{2}$ rd. wide. How many acres in it? (The width of a triangle is its perpendicular altitude.)

9. Find the areas of triangles having sides as follows:

(*a*) 16 ft., 28 ft., and 18 ft.
(*b*) 40 rd., 32 rd., and 24 rd.
(*c*) 30 ft., 24 ft., and 16 ft.
(*d*) 24.5 ft., 16 ft., and 13.5 ft.
(*e*) 60 yd., 45 yd., and 25 yd.
(*f*) 40 rd., 45 rd., 50 rd.

10. A farm, triangular in shape, has sides measuring 120 rd., 90 rd., and 70 rd. What is its value at $75 per acre?

11. A rectangular field is 45 rd. wide and 80 rd. long. What is the distance across the field diagonally?

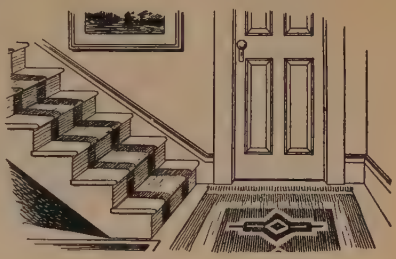
12. A telephone pole extends 35 ft. above the ground. A guy wire is fastened 3 ft. from the top of the pole, and is anchored in the ground 100 ft. from the pole. What is the length of the guy wire? (Make no allowance for slack in wire.)

* ch. = chain, surveyor's measure.

13. A derrick is 40 ft. high and is held in position by cables 200 ft. long. Making no allowance for slack, how far from the base of the derrick must the cables be anchored?

14. A baseball diamond is 90 ft. square. The first baseman throws the ball to the third baseman. How far does he throw it, if both are on their bases?

15. How long must be the side timbers of a stairway containing 16 steps each having a $7\frac{1}{2}''$ rise and a $10''$ tread?



16. A wheel is $3\frac{1}{2}$ ft. in diameter. What is its circumference?

17. An automobile wheel is 38 in. high. How many times will it turn in traveling one mile?

18. Find the diameter, in inches (to 2 decimal places), of a wheel that would make exactly 1000 revolutions in traveling 1 mile.

19. A machinist has three gears working together. The largest is 5 ft. in diameter. What must be the diameter of the second if it is to make two revolutions while the first makes one? What must be the diameter of the third if it is to make 5 revolutions while the second makes one?

20. How many square yards of concrete are necessary to cover the floor of a cylindrical silo 24 ft. in diameter, inside measurement?

21. A circular track is $\frac{1}{2}$ mile in circumference. Find the area of the space enclosed, expressed in highest terms.

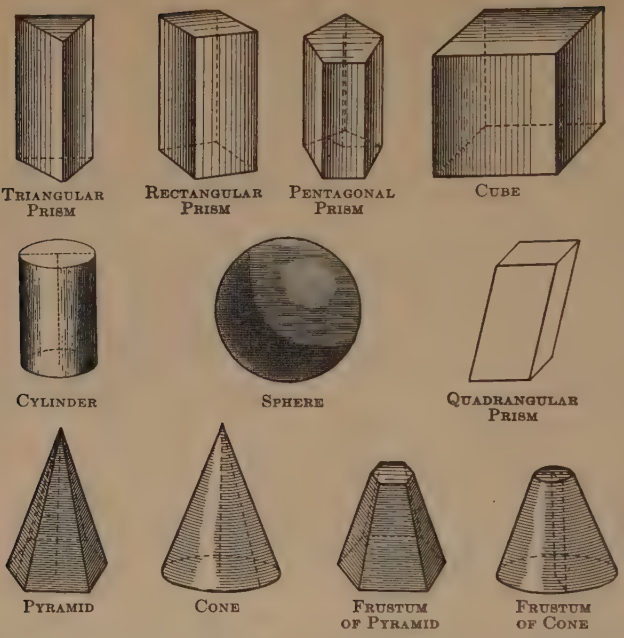
Surfaces and Volumes of Solids

A **prism** is a solid the bases of which are equal, similar, parallel figures, and the sides of which are parallelograms.

The **slant height** of a regular pyramid or regular cone is the shortest distance from apex to base, measured on the side of the figure (as contrasted with the altitude, or perpendicular height through the body of the figure).

The **lateral area of a cylinder** is the product of its circumference by its height; of a prism, its perimeter by its height.

Solids



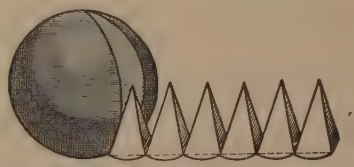
The **lateral area of a regular pyramid** is half the product of its slant height and its perimeter; of a regular cone, half its slant height by the circumference of its base.

The **volume of a prism** or of a **cylinder** is the product of its area on one base and its height.

The **volume of a pyramid** or of a **cone** is $\frac{1}{3}$ the product of the area of its base, and its altitude.

The **area of the surface of a sphere** is four times the square of its radius multiplied by 3.1416.

The **volume** of a sphere is $\frac{1}{3}$ of the area of its surface multiplied by its radius; or $\frac{4}{3} \times 3.1416 \times$ the third power of its radius.



The volume of a cylinder is .7854 (approximately, $\frac{3}{4}$ to $\frac{4}{5}$) times the volume of a cube into which it will exactly fit. The volume of a sphere is approximately $\frac{2}{3}$ that of a cylinder into which it will exactly fit. The volume of a ball is therefore about $\frac{1}{2}$ that of a cubic box into which it will exactly fit.

WRITTEN.

1. Find the areas of the lateral surfaces and total surfaces of the following prisms:

(*a*) A rectangular prism, ends 3″ x 5.5″; length, 8″.
(*b*) A rectangular prism, ends 4.5″ x 4.5″; length, 6″.
(*c*) A triangular prism, ends 3″ x 4″ x 5″; length, 10″.
(*d*) A hexagonal prism, six edges 4″ each; length, 11″.

SUGGESTION: A hexagon may be divided into 6 equilateral triangles.

2. Find the lateral area of a can with a rectangular bottom 5″ x 8″, if its height is $12\frac{1}{2}$″.

3. The sides of a belfry are to be painted. The belfry is in the shape of an octagonal prism 10 ft. high. The width of each face of the prism is 8 ft. How many square yards are to be painted?

4. What is the total outside area of a cylindrical tank 5 ft. in diameter and 8 ft. high?

5. A circular smoke stack has an average diameter of 9 ft. and is 52 ft. high. How many square yards of painting surface has it?

6. A conical funnel made of tin is 4 ft. in diameter at the base and is 6 ft. from the edge of the base to the tip. How many sq. yd. of painting surface has it?

7. A grain hopper in the form of a pyramid is 8 ft. square at the base and its slant height is 8 ft., inside measurements. How many sq. ft. of galvanized iron will there be in a lining for this hopper?

8. A steeple is a hexagonal pyramid in form. Each side of the steeple is 4 ft. at the base and 12 ft. in slant height. How many square feet of material will there be in the roof?

9. What is the capacity, in cu. ft., of a rectangular water trough 18 in. wide, 18 in. deep, and 16 ft. long?

10. A farmer has a cylindrical water tank 12 ft. in diameter and 4 ft. deep. How many gallons will it hold if $7\frac{1}{2}$ gallons equal 1 cu. ft.?

11. A bushel contains 2150.4 cubic inches. If a box is 14 in. square, how deep should it be to contain one bushel?

12. A galvanized iron pail is 12 in. in diameter. How deep should it be to contain $\frac{1}{2}$ bushel?

13. An oil tank on a car is 5 ft. in diameter and 30 ft. long. How many barrels of oil will it contain if there are approximately 4.25 cu. ft. in 1 bbl.?

Measuring Lumber

The unit of lumber measure is the board foot, which is 1 foot long, 12 inches wide, and 1 inch thick. A board that is less than 1 inch thick is measured as though it were 1 inch thick.

Certain kinds of finishing lumber, such as moldings and quarter rounds, are sold by the linear foot, but with these exceptions all lumber is sold by the board foot.

EXAMPLE. How many board feet in 80 joists each 2″ x 6″ x 18′?

$$\frac{2\times6\times18\times80}{12}=1440 \text{ bd. ft.}$$

To find the number of board feet in one piece 1 inch thick, multiply the length in feet by the width in inches and divide by 12.* To find the number of board feet in 80 pieces 2 in. thick, include the 80 and the 2 as factors, as shown in the solution given. The problem is solved by cancellation.

ORAL. Find the number of board feet in each of the following:

1. 24 pieces, 2″ x 10″ x 16′.
2. 36 pieces, 2″ x 4″ x 10′.
3. 60 pieces, 4″ x 4″ x 18′.
4. 48 pieces, 3″ x 5″ x 20′.
5. 18 pieces, 2″ x 4″ x 16′.
6. 60 pieces, 2″ x 8″ x 14′.
7. 12 pieces, 2″ x 8″ x 12′.
8. 120 pieces, 1″ x 6″ x 16′.
9. 75 pieces, 2″ x 5″ x 12′.
10. 6 pieces, 4″ x 6″ x 18′.

WRITTEN

1. Find the total number of board feet in the following: 80 pcs. 2″ x 4″ x 18′; 60 pcs. 2″ x 4″ x 12′; 120 pcs. 2″ x 6″ x 12′.

2. Find the cost at $22.50 per M of the following bill of rough oak lumber: 120 pcs. 2″ x 4″ x 16′; 60 pcs. 4″ x 4″ x 12′; 80 pcs. 2″ x 8″ x 14′; 4 pcs. 8″ x 8″ x 20′; 6 pcs. 4″ x 6″ x 20′.

3. Find the cost of the following at $27 per M: 40 pcs. 2″ x 8″ x 12′; 16 pcs. 4″ x 4″ x 16′; 30 pcs. 1″ x 6″ x 12′.

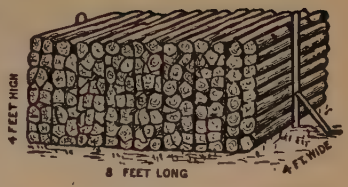
4. Find the cost at $33 of 40 pcs. 2″ x 5″ x 18′; 80 pcs. 2″ x 10″ x 14′; 20 pcs. 3″ x 4″ x 20′.

5. Find the cost of the following, at $27.50: 80 pcs. 1″ x 12″ x 16′; 140 pcs. 2″ x 4″ x 16′; 4 pcs. 2″ x 5″ x 20′; 20 pcs. 2″ x 6″ x 14′; 40 pcs. 2″ x 8″ x 14′.

* This gives the same result as multiplying the length in feet by the width in feet, but is simpler because the width of a board is always stated in inches.

Wood Measure

Wood in the pile is measured by the cord of 128 cu. ft. The solid contents of the pile, in cu. ft., divided by 128, gives the number of cords.



A Cord of Wood

A convenient plan in measuring cordwood is to think of each cord as being 4′ x 4′ x 8′. If a pile is 4′ high and the pieces are 4′ long, its length divided by 8 is the number of cords in the pile. If the pile is 6′ high and the pieces are 4′ long, divide the length by 8 and increase the result $\frac{1}{2}$. And so on.

Written

1. How many cords in a pile of wood 4′ x 4′ x 16′?
2. How many cords in a pile of wood 4′ x 6′ x 20′?
3. How many cords in a pile of wood 3′ x 5′ x 19′?
4. How many cords in a pile of wood if the pieces are 4′ long and the pile is 9′ high and 24′ long?
5. How many cords in a pile of wood if the pieces are 2′ long and the pile is 6′ high and 20′ long?

Roofing

Gable, Shed, and Gambrel Roofs

A roof is supported by timbers called rafters. These are of various sizes.

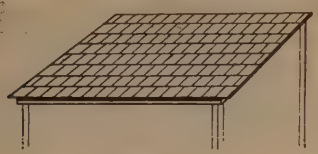
The lower ends of the rafters rest on horizontal timbers, called plates, which lie on the wall or on the posts which support the wall. The upper ends rest on the ridgepole.

Gable Roof, 12-16 Pitch

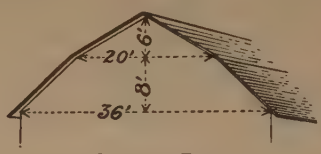
The slant of a rafter is known as the pitch of the roof. The pitch is the perpendicular distance between the ridge and the plates divided by the width of the building.

Thus, if the width of a building is 36 ft. and the ridge is 12 ft. higher than the plates, the pitch would be $\frac{1}{3}$. Carpenters also indicate the pitch of a roof

by numbers representing the height of the ridge and half of the width of the building, in feet. Thus the pitch in the preceding case would be given as 12-18.



Shed Roof



Gambrel Roof

In a true gambrel roof the rafters on the lower slant are the same length as the rafters on the upper slant. The upper and lower rafters come together at the purline, which is the board on which the upper rafters rest. There are many variations of the gambrel roof, however.

Written

1. What length of rafter is needed for a gable roof 28 ft. wide, if the ridge is 12 ft. higher than the plates and the rafter projects 1 ft. beyond the plate of the building?

Note. The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

2. Draw a diagram of the above roof to the scale of $\frac{1}{4}$ in. to 1 ft. and express numerically the pitch of the roof.

3. What is the pitch of the upper part of the gambrel roof shown in the illustration? What is the length of the upper rafters? What is the length of the lower rafters, if the rafters project 16″ over the plate?

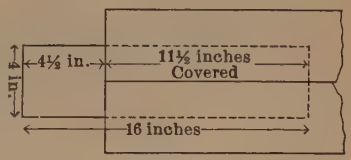
4. One of the rafter plates of a shed roof of a building 18′ x 24′ is 6 ft. lower than the other and the rafters project 1 ft. beyond the lower plate. What length should they be? How many feet of $\frac{7}{8}$ in. board laid flat are required for the roof?

5. Measure the width of the house you live in and then go to the attic and measure the height of the ridge above the plates. Determine the pitch of the roof and the length of rafter required.

6. How many feet of 2 x 4 studding will be required for a partition 23′ 9″ long and 10′ high, if the studs are set 15′ from center to center? If the studding can be bought only in 22 ft. lengths, what will the lumber cost at $19.50 per M.? A fraction of a foot is billed as a whole foot.

Shingling

Shingles are estimated as having a uniform width of 4 in. They are usually 16 in. long, but their length is not material in figuring the number required to cover a given space, since a large part of the surface of each shingle is covered by other shingles, with only 4, $4\frac{1}{2}$, 5, or $5\frac{1}{2}$ inches exposed to the weather.



A shingle laid 4 in. to the weather has 4 x 4 in. or 16 sq. in. of exposed surface. It would take 900 of such shingles to exactly cover a square (10 ft. x 10 ft.), but in practical calculations 1000 are estimated, to allow for waste.

Get a shingle. Measure it. Go to a house under construction and note how the shingles are laid.

Length Exposed to the Weather	Number of Shingles per Square (100 sq. ft.)
4 inches	1000
$4\frac{1}{2}$ inches	900
5 inches	800
$5\frac{1}{2}$ inches	700

Shingles are sold in bundles of 250 each. Only whole bundles can be purchased.

Extra allowance should be made for waste in shingling a hip roof, the opposite edges of which are not parallel.

The amount of exposure depends upon the pitch of the roof — the steeper the roof, the more exposure.

EXAMPLE. How many bundles of shingles laid $4\frac{1}{2}''$ to the weather will be required to cover a roof $18\frac{1}{2}$ ft. x 30 ft.?

$18\frac{1}{2} \times 30 = 555$, no. of sq. ft., or 5.55 squares.
$5.55 \times 900 = 4995$, no. of shingles, or 20 bundles.

WRITTEN

1. Find how many bundles of shingles laid $4\frac{1}{2}$ inches to the weather will be required for roofs of the following sizes:

(*a*) 30′ x 60′ (*c*) 34′ x 54′ 8″ (*e*) 16′ x 42′
(*b*) 22′ x 36′ (*d*) 12′ x 21′ 4″ (*f*) 14′ x 34′ 6″

2. How many bundles of shingles, laid 5 inches to the weather, will be required for the two sides of a gable roof each measuring 17′ 6″ x 54′?

3. If the pitch of a gable roof 42 ft. long is 8-18, and the eaves project 6 in., how many bundles of shingles laid $4\frac{1}{2}''$ to the weather will be required to cover both sides of it?

4. A roof 21′ 6″ x 48′ has one slope only. If the shingles are laid $4\frac{1}{2}''$ to the weather, how many bundles will be required?

5. How many bundles of shingles laid $5\frac{1}{2}''$ to the weather would be required for a shed roof laid on parallel plates 20′ apart, the lengths of the two plates being 40′ and 46′ respectively?

6. Consult the diagram of a gambrel roof shown near the first of this section. The eaves project 1 ft. The ridge pole, purlines, and wall plates of the house are 50 ft. long. Shingles are laid $5\frac{1}{2}''$ to the weather on the lower slope of the roof and $4\frac{1}{2}''$ to the weather on the upper slope. At $4.75 a thousand, what will the shingles for this house cost?

7. The second story of a house 24′ x 52′ is covered with shingles laid $5\frac{1}{2}''$ to the weather. The shingling measures 10 ft. from top to bottom. Allowing for 8 windows $3' \times 4\frac{1}{2}'$ and 4 windows $3\frac{1}{2}' \times 6\frac{1}{2}'$, what is the cost of shingling at $6.50 per M for the shingles, laid on?

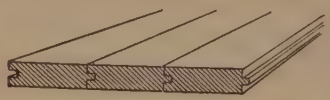
8. What will be the cost, at $4.90 a thousand, for shingles to cover a shed roof 22′ x 50′, if the shingles are laid $4\frac{1}{2}''$ to the weather?

9. A gable roof is 52′ long. The width of the house is 24′ 6″. The ridge of the roof is 9′ higher than the plates. The eaves project 8″ beyond the plates. What is the cost of shingles for this roof, laid $4\frac{1}{2}''$ to the weather, at $5.10 a thousand?

10. What would be your answer to the above problem if the pitch of the roof had been 12-18?

Flooring and Siding

Matched lumber is generally used for flooring. Each board has a tongue along one long edge and a groove along the other long edge. The tongue of one board fits into the groove of the next.



Siding may be matched, like flooring, or may be lapped, each board lapping over the one below it.

Whether boards are matched together with tongue and groove, or lapped, there is a waste that must be accounted for, for the full

width of a board is measured, and some of this width is lost in matching or lapping.

In solving the problems of this section, allow as follows:

One fifth (over actual measurement) for 4-inch matched flooring.
One fourth (over actual measurement) for 3-inch matched flooring.
One sixth (over actual measurement) for matched siding.
One sixth (over actual measurement) for ship lap siding.
One inch of the width of each board, in ordinary lap siding.

Bring a piece of siding to class if possible. What kind of siding is it? What kind of wood? What is it worth per thousand board feet?

EXAMPLE. How much 3-inch flooring is required for a room $18\frac{1}{2}' \times 15'$?

$18\frac{1}{2} \times 15 = 277\frac{1}{2}$, no. of board ft. in floor.
$\frac{1}{4}$ of $277\frac{1}{2} = 69\frac{3}{8}$, no. of board ft. allowed for matching.
$277\frac{1}{2} + 69\frac{3}{8} = 347$, total board ft. required, counting the fractional part of a foot as a whole foot.

WRITTEN

1. How many board feet of siding will be required for a house $24' \times 48'$, if the siding covers $18'$ all around the house, using ship lap siding, with no allowance for openings?

2. How many board feet of 3-inch flooring will be required to cover a floor $21\frac{1}{2}' \times 26'$? If this flooring can be purchased only in multiples of 10 board feet, what will it cost at $45.00 a thousand?

3. How many board feet of 4-inch flooring will be required for a floor $18' \times 28'$, allowing a deduction $3' \times 8\frac{1}{2}'$ for the fire-place?

4. How many board feet of ship lap siding will be required for a house $28' \times 48'$, if the house measures $22'$ from foundation to roof-plates and has a gable roof with a ridge 8 ft. high, allowing 150 sq. ft. for openings?

5. How many board feet of ship lap siding will be required for a house $24' \times 45'$, if the house measures $20'$ from foundation to roof-plates and has a gable roof, $\frac{1}{3}$ pitch, allowing 135 sq. ft. for openings?

6. A house $28' \times 54'$, and $20'$ from the foundation to the eaves, is sided from foundation to eaves with matched siding, over a sheeting of southern pine ship lap. Allowing 15 sq. ft. each

for 12 windows and 4 doors, what will the siding cost, at $32.50 for the southern pine, and $42.00 for the outer siding?

7. A barn 40′ x 70′, and 20′ to the eaves, is sided with ship lap. The ridge is 15′ higher than the eaves. No allowance for openings. What is the cost of the siding, at $27.50 per thousand?

8. An order calls for 3000 ft. of 4″ Y. P. flooring as follows: 1200 ft. of 12-ft. pieces; 800 ft. of 14-ft. pieces; and 1000 ft. of 16-ft. pieces. Pieces will not be cut. The price is $27.50. What is the amount of the bill?

9. A bathroom 9′ 6″ x 12′ 3″ is to be floored with tile. What will this cost at 75¢ per sq. ft.?

Plastering

The cost of plastering is generally estimated by the square yard. The square (100 sq. ft.) is sometimes used, but not often. Estimates generally include labor and materials. The estimator must know the number of coats desired; whether lath is to be used, and what kind; and whether furring is necessary.

Furring is attaching to brick or tile walls wooden strips to which the lath can be nailed. Furring is usually charged separately, and metal lath is usually charged separately, but with these two exceptions the prices quoted per sq. yd. usually include all labor and materials used.

Methods of calculation vary. The process followed in the solution below is typical, however, and will be used by you.

EXAMPLE. At 45¢ per square yard, what will it cost to plaster a room 11′ 6″ x 14′ x 9′, allowing as deductions one-half the area of three windows each 3′ 6″ x 6′ and two doors each 4′ x 7′?

$(11'\ 6''+14')\times 2 = 51'$, perimeter of room

$51\times 9 = 459$, no. of sq. ft., area of walls
$11\frac{1}{2}\times 14 = 161$, no. of sq. ft., ceiling
620, no. of sq. ft., walls and ceiling

$(3\frac{1}{2}\times 6)\times 3 = 63$, no. of sq. ft., area of windows
$(4\times 7)\times 2 = 56$, no. of sq. ft., area of doors
$2\overline{)\,119}$, no. of sq. ft., all openings
$59\frac{1}{2}$, no. of sq. ft. deducted for openings

$620-59\frac{1}{2} = 560\frac{1}{2}$, no. of sq. ft. to be plastered
$560\frac{1}{2}\div 9 = 62$, nearest no. of sq. yd.
$62\times 45¢ = \$27.90$, cost.

Papering

The unit in wall paper measure is the roll, which is 18 in. wide and 8 yd. long. American papers come in double rolls, or bolts 18 in. wide and 16 yd. long. These bolts are cut into whole strips each as long as the height of the wall to be papered, the remainder of each bolt, if any, being wasted. A half of a double roll can be bought. French and English papers come in bolts 12 yd. long, equivalent to a roll and a half.

The prices quoted on wall paper include putting the paper on the walls.

In figuring the allowance for an opening, in an ordinary room in a residence, it is customary to multiply the entire height of the wall from floor to ceiling by the width of the opening.

There are usually some spaces above doors, to be papered, but the waste ends of rolls can be utilized for this purpose in many cases. In case of an unusually high ceiling, the spaces above doors and windows are figured.

In figuring ceilings it is customary to run the paper either the long way or the short way of the ceiling, as may be most economical.

When the length of a wall is not an even multiple of 18″, one strip has to be split lengthwise, and only a part of it used. In this case a whole strip is figured.

Waste occurs from matching patterns, papering around irregular woodwork, etc. The student will not be required to figure this.

EXAMPLE. How many double rolls of paper will be required for the walls of a room 11′ 6″ x 14′ x 9′, with 3 windows each 3′ 6″ wide, and 2 doors each 4′ wide?

51′ (perim.) − 18′ 6″ (openings) = 32′ 6″, net perim.
32′ 6″ ÷ 18″ = $21\frac{2}{3}$, or 22, no. of strips for walls
48′ ÷ 9′ = 5, no. of strips from 1 double roll
22 ÷ 5 = $4\frac{2}{5}$, no. of double rolls, or 4 double rolls and 1 single roll required.

EXAMPLE. How many double rolls of paper will be required for a ceiling 11′ 6″ x 14′?

Running the paper the short way:
48′ ÷ $11\frac{1}{2}$′ = 4, no. of strips from 1 double roll.
14′ ÷ $1\frac{1}{2}$′ = 10, no. of strips across ceiling.
10 ÷ 4 = $2\frac{1}{2}$, no. double rolls, or 2 double rolls and 1 single roll required.

Figure for yourself the number of double rolls that would be required if paper were run the long way of the room.

Carpeting is sold by the linear yard, whatever the width of the carpet. Ingrain carpeting is usually 1 yd. wide, but the finer grades, such as Brussels, Axminster, Wilton, and Moquette, are usually 27″ wide. A fraction of a linear yard can be purchased, but split widths can not be bought.

Figure the carpet as running the long way of the room unless otherwise instructed.

EXAMPLE. How many yards of Axminster carpet will be required for a floor 14′ x 11′ 6″?

$11\frac{1}{2}' \div 2\frac{1}{4}' = 5\frac{1}{9}$, or 6, no. of strips required
$14' \div 3' = 4\frac{3}{4}$, no. of yd., length of each strip
$6 \times 4\frac{2}{3} = 28$, no. of yd. of carpet required.

WRITTEN

1. Find the amount of plastering surface in a room 12′ x 14′ x 9′, allowing for $\frac{1}{2}$ of openings as follows: 2 windows, each 3′ 6″ x 6′; and 2 doors, each 3′ 6″ x 6′ 6″.

2. Find the amount of plastering surface in the walls of a store building 30′ x 54′ x 16′, with full allowance for front door and windows, full width of store to a height of 14′; rear door 6′ x 8′; and wainscoting 2′ 6″ on the two sides.

3. The ceiling of the above store is to be covered with metal and painted. The metal costs 8¢ a sq. ft. put up in place. Painting, including cost of paint, $3.50 per square. Find the total cost.

4. A kitchen 10′ 6″ x 12′ x 9′ is to be calcimined, walls and ceiling. There is a 3′ 6″ wainscoting around the room. There are 2 doors, each 4′ x 6′ 6″ and 2 windows, each 3′ 6″ x 5′ 6″, with sills 2′ 6″ from the floor. Allow for $\frac{1}{2}$ of openings above the wainscoting. What will the calcimining cost at $1.50 a square?

5. A room 12′ x 14′ x 8′ is to be papered, walls and ceiling. What will the papering cost at 70¢ a double roll, if there are 3 windows each 3′ wide and 2 doors each $3\frac{1}{2}'$ wide to be allowed for?

6. The above room is to be carpeted with Moquette at $2.20 per yd. What will the carpeting cost, including 3 hours' labor at 60¢?

7. Measure your bedroom and determine the quantity of wall paper required for walls and ceiling.

8. How many double rolls of paper must be used to cover the ceiling of a room 16 ft. by 21 ft., the entire length of each roll except the last being used?

9. How many double rolls of paper will be used for the walls and ceiling of a room 17 ft. by 23 ft. and 10 ft. high, allowing 9″ for border, 2 ft. of paper on each strip for matching, and for 2 doors and 4 windows, each 4 ft. wide?

10. Find the cost of the border in the preceding problem, at 35¢ per yard for material and 7¢ per yard for application.

11. Find the cost of painting the outside of a house 32′ x 50′, if it measures 21′ from foundation to eaves and the two gable ends are 9′ high, at $4.00 per square.

12. If 1400 four-foot laths cover 100 sq. yd., and laths are bought in whole bundles only, containing 50 laths each, what is the cost at $5.00 per M, for laths for the walls and ceilings of the following rooms: One room, 14′ x 12′ x 9′, two rooms each 12′ x 10′ x 9′, and one room 18′ x 14′ x 9′, allowing $\frac{1}{2}$ for 11 openings averaging 18 sq. ft. each?

13. A man having the contract for excavating a cellar 6′ 9″ deep for a house 35′ 4″ wide and 62′ 8″ long, at $1.15 per cu. yd., finds on beginning the job that the digging and hauling will cost him $1.30 per cu. yd. but he can sell the dirt for $.50 a yard. In determining the amount of earth sold, assume that its volume increases one-third by swelling when it is loosened in digging. How much will he make on his contract?

14. What will it cost to wainscot both sides of a hall 28′ 8″ long, to the height of 4′ 6″, at $27\frac{1}{2}$¢ per sq. ft.?

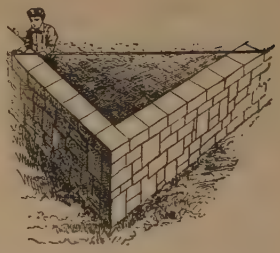
15. A skylight is 24′ long and 14′ wide. It is gable roof shaped, the ridge being 6′ high. What will it cost to glaze the two sides and the two ends, at 77¢ per sq. ft.?

16. A building 50′ x 160′ has a shed roof. The front roof plate is 9′ higher than the rear roof plate. Eaves project 15″ below lower roof plate only. Allowing for a court 20′ x 45′, what will it cost to gravel this roof at $5.25 per square?

17. Two workmen put up 160 sq. yd. of metal lath in 7 hours. The workmen were paid $7.00 each per 8-hour day. What was the cost of this labor per sq. yd.? What would it have been had half of the time been overtime, at time and a half?

Brick, Stone and Concrete

Brick Work



Bricks vary in size. A "common brick," however, is generally 8″ x 4″ x 2″.

Customs in estimating brick are different in different localities. Generally, however, 22 bricks are allowed for each cubic foot of brick masonry.

The student can figure this for himself as follows: Add $\frac{1}{4}$″ to each dimension of a common brick, to allow for mortar, and find how many times the product is contained in 1728 cu. in.

In estimating the number of bricks required for a given job, actual measurements are taken, with full allowance for all openings, and with corners counted only once.

If the full outside measurement of a foundation were taken, it would result in counting each corner twice.

In estimating the labor of laying brick, full outside measurements are taken, and deductions are not allowed for openings unless specified in the contract.

WRITTEN

1. How many bricks are required for a wall 40 ft. long, 5 ft. high, and 1 ft. thick?

2. How many bricks are required for a 16-inch foundation 24′ x 28′, and 6′ high, with 6 openings $2\frac{3}{4}$′ x 4′ and one $3\frac{1}{2}$′ x $6\frac{1}{2}$′? What will be the cost, at $9.25 per thousand, if full thousands only can be bought?

3. What will be the cost for brick for an 8-in. wall 28′ x 36′, and 12′ high, with 120 sq. ft. of openings, at $8.50 per M, if full thousands only can be bought?

4. What would be the cost of brick for a wall as in the preceding problem if for the facing a $14.00 pressed brick were used, which could be bought in the exact quantity required?

5. The excavation for a basement is 38′ x 44′. The wall is to be 12 in. thick, $8\frac{1}{2}$ ft. high and built of common brick. Openings, 120 cu. ft. What will the brick cost at $8.25 per full M.?

6. The walls above the basement in the foregoing problem are to be 8 in. in thickness and 24 ft. high, on the 44′ sides and rear wall. What will the material for these three walls cost at $8.00 per full M. if there are to be 10 openings, each 15 sq. ft.?

7. A house 26′ x 30′ x 18′ is to be veneered* with brick. The two 30′ walls and the rear wall, each with 45 sq. ft. of openings, are to be veneered with common brick, and the front wall, which has four openings of 18 sq. ft. each, is to be veneered with pressed brick having the same dimensions as the common brick. How many of each kind of brick will be required?

8. What will the material in the above problem cost with common brick at $8.50 per M and pressed brick at $19.75 per M, if the common brick can be bought in thousand lots only?

9. How many bricks will be required to construct a wall around a lot 254 ft. x 310 ft., if it is to be 10 ft. high and 16 in. thick? The openings total 46 sq. ft.

Stone Work

Perch. Stone work is measured either by the cubic yard or by the perch. A perch is equivalent to a mass $1\frac{1}{2}$ ft. thick, 1 ft. high, and a rod long, containing $24\frac{3}{4}$ cu. ft.

In estimating masonry, outside measurements are taken, and openings are not usually allowed for.

The difficulty of laying the corners justifies the double charge and the extra work of laying around the openings offsets the saving in straight work.

WRITTEN

1. How many perches of masonry in the four walls of an 18-in. foundation 42′ x 28′ and 9′ high?

2. How many perches of masonry in a wall $2\frac{1}{2}$ ft. thick, 8 ft. high, and 150 ft. long?

3. How many cubic yards of masonry in a garden wall 120′ x 180′ if the wall is 18″ thick and 8′ high?

4. Three piers of a bridge are each 8′ wide, 24′ long, and 28′ 6″ high. They are made of rubble stone costing $12.35 a perch. The quantity of cement used is 10% of the quantity of stone. What is the cost of the stone?

* A brick veneer is generally one layer of brick built against a rough siding. The veneering is 4 in. thick, the width of one brick.

Concrete

Concrete is a mixture of cement and sand, or of cement, sand, and crushed stone or gravel.

Cement is bought by the barrel, estimated at 400 lb. gross and 376 lb. net. It is shipped in bags holding $\frac{1}{4}$ bbl. each. A bag contains approximately 1 cu. ft.

Sand and stone are bought by the cubic yard.

Mixtures. The materials to make concrete are mixed in varying proportions, depending upon the quality of cement desired.

A mixture is referred to by naming in turn the number of parts of cement, sand, and gravel required to make it. Thus a "1, 2, 4 mixture" is a mixture containing 1 part cement, 2 parts sand, and 4 parts gravel.

Voids. In mixing concrete the small particles fill in the voids or spaces between the larger pieces. It has been estimated that 1.45 cu. yd. of unmixed materials must be provided to make 1 cu. yd. of mixture. When one of the materials used is gravel or stone, an extra 8% or 10% of the whole must be provided.

Assuming that for the case in hand it is proper to figure on 45% voids, and to add 8% to total material because gravel is used, note the following solution.

Example. How much cement shall be used in making 1 cu. yd. of a 1, 2, 4 mixture?

In a 1, 2, 4 mixture there are 1+2+4 parts, or 7 parts, of which 1 is cement. Multiplying $\frac{1}{7}$ of 1.45 by 27 (the no. of cu. ft. in 1 cu. yd.), then dividing by 4 (the no. of cu. ft. in 1 bbl.), and increasing the total quantity 8%, we have $\frac{1}{7}\times\frac{1.45}{1}\times\frac{27}{1}\times\frac{1}{4}\times\frac{1.08}{1}=1.51$, the no. of bbl. cement required.

Written. Using the same basis as in the above solution (45% voids and 8% added to total when stone or gravel is used), solve the following problems.

1. How many barrels of cement are needed to make one cubic yard of a 1, $2\frac{1}{2}$, 5 mixture of concrete?

2. How many barrels of cement are needed to make one cubic yard of a 1, 3, 5 mixture of concrete?

3. How many barrels of cement are needed to make one cubic yard of a 1, 3, 6 mixture of concrete?

4. How many barrels of cement must a dairyman order for a floor 22′ x 50′, using a 1, 3, 6 mixture 3″ thick?

Table

The per cent of voids and the per cent allowed for waste are not the same for all mixtures. The following table shows the number of barrels of cement required for 1 cu. yd., for each of a dozen mixtures.

Proportion of Volume			Amount in One Cubic Yard of Concrete		
Cement	Sand	Stone or Gravel	Cement bbl.	Sand cu. yd.	Stone or Gravel cu. yd.
1	1		4.80		
1	$1\frac{1}{2}$		3.87		
1	2		3.21		
1	$2\frac{1}{2}$		2.74		
1	$1\frac{1}{2}$	2	2.30		
1	$1\frac{1}{2}$	$2\frac{1}{2}$	2.09		
1	$1\frac{1}{2}$	3	1.91		
1	2	3	1.74		
1	2	4	1.51		
1	$2\frac{1}{2}$	5	1.24		
1	3	5	1.16		
1	3	6	1.06		

Note that the spaces in the above table for the amount of sand and the amount of gravel have been left blank. These are for you to compute. Note the following solution.

Example. How many cubic yards of sand and gravel are required for one cubic yard of a 1, 2, 4 mixture of concrete?

1.51 bbl. cement $= 1.51 \times 4 = 6.04$ (cu. ft.) $\div 27 = .2237$ (cu. yd.)
$2 \times .2237 = .4474$, or approximately .45, no. of cu. yd. of sand
$4 \times .2237 = .8948$, or approximately .89, no. of cu. yd. of gravel

Written

1. Prepare a table as above. Fill blank spaces to two places.

2. A garage is 27′ x 40′. How much cement, sand, and gravel are required for a 3″ concrete floor of a 1, 2, 3 mixture?

3. How much of each material is required for the six walls of a fire-proof vault 13′ square, and 6′ 9″ high, outside measurements, if the walls are 1 foot thick and a 1, 2, 4 mixture is used?

4. The excavation for a cellar is 26′ x 52′ x 9′. How much of cement, sand, and crushed rock are needed to build 12-in. walls of a 1, 3, 6 mixture of concrete?

5. Find the quantity of each material of a 1, 2, 4 concrete mixture for the floor of the cellar mentioned in the preceding problem, if the floor is to be 4 inches thick.

6. A farmer builds a cylindrical silo 20 ft. in diameter, and 34 ft. in height, outside measurements. The wall is 9 in. thick and the floor 1 ft. thick. A 1, $2\frac{1}{2}$, 5 mixture of concrete is used. The cement costs $2.15 per barrel; the sand, $1.15 per cu. yd.; and the gravel 95¢ per cu. yd. The cement is bought by the full sack and the sand and gravel by the full load. Cost of roofing materials, labor, and all other expenses, $145.00. Find the total cost.

7. Find the cost of the materials necessary to make 200 posts, each 8 ft. long and 1 ft. square and reinforced with an iron bar 8′ x 1″ x 1″ weighing 338 lb. per 100 ft. A 1, $2\frac{1}{2}$, 5 concrete mixture is used. Allow for the space occupied by the iron bar. The cement costs $1.60 per bbl.; sand, $1.00 per yd.; stone, $2.00 per yd.; and iron, 3¢ per lb.

8. A barn floor 10′ x 20′ is to be laid, using a 5-in. base of a 1, 3, 6 mixture and a 1-in. top-coat of a 1, $1\frac{1}{2}$ mixture. At $2.10 for cement, $1.25 for sand, and $1.40 for crushed stone, what will the concrete cost?

Paving

1. A street is being prepared for paving. The part to be paved is 759 ft. long and 30 ft. wide. The excavating will average 12 in. in depth. How many cu. yd. of earth must be moved?

2. If earth expands $\frac{1}{3}$ in digging, how much will be received for the earth taken out in problem 1, at 45¢ a load? A load is a cubic yard. (Use full loads in all these problems.)

3. What will be the cost of a 6 in. concrete base for the paving of the above street, a 1, 3, 6 mixture being used, if cement costs $1.40 per bbl., sand $1 per cu. yd., and stone $1.90 per cu. yd.?

4. The paving blocks used are 4″ x 4″ x 8″. How many will be required, laid flat, to pave the street, and what will they cost at $23.50 per thousand?

5. The contract for paving the street in problem 4 was let by the city for $6425. How much of an assessment was levied against Mr. Thatcher, whose lot fronted 40 ft. on the west side of this street, to pay for his frontage only?

Procure all the information obtainable in regard to the cost of paving in your city. Find out whether adjoining property owners pay for the paving or whether the paving is paid by a general tax levy.

Good Roads

It was officially estimated that in one year 42,743,500 tons of certain farm products were hauled to shipping points in the U. S. (Bulletin 49, U. S. Office of Public Roads). The average cost of hauling was 25¢ per ton per mile. The average distance was 9 miles. The cost over good stone roads was 8¢ per ton per mile; over ordinary stone roads, 11.09¢; and over dirt roads, 39¢. Use the foregoing data in solving the problems following.

WRITTEN

1. How much would the farmers of the United States have saved that year if they had used good stone roads in hauling all their crops to market? What per cent of the money paid was loss if all costs above 8¢ per mile are considered losses?

2. If the farmers of Brown County hauled 150,000 tons of farm products to market last year at an average cost of 25¢ per ton per mile, and an average distance of 7 miles, what did they lose by using poor roads, if all costs above 8¢ per ton per mile are figured as losses? What per cent was their cost for hauling in excess of the cost required to haul over good stone roads?

3. The assessed valuation of the property in Brown County is $20,000,000. What tax (in mills) would have to be levied in order to build 100 miles of stone road at an average cost of $5000 per mile? How much would James Smith's taxes be increased by this levy if the assessed valuation of his property is $10,000? How much would he save each year if he should haul 35 tons of farm products to market, a distance of 8 miles, at 12¢ per ton per mile, if he had formerly been hauling over dirt roads? How long would be required for him to pay the amount levied on his property in taxes for building the improved roads if the amounts he saved were used for this purpose?

4. One hundred miles of gravel road could be built in Franklin County at an average cost of $2000 per mile, which would reduce the cost of hauling a ton one mile from 25¢ to 15¢. How much would the farmers of this county save in one year by the use of the improved roads if the average total haul per year is 125,000 tons, and the average distance is 9 miles? At this rate, how long would be required by the farmers of the county to pay for the improved roads if they contributed their savings resulting from the use of the gravel roads to pay this indebtedness?

5. How much would the farmers of Franklin County save above the expense of building the improved roads, in 15 years, if the hauling should average during this period 125,000 tons a year, and the distances should average 9 miles?

6. It costs a farmer $2.25 per ton to haul his wheat to a shipping point 9 miles from the farm. A commission merchant in New York ships a ton of wheat to Liverpool, 3100 miles, at $1.75 per ton. How much more per mile does it cost the farmer to haul a ton of wheat on the farm road than the commission merchant pays in shipping from New York to Liverpool? What per cent is the cost of shipping on the water of the cost of hauling on the farm road?

Heating

Steam. A steam radiator consists of sections (coils) through which the steam passes. Heat is radiated from the coils in proportion to the number of sq. ft. in their total surface. A square foot of radiation is one square foot of coil surface.

The quantity of radiation required for heating a room depends upon the construction of the building, the amount of exposed wall surface of the room, the number and size of outside openings (glass), the outside temperature, the degree of heat desired, and many other factors.

Contractors have various methods of estimating the quantity of radiation. In solving the problems of this section, however, you will use the following schedule, which is recognized as standard by the Chicago Master Steam Fitters' Association.

Schedule for computing minimum quantities of steam radiation at 70° F. with outside temperature at minus 10° F.

(*a*) 1 foot of radiation for every 300 cu. ft. of air space, plus

(*b*) 1 foot of radiation for every 15 sq. ft. net exposed wall surface, plus

(*c*) 1 foot of radiation for every 2 sq. ft. of glass surface.

In measuring glass surface the full opening in the wall shall be figured. Outside door openings shall be taken as glass.

For **hot water** radiation add 60% to quantity necessary for steam radiation.

For **vapor systems** add 20% to quantity necessary for steam.

For heating to 65°, estimate as for 70° and multiply by .94.

For heating to 60°, estimate as for 70° and multiply by .87.

For heating to 55°, estimate as for 70° and multiply by .81.

For heating to 50°, estimate as for 70° and multiply by .75.

EXAMPLE. How much steam radiation is necessary for heating to 70° a room 16 ft. x 12 ft. x 9 ft., with the front and one side wall exposed, if the room has 2 windows each $3\frac{1}{2}$ ft. x 6 ft. and one window 5 ft. x 6 ft.?

$16 \times 12 \times 9 = 1728$, no. of cu. ft., air space.
$1728 \div 300 = 5.76$, or 6, no. of sq. ft. radiation (*a*) nearest sq. ft.
$(16+12) \times 9 = 252$, no. of sq. ft., total exposed wall surface.
$2 \times (3\frac{1}{2} \times 6) + (5 \times 6) = 72$, no. of sq. ft., glass.
252 sq. ft. $-$ 72 sq. ft. $=$ 180 sq. ft., net exposed wall surface.
180 sq. ft. $\div$ 15 sq. ft. $=$ 12, no. sq. ft. radiation (*b*).
72 sq. ft. $\div$ 2 sq. ft. $=$ 36, no. sq. ft. radiation (*c*).
$6+12+36 = 54$, no. of sq. ft. total radiation.

NOTE. In all solutions compute on the basis of 10° below zero, outside temperature.

WRITTEN

1. A dining room is 15′ x 12′ x 8′ 6″. It has one 15-ft. side exposed, with 3 windows, one 5′ x 6′ and the others 2′ 6″ x 6′. How much steam radiation is required for heating this room to 70°?

2. How many sections each having 5 sq. ft. of radiation should there be to a steam radiator in a room 15′ 6″ x 11′ 6″ x 9′, in order to get an inside temperature of 65°, if one 15′ 6″ wall and one 11′ 6″ wall are exposed, and there are 3 windows each 3′ x 5′ 6″?

3. How much steam radiation is required for heating to 60° a building 24′ x 60′ and 12′ to the ceiling, if the entire front is of glass and the building joins another for its entire length on one side, and there are 5 windows, each 3′ 6″ x 6′?

4. A steam pipe is $2\frac{1}{2}$″ in diameter. How many feet of this pipe are necessary to supply 100 sq. ft. of radiation? (Do not attempt to figure on the thickness of the material of which the pipe is made.)

5. A greenhouse is heated by steam pipes $2\frac{1}{2}$″ in diameter. It is 16′ wide, 80′ long, 9′ high on one side and 12′ high on the other. The roof is made of glass and all the walls except one end wall are exposed. How many feet of pipe are required to keep this greenhouse at 70°?

6. Which will give the greater heating surface and how much, 100′ of pipe 2″ in diameter or 200′ of pipe 1″ in diameter?

7. How many feet of 1″ pipe would be required to heat by steam to 70° a greenhouse of the dimensions given in problem 5?

8. A garage is 40′ x 80′ x 12′. There are four windows each 8′ x 5′ and four windows each 3′ x 5′. The structure is joined on both sides by other buildings and it is estimated that $\frac{3}{5}$ the usual

amount of radiation will be sufficient. How many sq. ft. of steam radiation will be required to heat to 50°?

9. How many sq. ft. of steam radiation must a contractor supply to heat to 55° a room 12′ x 18′ x 10′, which has one 12′ x 10′ wall and one 18′ x 10′ wall exposed, if there are two windows each 4′ x 8′?

10. How much hot water radiation will be required to heat to 70° a living room 14′ x 14′ x 9′ having 2 exposed sides in each of which are 2 windows 2′ 9″ x 5′ 6″?

11. How many sections, each having 5 sq. ft. of radiation, will be required for a hot water radiator to heat to 60° a room 15′ x 12′ x 8′ 6″, having one 15-ft. side exposed, with 3 windows, one 5′ x 6′ and the others 2′ 6″ x 6′?

12. A room 12′ x 11′ x 8′ 6″ has one 12′ side exposed, with a window 5′ x 5′. How much hot water radiation (sections of 5 sq. ft. radiation each) will be required to heat this room to 70°?

13. A residence has 5 rooms and 2 halls on the second floor all 8′ 6″ high, as follows:

Southwest bedroom, 12′ x 12′, 2 windows each 2′ 6″ x 4′ 6″.
Southeast bedroom, 12′ x 10′, 1 window 26″ x 56″, 1 window 24″ x 24″.
Northwest bedroom, 12′ x 11′ 6″, 2 windows each 2′ 6″ x 5′.
Northeast bedroom, 10′ x 10′, 2 windows each 2′ 6″ x 5′.
Bathroom, 7′ 9″ x 6′, 1 window 2′ x 3′ in 7′ 9″ wall.
Halls, 12′ x 6′ 3″ and 7′ 9″ x 4′, 1 window 30″ x 3′ in 6′ 3″ wall.

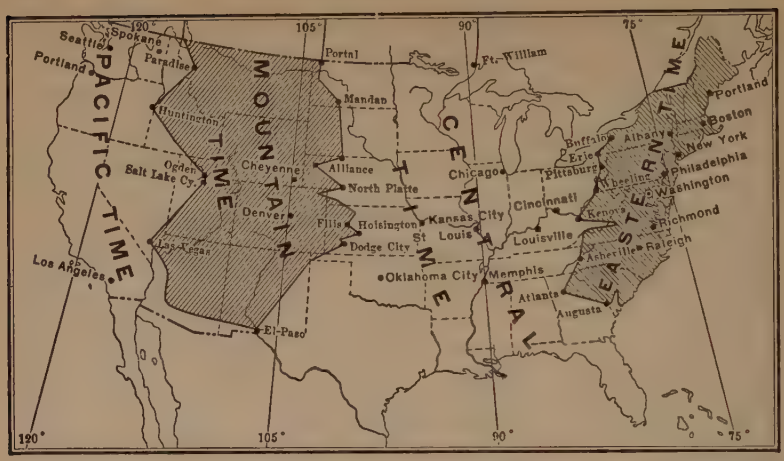
How many coils of $2\frac{1}{2}$ sq. ft radiation each should be placed in each room and in the halls, to heat them to 70° by a vapor system?

14. How many sections of hot water radiation of 5 sq. ft. each should a contractor provide to heat to 65° a room 16′ x 14′ x 9′ if on one 16-ft. side there are two windows each 24″ x 60″, and a glass door 36″ x 7′?

Standard Time

The earth makes one rotation in 24 hours. Since there are 360° in its circumference, a point anywhere on its surface must pass through 15° of longitude in one hour of time.

The circumference of the earth at the equator is approximately 25,000 miles. A point near the equator would therefore pass through about 1000 miles of space in an hour.



Pacific Belt Mountain Belt Central Belt Eastern Belt

The distance between two meridians of longitude diminishes in a constantly increasing ratio as they get farther from the equator and nearer the pole, but roughly speaking, throughout the United States an hour of time is equivalent to about 1000 miles of distance.

In order to reduce the keeping of time to an exact system in this country, the principal railroads of the United States, in 1883, agreed upon the following plan. The United States was divided into four longitudinal sections, each 15° from east to west, called time belts. The meridian of 75° west from Greenwich became the center of the eastern belt, and the time kept in the eastern belt was called Eastern time. The meridians of 90° W, 105° W, and 120° west, became centers respectively, of the second, third, and fourth belts. The times in these three belts were called, respectively, Central time, Mountain time, and Pacific time. Each belt extends, roughly speaking, about $7\frac{1}{2}°$ east and $7\frac{1}{2}°$ west of its central meridian.

Eastern time is one hour later than Central time, Central time is one hour

International Date Line

later than Mountain time, and Mountain time is one hour later than Pacific time.

This system of time is called *standard time*. It prevails in Canada as well as in the United States. Locally in Michigan, use is made of "Michigan time," which is between Eastern time and Central time. The boundaries of the various time belts are very irregular, as shown by the map.

WRITTEN

1. When it is 9 A.M. in New York City what time is it in Chicago? in Denver? in San Francisco?

2. How often would a traveler change the time shown by his watch in going from Philadelphia to Los Angeles? Where would he make these changes, and what changes would he make?

3. A train going east reaches Wheeling, W. Va., at 5:54 P.M. Central time and leaves at 7:06 P.M. Eastern time. On the return trip the train arrives 7:30 P.M. and stops the same length of time. What time does it leave?

4. A telegram is received by the telegraph company at 10:30 A.M. in Chicago to be transmitted to Denver. The records of the company show that the telegram was received in Denver at 10:22 A.M. the same day. How much time was used in transmitting the message?

5. A traveler went from San Francisco to St. Louis and forgot to set his watch *en route*. Is his watch ahead or behind at St. Louis? How much?

6. If you went over the Southern Pacific R. R. from New Orleans to Los Angeles via El Paso, what change would you make in your watch at El Paso?

7. Would you change your watch in going from Chicago to New Orleans? Why?

8. When it is 12 M. at Denver, what is the time at Boston?

9. When it is 2 P.M. at Portland, Me., what is the time at Charleston, S. C.?

10. A limited train leaves Chicago at 8:05 P.M. and arrives at Las Vegas the second morning at 6:20, mountain time. If the distance is 1221 miles, what is the average speed?

11. Determine from the following time table what is the best time, in days, hours, and minutes going west, between Chicago

and Omaha; Chicago and Kearney, Nebr.; Laramie, Wyo., and Tacoma, Wash.; Chicago and Seattle; and between Pocatello, Idaho, and Council Bluffs, Iowa, going east.

UNION PACIFIC SYSTEM

WESTBOUND Read downward		MILES	BETWEEN CHICAGO OMAHA PORTLAND AND SEATTLE	ROUTE	TIME	ELEVATION	EASTBOUND Read upward		
Oregon, Washington Limited, 17, daily	Portland and Puget Sound Express, 19, daily						Omaha Express 6, daily	Oregon, Washington Limited, 18, daily	Portland and Puget Sound Express, 4, daily
9.35	10.30	0	Lv. Chicago, Ill. . . . . Ar.	C. & N. W.	C	590	7.34	11.00	**8.45**
9.49	**11.35**	488	Ar. Council Bluffs, Ia. . . . . Lv.	"	C	981	**6.24**	**8.57**	7.55
10.15	**11.59**	491	Ar. Omaha, Neb. . . . . Lv.	"	C	1034	**6.00**	**8.32**	7.30
10.30	12.40	0	Lv. Omaha, Neb. . . . . Ar.	Union	C	1034	**5.00**	**8.15**	6.50
11.33	1.40	46	Lv. Fremont, Neb. . . . . Lv.	Pacific	C	1196	**3.52**	**7.01**	5.34
12.36	2.43	91	Lv. Columbus, Neb. . . . . Lv.	System	C	1444	**2.49**	**5.53**	4.20
1.36	3.44	132	Lv. Central City, Neb. . . . . Lv.	"	C	1699	**1.51**	**4.52**	3.15
2.10	4.20	153	Ar. Grand Island, Neb. . . . . Lv.	"	C	1861	**1.20**	**4.20**	2.45
3.15	5.45	196	Lv. Kearney, Neb. . . . . Lv.	"	C	2146	11.57	**3.01**	1.19
5.45	8.45	291	Ar. N. Platte, Neb. . . . . Lv.	"	C	2800	9.30	**12.40**	**10.50**
8.15	11.50	414	Lv. Sidney, Neb. . . . . Lv.	"	M	4090	4.35	8.30	**6.10**
11.45	**3.15**	516	Ar. Cheyenne, Wyo. . . . . Lv.	"	M	6058	2.00	5.55	**2.35**
2.05	**5.20**	573	Ar. Laramie, Wyo. . . . . Lv.	"	M	7145	**11.45**	3.40	**12.22**
5.40	**8.50**	690	Lv. Rawlins, Wyo. . . . . Lv.	"	M	6741	**7.50**	**11.55**	8.30
9.35	12.45	824	Ar. Green River, Wyo. . . . . Lv.	"	M	6082	**3.30**	**7.30**	4.15
12.12	3.20	879	Lv. Opal, Wyo. . . . . Lv.	"	M	6666	**12.50**	**5.40**	2.29
12.42	3.44	893	Lv. Diamondville, Wyo. . . . . Lv.	"	M	6883	**12.27**	**5.17**	2.03
12.52	3.55	894	Lv. Kemmerer, Wyo. . . . . Lv.	"	M	6908	**12.20**	**5.09**	1.59
3.30	6.00	969	Ar. Montpelier, Idaho. . . . . Lv.	"	M	5943	9.30	**2.45**	**11.42**
5.17	7.35	1016	Lv. Bancroft, Idaho. . . . . Lv.	"	M	5425	7.35	**1.15**	**10.11**
7.10	9.10	1068	Ar. Pocatello, Idaho. . . . . Lv.	"	M	4461	5.15	11.30	**8.30**
8.35	10.20	1093	Lv. American Falls, Idaho. . . . . Lv.	"	M	4330	3.40	10.20	**7.20**
9.38	11.21	1126	Lv. Minidoka, Idaho. . . . . Lv.	"	M	4280	2.40	9.30	**6.33**
10 57	**12.45**	1176	Lv. Shoshone, Idaho. . . . . Lv.	"	M	3968	1.10	8.15	**5.16**
12.20	**2.10**	1228	Ar. Glenn's Ferry, Idaho. . . . . Lv.	"	M	2556	**11.00**	6.30	**3.40**
3 29	**5.35**	1322	Lv. Caldwell, Idaho. . . . . Lv.	"	M	2367	**7.45**	3.41	**12.55**
5.40	**8.20**	1395	Ar. Huntington, Ore. . . . . Lv.	"	M	2108	**4.55**	1.45	11.05
9.00	12.05	1493	Lv. LaGrande, Ore. . . . . Lv.	"	P	2786	11.35	**8.30**	5.45
11.50	3.20	1568	Ar. Pendleton, Ore. . . . . Lv.	"	P	1070	7.30	**5.15**	2.30
4.05	8.30	1710	Ar. Dalles, Ore. . . . . Lv.	"	P	106	1.55	**12.45**	**9.55**
7.00	**12.15**	1799	Ar. Portland, Ore. . . . . Lv.	"	P	29	**11.20**	10.00	**6.15**
4.40	**6.45**	1940	Ar. Tacoma, Wash. . . . . Lv.	"	P	26	**12.15**	12.45	**12.15**
6.15	**8.10**	1979	Ar. Seattle, (b) Wash. . . . . Lv.	"	P	10	10.45	**11.15**	10.45

How far is it from Chicago to La Grande, Ore.? What is the highest point of all those listed above? The average elevation of the seven points in Wyoming named? The best rate of speed in miles per hour, between any two stations from Columbus, Nebr., to Shoshone, Idaho, on the limited?

Land Surveys

Originally, farm lands of the United States had no regularity as regards boundaries or size. This is largely true today of the eastern states; that is, the states east of the Appalachian mountains and parts of the states adjacent to the western slopes of these mountains.

When Virginia ceded her claims to the great Northwest territory, the federal government instituted a method of regular surveys for all government land. This method has been followed generally throughout the country ever since. The plan of surveying as it now exists follows:

A **principal meridian** is a true north and south line established as a basis for government surveys. There are in the United States twenty-four principal meridians.

The meridians established for the purpose of government survey must not be confounded with meridians of longitude used by astronomers and navigators, as they are entirely different. The principal meridians used in surveying land are usually located with reference to some natural landmark, such as the mouth of a river, and are not equi-distant from each other.

The first principal meridian begins at the junction of the Miami and Ohio rivers, and forms the boundary between Indiana and Ohio; the second begins at a point two and a half miles west of the junction of the Little Blue and Ohio rivers; the third begins at the mouth of the Ohio and runs north to the northern boundary of Illinois; the fourth begins at the mouth of the Illinois river and extends north through Wisconsin and Minnesota, governing surveys in Wisconsin, Northern Minnesota and Western Illinois; the fifth begins at the mouth of the Arkansas and governs surveys in Arkansas, Missouri, the greater part of Minnesota, and Dakota east of the Missouri river; the sixth is the meridian of 97 deg., 22 min. west, extending south to latitude 37 deg. and north to the Missouri, and controlling the surveys in Kansas, Nebraska, the Dakotas south and west of the Missouri, Wyoming, and all of Colorado except a portion along the Rio Grande, the survey of which is governed by the New Mexico meridian.

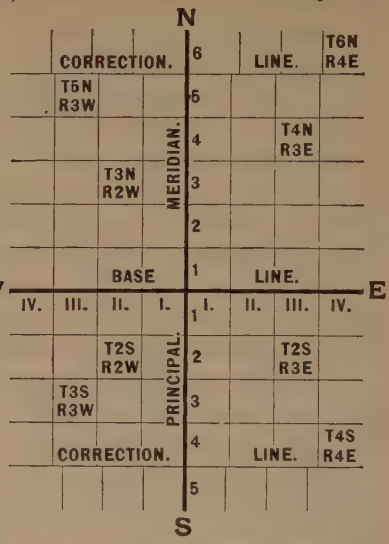
Besides these there are eighteen principal meridians in the United States survey known by names instead of numbers.

A **base line** is a line which crosses the principal meridians at right angles to them, and is used as a basis of measurements north or south.

Intersections of meridians and base lines are marked by stone monuments.

Across the principal meridians and base lines at right angles, other lines are run six miles apart, which divide the territory into square tracts six miles square.

These tracts are called **townships.**



Correction Lines. Since the surface of the earth is convex, all meridians converge as the latitude increases. Townships being exactly retangular, their east and west boundaries are not exactly parallel with the principal meridian by which they are governed. This creates a necessity for occasional adjustments, which are made upon correction lines, as shown by the diagram.

The townships are numbered north or south from the base line. Thus "town 1 north," "town 1 south," etc.

A **range** is a line of townships running north and south, and is designated by its number east or west from the principal meridian. Thus the range of townships lying next a prime meridian is known as range 1, east or west, the next range 2, east or west, etc.

A complete description of the location of a township states both its north and south location and its east and west location, thus: 1 N, R 2 E, to which is added in legal descriptions "of the third principal meridian, county of Morgan."

The above would be read, in full, "Township one north, range two east of the third principal meridian, county of Morgan."

Sections. Each township is divided into thirty-six sections, each 1 mile square and containing 640 acres.

These sections are numbered serially from 1 to 36 inclusive, beginning at the N. E. cor. and running W. in the first tier, E. in the second, etc. All fractional sections except those due to lakes, rivers, and other natural boundaries are thrown into the northern and western rows. Sections are further divided into halves, quarters, and eighths. An eighth section is also called a "forty," because it contains 40 acres.

WRITTEN

1. Make a drawing of a township on a scale of $\frac{1}{2}$ in. to 1 mi., and number the sections as they should be. What is the area of each square?

2. Draw a line from the center of the north boundary to the center of the south boundary of any one of these sections. What is the area of the part on each side of the line?

3. Draw a line from the center of the east boundary to the center of the west boundary of the section marked as above. What is the area of each quarter?

4. On the map you have made, mark the N. $\frac{1}{2}$ of section 9; the S. E. $\frac{1}{4}$ of Sec. 9; the E. $\frac{1}{2}$ of the S. W. $\frac{1}{4}$ of sec. 9; the S. E. $\frac{1}{4}$ of the N. W. $\frac{1}{4}$ of Sec. 21.

5. What is the value of the S. W. $\frac{1}{4}$ of the N. W. $\frac{1}{4}$ of Sec. 21, at \$32.50 per A.?

6. A man buys the following tracts of land: the E. $\frac{1}{2}$ of the S. E. $\frac{1}{4}$ of Sec. 30; the S. W. $\frac{1}{4}$ of Sec. 29; the E. $\frac{1}{2}$ of the N. W. $\frac{1}{4}$ of Sec. 32; the N. W. $\frac{1}{4}$ of the N. W. $\frac{1}{4}$ of Sec. 32 and the N. E. $\frac{1}{4}$ of the N. E. $\frac{1}{4}$ of Sec. 31. How many acres in all does he buy?

7. Mr. Hamilton owns the E. $\frac{1}{2}$ of the S. W. $\frac{1}{4}$ of the S. E. $\frac{1}{4}$ of Sec. 12. There is an assessment of \$200 for road improvements against Mr. Hamilton's farm. How much per acre is this assessment? How much per rod on the road?

8. What will it cost for material to inclose the E. $\frac{1}{2}$ of the S. W. $\frac{1}{4}$ of Sec. 17, T 4 N, R 2 W, with wire fencing that costs 38¢ per rod, and posts 1 rd. apart that cost $12\frac{1}{2}$¢ each?

9. A farmer owns the SW$\frac{1}{4}$ of the NW$\frac{1}{4}$ and the NW$\frac{1}{4}$ of the SW$\frac{1}{4}$ of Sec. 14. It is valued at \$55 an acre. If it is assessed at $\frac{2}{3}$ of its value and the tax rate is 10.8 mills per dollar of assessed valuation, what tax does he pay on this farm?

CHAPTER XX

APPROXIMATE MEASUREMENTS

Common grains and vegetables in large quantities are usually sold by weight. Each of the states has fixed, by statute, the number of pounds per bushel which shall be considered legal, for different kinds of produce. Dividing the total weight of a given lot by the weight per bushel gives the number of bushels, exact measurement.

A table of statutory weights of produce is given in the appendix.

For the approximate measurement of quantities of produce, estimates are made by cubic measurement, and in other readily applied ways, described in this chapter.

These rough measurements are of practical value in estimating the quantity of produce in a bin or pile, and in figuring the capacity of a bin or other container.

Grains are measured by the stricken bushel, which contains 2150.42 cu. in. Larger bodied products, such as apples and potatoes, are measured by the heaped bushel of 2747.71 cu. in.

To find the number of bushels a bin will hold.

A cubic foot contains 1728 cu. in., and is therefore about .8 of a stricken bushel or .63 of a heaped bushel.

RULE. To find how many bushels of grain a bin will hold, multiply its capacity in cubic feet by .8.

RULE. To find the number of bushels of bulky products a bin will hold, multiply its capacity by .63.

WRITTEN. Find the approximate capacity in bushels of shelled corn, of containers having the following dimensions:

1. Box, 2′ x 2′ x 4′.
2. Box, 3′ x 4′ x 4′.
3. Bin, 6′ x 10′ x 12′.
4. Car, 6′ x 8′ 6″ x 40′.
5. Bin, 6′ x 10′ x 15′.
6. Wagon, $3\frac{1}{2}$′ x 10′ 6″ x 24″.
7. Wagon, 3′ 6″ x 10′ 6″ x 26″
8. Wagon, 4′ x 10′ 6″ x 30″.
9. Car, 6′ 6″ x 8′ 9″ x 42′.
10. Box, 3′ 6″ x 4′ x 2′ 9″.

11-20. Find the approximate capacity in bushels of potatoes, of each of the above bins.

21. Find the exact capacity, in bushels of grain, in problem 3 above; in bushels of bulky products.

22. How many bushels (to 2 decimal places) should be added to or deducted from the capacity, estimated by the approximate method, to give the exact capacity, for each 1000 cu. ft. of grain? of bulky products?

23. A bin 12′ long and 5′ 6″ deep is square on all sides except the front, which slopes in. The width of the bin at the top is 6 ft.; at the bottom, 4′ 6″. How many bushels of wheat will it hold?

24. A grain bin is 12 ft. wide and 30 ft. long. If it is filled with oats to a depth of 8 ft., how many bushels are in it?

25. A bin of the same dimensions as that in problem 24 is filled to the same depth with potatoes. How many bushels are in it?

SUGGESTION: Figure the relation between 1 bu. exact and 1 bu. approx., and preserve this for future use. Do the same for 1 gal.

26. A car is marked "Capacity 77,000 lb." How many bushels of oats may be shipped in it, if it is permitted to load it to 10% more than its marked capacity?

27. A bin 5 ft. deep and 8 ft. wide is being built. How long must it be made to hold 1000 bu. of wheat?

28. Mr. Williams is building a bin to hold 8 tons of coal. He has 60 sq. ft. of floor space available. How high must the bin be if the kind of coal he uses measures 35 cu. ft. to the ton?

29. What is the weight, estimated approximately, of oats in a bin 10′ x 16′, filled to a depth of 5′ 6″?

30. A farmer in North Dakota sold 4 loads of wheat, weighing respectively 4260, 4520, 4440, and 5120. What was it worth at $82\frac{1}{2}$¢ per bu.?

31. If wheat is worth 84¢ per bushel, what is it worth per cwt.? What is the value of a carload of wheat weighing 46,000 lb.?

32. A wagon loaded with wheat weighed 3529 lb. The empty wagon weighed 1530 lb. What was the value of the contents, at $\$1.92\frac{1}{2}$ per bu.?

33. Measure a berry box or other container and determine whether it will hold the quantity represented to be in it when it was sold.

Corn in the Ear

Corn in the ear occupies approximately twice the space that it will occupy when shelled. To find how much shelled corn in a wagon box full of corn in the ear, apply the rule for finding the number of bushels of grain in a bin and divide by 2.

NOTE. Some farmers estimate that it requires $2\frac{2}{5}$ cu. ft. of corn in the ear instead of $2\frac{1}{2}$ cu. ft., to produce one bushel of shelled corn. If this basis be used, it is necessary to add to the number of bushels found by the above $\frac{1}{24}$ of itself. The rule followed in this book is the one first stated.

WRITTEN. Solve problems 1 to 10 in the preceding exercise, for corn in the ear.

Corn in the Husk

To find the number of bushels of corn in the husk in a given space, divide the number of cubic feet by $3\frac{1}{4}$.

WRITTEN. Solve problems 1 to 10 in the preceding exercise, for corn in the husk.

Bushels in a Round Pile of Bulky Products

Square $\frac{1}{2}$ of the diameter of the pile in feet, multiply by $3\frac{1}{7}$, and multiply the product by $\frac{1}{2}$ the height of the pile in feet, to get the contents in cubic feet. Reduce to bushels by multiplying by $\frac{5}{8}$.

EXERCISE. How many bushels of potatoes are there in a round pile 18 ft. in diameter and 6 ft. high?

$$9^2 \times 3\tfrac{1}{7} \times (6 \div 2) \times \tfrac{5}{8} = \tfrac{81}{1} \times \tfrac{22}{7} \times \tfrac{3}{1} \times \tfrac{5}{8} = 477\tfrac{9}{28}, \text{ no. of bu.}$$

WRITTEN

1. A farmer has for sale a pile of apples the base of which is 22 ft. in circumference and the apex of which is $4\frac{1}{2}$ ft. high. What is the pile worth at \$1.10 a bushel? (Diam. = circum. $\div 3\frac{1}{7}$.)

2. What is a pile of potatoes worth at 45 cents a bushel if it is $3\frac{1}{4}$ ft. high and if the base is 6 ft. in diameter?

Shelled Corn in a Round Pile of Corn in the Husk

RULE. Square $\frac{1}{2}$ of the diameter of the pile in feet and multiply by $3\frac{1}{7}$, then multiply by $\frac{1}{2}$ of the height in feet, to get the number of cubic feet in the pile. Divide the product by $3\frac{1}{5}$ (or multiply by $\frac{5}{16}$, which is $\frac{1}{2}$ of $\frac{5}{8}$) to reduce to bushels of shelled corn.

EXERCISE. How many bushels of shelled corn are there in a pile of corn in the husk if the pile is round, has a diameter of 16 ft. and is 6 ft. high?

$(16 \div 2)^2 \times 3\frac{1}{7} \times (\frac{1}{2} \text{ of } 6) \div 3\frac{1}{5} = \frac{64}{1} \times \frac{22}{7} \times \frac{3}{1} \times \frac{5}{16} = 188\frac{4}{7}$, no. of bu.

NOTE. Some farmers multiply the cubic contents by $\frac{4}{13}$.

Hay in the Mow and in Stacks

The only accurate method of determining the number of tons of hay is by weighing. When the hay is in the mow or stack, however, we can determine the amount approximately by measurement.

Hay varies from 400 to 600 cu. ft. to the ton. Rough, coarse hay requires about 400 cu. ft. to the ton. Timothy requires 450 to 550 cu. ft. to the ton. Clover, alfalfa, or cow pea hay requires 550 to 600 cu. ft. to the ton. In low stacks that have not much weight near the top the space occupied by a ton is greater than in tall stacks and weighted stacks. The hay in stacks that have been standing a long time is packed more closely than in fresh stacks. Knowledge of the situation in each case, is required, to know what figure to use. When in doubt, use 400 cu. ft.

To find the approximate number of tons of hay in a stack, proceed as follows:

RULE. Measure around the stack half-way to the top, and divide by 4. Square the result and multiply by the height of the stack. Divide by the number of cu. ft. to the ton.

EXERCISE. How many tons of timothy hay are there in a tall, well-weighted stack which has been standing a long time, if the stack is 10 ft. high and measures 60 ft. around at a point 5 ft. from the ground? Circumference $= 3\frac{1}{7} \times$ diameter.

NOTE. Under the conditions stated, use 450 cu. ft. to the ton.

$60 \text{ ft.} \div 4 = 15 \text{ ft.}$ $15^2 = 225$.

$225 \times 10 \div 450 = 5$, the no. of tons.

WRITTEN. Find the no. of tons of hay in stacks measuring as follows. The diameters and circumferences given are half way to top in each case.

1. Circum., 50 ft.; height, 10 ft.; 450 cu. ft. to ton.
2. Diam., 18 ft.; height, 11 ft.; 500 cu. ft. to ton.
3. Circum., 60 ft.; height, 12 ft.; 400 cu. ft. to ton.
4. Diam., 20 ft.; height, 9 ft.; 550 cu. ft. to ton.

Hard Coal in the Bin

RULE. Multiply together the three dimensions of the bin in feet and divide by $34\frac{1}{2}$.

EXERCISE. How many tons of anthracite coal in a bin 8′ x 10′, filled to a depth of $4\frac{1}{2}$′?

$8\times10\times4\frac{1}{2}=360$, no. of cu. ft. in bin.

$360\div34\frac{1}{2}=10\frac{3}{7}-$, no. of tons in bin.

NOTE. Various kinds of coal run from 32 cu. ft. to 37 cu. ft. to the ton.

To Measure Lumber in the Log

RULE. Take 4 inches off the diameter of the log, multiply the remainder (in inches) by one-half of itself, then by the length of the log in feet, and divide the product by 8.

EXERCISE. How many feet of lumber in a log 12 ft. long and 36 in. in diameter?

$$\frac{32\times16\times12}{8}=4\times16\times12=768 \text{ ft.}$$

EXPLANATION. The diameter is reduced 4 inches to account for slabs cut off the sides, which is waste. The remainder is a rough, round piece. The area of a cross-section of this round piece could be found exactly by squaring the diameter (in inches) and multiplying by .7854. An inch cut off the length of the board would contain ($D^2\times.7854\div144$) board feet. A foot of it would contain ($D^2\times.7854\div12$) board feet. This calculation would be difficult, and more exact than is necessary or desirable. The decimal .7854 is approximately $\frac{3}{4}$, and it is more conservative measuring to use $\frac{3}{4}$ than .7854 because it is a little smaller. $D^2\times\frac{3}{4}\div12$ would give the approximately correct result, but it has been found easier to get the same result by applying the formula $\frac{D^2}{2}\div8$, which gives the same result, for one foot of length.

$$\left(D^2\times\frac{3}{4}\times\frac{1}{12}=D^2\times\frac{1}{16}=\frac{D^2}{2}\times\frac{1}{8}\right).$$

Multiply this by the length in feet. Note that $\frac{D^2}{2}$ is the same as $D\times\frac{D}{2}$.

WRITTEN. Find the number of board feet in logs of the following dimensions:

1. Length, 15′; diam., 30″.
2. Length, 18′; diam., 36″.
3. Length, 12′; diam., 32″.
4. Length, 16′; diam., 30″.
5. Length, 18′; diam., 32″.
6. Length, 17′; diam., 36″.
7. Length, 20′; diam., 32″.
8. Length, 20′; diam., 36″.
9. Length, 12′; diam., 30″.
10. Length, 14′; diam., 32″.

11. What is the total board feet in the logs given in the first 10 problems?

12. Prove your answer to problem 11 by combining the first 10 problems in such a way as to make three problems, adding the results of the three.

Cisterns, Tanks, and Reservoirs

A U. S. standard gallon contains 231 cu. in. One cubic foot, therefore, is equivalent to about $7\frac{1}{2}$ gal.

RULE. To find how many gallons a container will hold, multiply its contents in cu. ft. by $7\frac{1}{2}$.

WRITTEN. Find the approximate contents, in gallons, of each of the following:

1. A stone jar 12″ in diameter and 18″ deep.

2. A fish bowl 20″ in diameter and 14″ deep.

3. A barrel 3′ high, having an average diameter of 1′ 4″.

4. A natatorium 20′ x 30′; depth 2′ 6″ at one end, gradually increasing to 10′.

5. A barrel 2′ 4″ high, average diameter, 1′ 6″.

6. A silo 12′ in diameter and 21′ high.

7. A mason jar 5″ in diameter, filled to a depth of 7″.

8. A well 5′ in diameter, 12′ deep.

9. A circular reservoir with sides cut straight down, 50′ in diameter and 12′ deep.

10. A galvanized iron tank 8′ in diameter and 4′ deep.

11. Find the contents in gallons, exact measurement, for problems 1, 2, and 7.

Why, in your opinion, were problems 1, 2, and 7 selected for exact measurement?

12. How many gallons (to 2 decimal places), must be deducted from the quantity found by approximate measurement, for every 1000 cu. ft., to give an accurate result?

13. How many cu. ft. in a barrel of $31\frac{1}{2}$ gal.?

14. A tank on a street sprinkler is 5 ft. in diameter and 12 ft. long. How many bbl. will it contain, if there are $31\frac{1}{2}$ gal. to a bbl.?

15. In a certain heating system there are approximately 240 ft. of pipes 2″ in diameter, inside measurement, and 520 ft. of

pipes 1″ in diameter, inside measurement. How many gallons of water will be required to fill the pipes of the system?

16. The water tank of an office building is 10 ft. in diameter and 12 ft. high. If water weighs $62\frac{1}{2}$ lb. to the cu. ft., how many tons of water will the tank contain?

17. A man wants to build a rectangular tank with a capacity of 75 bbl. The tank is to be 5 ft. wide and 3 ft. deep. How long must it be?

18. The inside dimensions of an electroplating tank are: Depth, 3 ft.; width, 5 ft.; and length, 12 ft. How many bbl. of solution will it contain when filled to within 8 in. of the top?

CHAPTER XXI

SPECIFIC GRAVITY

Specific gravity, or relative weight, is the ratio of the weight of a substance to that of an equal volume of another substance taken as the standard.

The standard usually taken is one cubic foot of water, which weighs 1000 ounces avoirdupois, or $62\frac{1}{2}$ lb., under the conditions usually specified.

A cubic foot of cork weighs 15 lb. and a cubic foot of water weighs $62\frac{1}{2}$ lb. The specific gravity of cork is $15 \div 62\frac{1}{2} = .24$.

A body floating in water displaces a weight of water equal to its own weight. The weight of a submerged body is diminished by the weight of the water it displaces.

WRITTEN. Find the specific gravity of each of the following substances:

METALS	Weight (lb.) per cu. ft.	WOOD	Weight (lb.) per cu. ft.
1. Platinum	1415.232	5. Ebony, American	67.204
2. Bismuth	608.256	6. Lignum-vitae	83.116
3. Bronze	547.776	7. Spruce	31.276
4. Iron, cast	485.568	8. Walnut	38.016

STONE AND EARTH	Weight (lb.) per cu. ft.
9. Emery	250.560
10. Mica	174.873
11. Quartz	166.060
12. Loose Earth	84.844

SPECIFIC GRAVITIES OF SOME COMMON SUBSTANCES

METALS		STONE AND EARTH		WOODS	
Gold, pure	19.258	Sand	2.650	Ash	.845
Lead, pure	11.330	Limestone	2.700	Cedar	.561
Silver, pure	10.474	Marble, common	2.686	Cork	.24
Copper, pure	8.788	Granite	2.704	Elm	.560
Brass, common	8.500	Slate	2.800	Mahogany	.560

Metals		Stone and Earth		Woods	
Steel, cast	7.919	Salt, common	2.130	Maple	.790
Iron, cast	7.40	Clay	1.900	Oak	.950
Tin	7.201	Brick	2.000	Poplar	.383
Zinc	7.101	Coal, anthracite	1.640	Pine, southern	.720
Aluminum	2.660	Coal, bituminous	1.355	Pine, white	.400

Liquids		Liquids		Liquids	
Cider	1.018	Milk	1.032	Lard	.947
Egg	1.090	Vinegar	1.08	Tallow	.924
Honey	1.45	Butter	.942	Water, sea	1.03
Ice	.93	Sugar	1.605		

Written. Using the foregoing table, find the weight of a cubic foot of each of the following substances:

1. Aluminum
2. Butter
3. Lard
4. Clay
5. Sea Water
6. Pure Gold
7. Sugar
8. Limestone
9. Cedar
10. Tin
11. Common brass
12. Anthracite coal
13. Cast Steel
14. Oak
15. Pure Silver

Written

1. What is the weight of the contents of a 10 gal. can of milk?

2. A white pine timber 6″ x 6″ x 20′ is floating in water. How much weight will it sustain?

3. A teamster hauls a block of granite containing 1 perch. How much does the load weigh?

4. Ten cubic feet of white pine floating in water will displace how many gallons of water?

5. An object 2 ft. 8 in. thick floats in water with 4 in. above the surface. What is its specific gravity?

6. A mill pond is frozen over with ice 15 in. thick. 280,000 sq. ft. of it can be cut. How many tons of ice can be harvested from this area?

7. A marble column is 3 ft. in diameter and 24 ft. high. What is its weight?

8. A granite column 2 ft. x 4 ft. x 18 ft. is on the bottom of a river. It is desired to raise it with a derrick. How many pounds must the derrick be capable of lifting to raise this granite column to the surface of the water?

9. The pattern for a casting is made of white pine and weighs 1 lb. 8 oz. The casting is to be of cast iron. How much cast iron will be needed for 100 such castings?

10. A casting for an automobile starter was made of brass. It was decided to use aluminum for the casting instead of brass. If the brass casting weighed 4 lb. 6 oz., what was the weight of the aluminum casting?

11. A rectangular piece of ice weighs 100 lb. If one side of it measures 12″ x 18″, how thick should it be?

Review Problems

1. A cylindrical container is 12 in. in diameter. How high must it be, if its capacity is 1 bu.?

2. A cylindrical container is 9 in. high. What must its diameter be, if its capacity is 1 bu.?

3. If a lath is 4 ft. long, and one lath is used for each 2 in. covered, how many lath will be required to lath a room 18′ x 30′ x 12′?

4. A cylindrical watering trough is 8 ft. 2 in. in diameter, and 2 ft. 5 in. in height. How many gallons does it hold?

5. A wagon box is 10 ft. 6 in. long, 3 ft. 2 in. wide, and 28 in. deep. How many bushels of shelled corn will it hold? How many bushels of potatoes?

6. A wooden berry box is 4 in. square. How deep should it be to hold a quart?

7. If cement is worth $1.65 per bbl. and sand and gravel are each worth $1.10 per load, what is the cost of materials for 10 cu. yd. of a 1, 2, 3 mixture of concrete? Use full loads.

8. A farmer made 10 concrete posts reinforced with metal. He used 1 cu. yd. of a 1, 2, 3 mixture. The cement cost $2.00 per bbl. The sand and gravel each cost $1.00 per cu. yd., and 46 lb. of $\frac{1}{4}$″ rods at $2\frac{1}{2}$¢ per lb. were required for reinforcing. What was the cost of the material for each post? (Use exact quantities.)

9. If milk costs $3.43 per hundred pounds, what does it cost per gallon?

10. If the rainfall is $\frac{1}{4}$ inch per hour, how many gallons will fall on one acre in one minute? How many barrels?

11. A farmer digs a ditch 15 in. wide, 3 ft. deep, and 45 rd. long. How many cu. yd. of earth does he remove, allowing for an increase of 35% due to loosening in digging?

12. A crib filled with ear corn is 16 ft. long, 8 ft. wide, and averages 10 ft. high. How many bushels should it shell, if a bushel of ear corn makes $\frac{1}{2}$ bu. shelled corn?

13. How many bushel of oats in a bin 18 ft. wide, 21 ft. long, and averaging 4 ft. deep? Get the accurate result and the approximate result, and find how much they differ.

14. How many bushels of barley in a wagon 3 ft. 2 in. wide, 10 ft. 6 in. long, and 26 in. deep?

15. In measuring articles sold by the heaped bushel, it is common practice to consider each 4 cu. ft. as 3 bu. A car is $8\frac{1}{2}$ ft. wide, 40 ft. long, and is filled with potatoes to a depth of $5\frac{1}{2}$ ft. How many bushels does it contain?

16. If this car were similarly filled with wheat, how many bushels would it hold?

17. Which is the heaviest, 23 bu. of wheat, 31 bu. of barley, 42 bu. of oats, or 32 bu. of rye?

18. An aeronaut ascended 11,642 ft. above Chicago. How many miles was this?

19. A man, asked to give his rule for finding the number of bushels of grain in a bin, replied: "Find the number of cubic feet in the bin, multiply this by 8, and divide the product by 10". Explain why this will give the approximate number of bushels.

20. The center of a carpet rug is $14\frac{1}{4}$ ft. long and 9 ft. wide. It is surrounded by a border 18 in. wide. How many yards of border will it take? If the center is made of four strips of carpet, 27 in. wide, how many yards were used?

21. The drive wheel of a locomotive is 7 ft. 3 in. in diameter. How many revolutions does it make in going a mile?

22. The plunger on a pump is 6 in. in diameter; the length of the stroke is 24 in. How many strokes will it take to fill a 100-gallon tank, if full 24 in. of water is brought up at every stroke?

23. A coal bin is 18′ 6″ x 11′ 9″ x 6′ 3″. How many tons of coal will it hold if the coal weighs 80 pounds to the bushel?

24. A passenger on a railway train by noting the time it requires to pass from one mile post to the next may determine the number

of miles per hour the train is traveling. One man made three observations, as follows: 42″, 53″, 1′ 6″. At what rate was the train traveling in each case?

25. If railway rails are 30 ft. long and weigh 90 lb. to the yard, how many rails will it take to lay a mile of railway? What is the cost at $32.00 per ton?

26. If coal weighs 80 lb. to the bushel, how many cu. ft. in a ton?

27. A railroad buys a right of way 100 ft. wide through a farm. The road is 100 rd. long. What is the value of the right of way at $82.50 an acre?

28. If railroad ties average $2\frac{1}{4}$ ft. apart, center to center, how many ties will be required per mile?

29. I have used 32″ x 44″ paper, weighing 120 lb. per ream. I wish to use paper of an equivalent weight, but in size 28″ x 42″. What will it weigh per ream?

30. If cloth is 32 in. wide, how many yards of ruffling, cut crosswise, 5 in. wide, can be made from a yard? How much waste is there?

31. How much cloth, cut crosswise, is needed to make four yards of ruffling 5 in. wide, allowing $\frac{1}{2}$ inch for hem, from cloth 40 inches wide? How much waste?

32. A ruffle 10 inches wide is to be put on $2\frac{1}{4}$ yards of cloth. The ruffle is one-third fuller. How much cloth 27 inches wide must be allowed for it?

33. A paper machine makes paper 144″ wide at the rate of 500 lineal feet per minute. How many reams of 24″ x 36″ paper will this machine make in twenty-four hours? How many lb., if it weighs 36 lb. to the ream? How much time to make two rolls 72″ wide, each weighing 792 lb.?

34. A printing press averages 2250 impressions per hour, and prints 64 pages at each impression. At $2.00 per hour, what will it cost to print 17,500 catalogs containing 384 pages?

35. A publisher needed for a book 50 reams of 25 x 38 — 60. Being unable to get it on time, he was forced to buy 28 x 42 — 74. If the stock was worth $6\frac{3}{4}$¢ per lb., how much did he lose?

CHAPTER XXII

PROFIT AND LOSS ON TRADING

The prime cost, or net cost, of goods purchased is the original purchase price as shown by the invoice.

The gross cost is the total cost after adding in-freight and cartage, insurance and storage *en route,* and any other direct charges entering into the cost of the delivered goods.

When these direct charges can not be applied integrally to units of purchase, it is usual to find the ratio of direct charges to purchases and express this as a per cent of the purchase price.

The cost of goods sold is found by adding to the purchase price of the goods sold the per cent necessary to cover direct charges.

To find the cost of goods sold during a given period, add to the inventory at the beginning of the period the amount of purchases, and deduct the inventory at the end of the period. Then add the per cent necessary to cover direct charges.

The gross sales is the total of sales at invoice price.

The net returns from sales is the amount remaining after deducting from gross sales the returns and allowances, sales commissions, and shipping expense. Sometimes a deduction is made for bad debts.

The profit or loss on trading is the difference between the gross cost of goods sold and the net returns from sales. The per cent of profit or loss is the amount of profit or loss expressed as a per cent; it may be expressed as a per cent of the prime cost, or of the gross cost, or of the net returns, or of the selling price.

The profit or loss on doing business is the profit or loss on trading less the cost of doing business.

WRITTEN

1. A dealer bought of Ebeling & Reuss, Philadelphia, Pa., a bill of glassware amounting to $44.28. The shipment weighed 480 lb., the freight rate was 53¢ per cwt., and drayage was $1.20. What was the gross cost? What per cent was added to the invoice price by the transportation charges?

2. A wholesale dealer imported from Canada a carload of beans weighing 42,180 lb. at \$2.29 per bu. of 60 lb. The tariff on beans was 45¢ per bu. of 60 lb. and the freight was 42¢ per cwt. Labor for unloading the beans and putting them in bags cost $22\frac{1}{2}$¢ per hour for 42 hours and bags to hold 2 bushels of beans cost $3\frac{1}{2}$¢ each. Find the gross cost per bushel; per pound. (Consider duty a direct charge.)

3. A lumber dealer purchased a carload of lumber containing 29,000 ft. at \$22.50 per M. The freight amounted to \$63.98 and the labor for unloading cost 25¢ per hour for $13\frac{1}{2}$ hours. What was the gross cost per thousand feet? What per cent was added to the invoice price by the direct charges?

4. A granary that cost \$950 will store 2500 bu. of wheat, 1500 bu. of oats, and 500 bu. of barley. If the interest rate on the cost of the granary is 6%, insurance 40¢ per \$100 on \$900, and general expense and repairs \$15.60 per annum, what will be the gross cost per bushel to store grain to the capacity of the granary? What will be the gross cost per bushel if only $\frac{3}{4}$ of the possible total quantity of each grain is stored?

5. A farmer employed a man for 9 months, agreeing to pay him \$34 per month for the first 4 mo. and \$40 per month for the last 5 mo. The hired man was to receive board and to have a horse and buggy kept. His board was estimated at \$12 per mo. and the keep of the horse and buggy at \$5.50 per month. During three months of the time he worked, on an average, 10 hours per day; during two months of the time, 12 hours per day; and for the remainder of the time, 9 hours per day. What was the cost per hour for his labor, if he worked 26 days per month?

6. The John Ena, a four-masted steel barkentine, has a registered net tonnage* of 2706 tons (capacity). The Panama canal measurement rates it at 2609 net tons. The expenses for passing through the canal were \$1.20 per ton of her entire capacity, canal measurement, and tug service amounting to \$302.15. The cargo carried was 4400 tons of petroleum and wax. What was the cost per ton of cargo carried for passing through the canal? What would be your answer had her registered tonnage been charged for?

* *Ton* here means 100 cu. ft. A *ton* of cargo is 40 cu. ft. Neither of these words has any relation to the *ton* of weight.

Per Cent of Profit

7. It cost a gas company .345 to manufacture a thousand feet of gas. Cost of maintenance and service was .183 for every thousand feet of gas used. Other expenses were, to every thousand feet: Taxes, .054; insurance, .013; depreciation, .028; rents, .026. If gas was sold at 80¢ per thousand, what was the per cent of profit over manufacturing cost? Over gross cost? What part of the 80¢ was profit?

8. Smith & Co.'s net profit during the last fiscal period was $17,800. Total sales for the period amounted to $128,000. Expenses were: Rent, $1200; clerk hire, $4200; light and heat, $160; sundries, $500. What was their gross profit, in per cent, based upon sales?

Selling Price

9. Determine the rate of profit in each of the following cases:

	Invoice Price Each	Add for Direct Charges	Selling Price	Find Rate of Profit Based on
(*a*)	$25.00	5%	$40.00 each	Prime cost
(*b*)	25.00	5%	40.00 each	Gross cost
(*c*)	25.00	5%	40.00 each	Selling price
(*d*)	25.00	$1.50	40.00 each	Prime cost
(*e*)	.423	10%	10.00 per doz.	Prime cost
(*f*)	.423	10%	10.00 per doz.	Gross cost
(*g*)	.423	10%	10.00 per doz.	Selling price
(*h*)	.423	$.0612	10.00 per doz.	Prime cost
(*i*)	73.40	$7\frac{1}{2}$%	100.00 each	Prime cost
(*j*)	73.40	$7\frac{1}{2}$%	100.00 each	Selling price

10. From the following facts determine my profit for the year and my rate of profit based upon original investment: Mdse. on hand Jan. 1, $12,500. Purchases during year, $30,000. Sales during year, $56,000. Inventory Dec. 1, $9450. Add 5% of invoice price to cover direct charges. Deduct 4% from sales for allowances. General expenses, $6500.

11. Determine the selling price in each of the following cases:

	Invoice Price	Add for Direct Charges	Rate of Profit Desired	Allowance for Rebates, Returned Goods, Bad Debts, etc.
(*a*)	$90.00	5%	10% of inv. price	4% of selling price
(*b*)	90.00	5%	10% of gross cost	4% of selling price
(*c*)	90.00	5%	10% of net returns	4% of selling price
(*d*)	90.00	5%	10% of sell. price	4% of selling price
(*e*)	5.50	$.78	25% of inv. price	2% of selling price
(*f*)	5.50	.78	25% of sell. price	2% of selling price
(*g*)	.075	$12\frac{1}{2}$%	20% of inv. price	2% of selling price
(*h*)	1.216	10%	15% of gross cost	5% of selling price
(*i*)	3.166	$.634	20% of gross cost	5% of selling price
(*j*)	75.00	$7\frac{1}{2}$%	$\frac{1}{3}$ of invoice price	2% of selling price

12. Determine the selling price per unit in each of the following cases, assuming that $7\frac{1}{2}$% must be added to cover direct charges, and that the dealer wishes to make 20% on gross cost.

	Invoice Price		Invoice Price
(*a*)	$12.00 per dozen	(*f*)	$10.00 per gross, less 20 and 10%
(*b*)	15.00 per dozen	(*g*)	27.00 per dozen, less 25 and 5%
(*c*)	16.00 per gross	(*h*)	75.00 per gross, less 10 and 10%
(*d*)	125.00 per thousand	(*i*)	39.00 per dozen, less 10 and 5%
(*e*)	9.00 per dozen	(*j*)	24.00 per dozen, less $33\frac{1}{3}$ and 10%

13. The following table shows the five highest records made by members of the Iowa Baby Pork Club in a contest held in 1915 under the auspices of the extension department of the Iowa State College of Agriculture:

Name of Owner	County	Weight at close of contest	Daily gain in lb.	Cost per lb. of gain	Age in days of pig at close of contest	Final score
Bernard Glotfelty....	Jefferson	275	1.76	.047	223	96.15
Wendell McCleary...	Jefferson	290	1.95	.055	205	95
Fred Klopfenstein...	Wash.	320	2.06	.058	222	90
Malcom Trout.....	VanBuren	210	1.44	.055	197	85.8
Lester Schuerman....	Henry	$262\frac{1}{2}$	1.57	.0276	226	85.4

What was the average weight of the five pigs?

What was the average daily cost of feeding?

The contest lasted 17 weeks. What was the weight of the first pig at the beginning of the contest? If feeding cost only half as much per day during the time before the start of the contest and if the pig was sold at \$7.30 per hundred, what was the profit on the pig?

Apportionment of Charges

Frequently charges must be apportioned over several articles that are not alike. Thus, if a carload of furniture contains several kinds of furniture, as rockers, tables, dressers, beds, and chairs, all incidental expenses of shipping must be divided and distributed among the different articles upon some basis. The basis of distribution may be cost, weight, size or number of articles contained in the shipment, or some other unit.

WRITTEN

1. A dealer in building supplies purchased 60 bbl. cement at \$1.92 per bbl., and 10 tons prepared plaster at \$16.45 per ton. Both kinds of material were shipped in the same car at 27¢ per cwt. The cost of labor for unloading this material was \$1.80 per hour for $9\frac{1}{2}$ hours. The weight of the cement averaged 400 lb. per bbl. Find the gross cost of 1 barrel of cement and 1 ton of plaster, after distributing the freight and labor (*a*) according to weight; (*b*) according to the invoice cost of each kind of material.

2. What per cent is added to the invoice cost of the different kinds of goods, in the first problem, by each method of distribution?

3. A furniture dealer bought 10 pianos at \$214.50 each and 36 dining tables at \$21.00 each. Cartage on the pianos cost \$13.85 and on the dining tables \$13.90. The total amount of freight was \$29.76. First, distribute the freight on these articles according to weight, assuming that the pianos weighed 750 lb. each, and the dining tables 75 pounds each. Second, distribute the freight according to the number of articles purchased. How much difference is made by the two methods of distribution, in the cost of each table? What per cent is added to the invoice cost of each table under each method of distribution?

4. A farmer lives three miles from the railway station, and in hauling grain makes four trips a day, each round trip taking $1\frac{3}{4}$ hours. Horse labor is worth 18.31 cents an hour. Man labor is worth 34.65 cents an hour. At each load he hauls 50 bu. with a two-horse team. If 40% of the grain hauled is oats weighing 32 lb. per bu. and the remainder wheat weighing 60 lb. per bu., what is the cost for hauling per bu. of each grain, if the hauling cost is distributed according to weight?

5. If in the above problem wheat is worth 130% more than oats, and the hauling price is distributed according to value, what is the hauling cost per bu. of each grain?

6. Another man lives five miles from the station, and can make but two trips a day. He hauls one kind of grain only. What is his charge per bu. for hauling if it takes $3\frac{1}{2}$ hours for each round trip?

7. A barn cost \$3000 and was built to shelter 12 horses, 30 cows, 200 sheep, and 150 hogs. Interest was charged on cost at 6%, repairs amounted to \$50 per year, and insurance on the building cost \$47.80 per year. These annual charges were distributed according to weight. The horses were estimated to average 1250 lb., the cows 1000 lb., the sheep 100 lb., and the hogs 125 lb. What was the annual cost per head to shelter each kind of stock?

Profit, Loss, and Selling Price

WRITTEN

1. A carload of cattle, 22 head averaging 1340 lb. each, was purchased at \$9.50 per C. They were sold at a profit of 10% of the cost. What was the profit?

2. An investor bought three 35 ft. lots at \$400 per lot. The sewer cost him \$1.35 per front foot, and the paving \$3.80 per front foot. He then sold the lots at a profit of $28\frac{1}{2}$% of the total cost. What was his profit?

3. A grocer bought blackberries at \$2.45 per crate of 1 bushel each. He paid 15¢ per crate for delivery to his store. At what price per quart must he sell them to gain 25% of the total cost?

4. A grocer bought a barrel of vinegar for \$8.25. It contained 32 gallons and he estimated a loss of 6% by waste and leakage. At what price per quart must he sell it to gain 50% of the cost?

5. A shoe dealer buys shoes at \$97.50 per case, 3 doz. pairs to a case. If his expenses in handling shoes average 49¢ per pair, at what price must he sell them to gain 25% of the total cost?

6. A jobber bought for \$1287.40 a lot of goods which, because of damage, he had to sell at a loss of 14% of the cost. The retailer who purchased these goods paid \$500 at the time of purchase and subsequently became bankrupt, paying 68¢ on the dollar. What was the total loss to the jobber?

7. A herder bought 500 sheep for \$4200. He sold 60% of the herd (second grade) at \$8.05 per head. At how much per head must he sell the remainder of the sheep to make a total gain of 15% on the original cost of the herd?

8. A grocer bought eggs at \$7.40 per crate, 30 doz. to a crate, and found 1 dozen eggs broken when he opened the crate. At what price per dozen must he sell the remainder to gain 30% on the cost of the crate?

9. A firm has determined that it costs 8% of the net cost of all goods sold to pay the expenses of doing business. If a barrel of flour costs \$7.35, for what must it be sold so there will be neither gain nor loss? To gain 10% of the total cost?

10. If corn costs 88¢ per dozen cans, direct charges are $2\frac{1}{4}$¢ per doz., and the cost of doing business is 12% of the invoice cost, for what price per can must it be sold to gain 20% of the gross cost?

Marking Goods

In marking goods a code is often used, so that the price may be known to the merchant but not to the customer. Thus a merchant desiring to mark both his cost and selling prices on his price tags might use the following symbols:

Cost Key										Selling Key									
1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0
d	o	n	t	g	i	v	e	u	p	o	x	y	g	e	n	a	c	i	d
repeater 1										repeater k									

The repeater is for use as the second of two figures which are alike, as in the cost price \$4.00, which would be written t.pl instead of t.pp.

A merchant using the codes shown, with his cost marked at the top of his price tag and his selling price below, would use the following marks on an article costing \$4.00 and selling for \$5.50:

$$\frac{\texttt{t.pl}}{\texttt{e.kd}}$$

Using the keys given on the previous page for cost and selling price, mark the following articles:

	Cost	Rate of Profit		Cost	Rate of Profit
1.	\$1.30	10% of selling price	9.	\$ 3.18	28% of cost
2.	1.45	15% of selling price	10.	3.72	8% of selling price
3.	1.67	20% of cost	11.	4.60	35% of cost
4.	1.86	25% of cost	12.	4.90	45% of cost
5.	1.92	$33\frac{1}{3}$% of cost	13.	12.50	20% of selling price
6.	2.04	40% of cost	14.	13.75	50% of cost
7.	2.27	30% of selling price	15.	14.83	$12\frac{1}{2}$% of selling price
8.	2.85	40% of cost	16.	15.40	58% of cost

Net Returns from Sales

1. My sales for 1919 amounted to \$17,500; goods returned by customers, \$629.50; other rebates allowed to customers, \$220.45. Allowing 2% for bad debts, what was the amount of net returns from sales?

2. My sales are handled entirely through an agent to whom I allow a commission of 10% on net collections after deducting 2% for bad debts. If sales for the year amount to \$25,000, and returns \$550, what is my net return from sales?

3. Total sales for Aug., \$35,600. Returns to us, \$550. Other rebates, \$67.50. Bad debts, $2\frac{1}{2}$% of sales. Agent's commission 3% on collections. Determine the net returns from sales.

Profit or Loss on Trading

As previously defined, profit or loss on trading is the difference between the gross cost of merchandise and the net returns from sales, without consideration of the general expenses of the business. This profit or loss may be stated as a per cent of prime cost, gross cost, net sales, or gross sales.

WRITTEN

1. State the per cent of profit or loss based on purchase price, in each of the following cases:

	PURCHASES	DIRECT CHARGES	DEDUCTIONS FROM SALES	SALES
(*a*)	$9426.50	$ 329.48	$542.50	$14,862.50
(*b*)	8534.25	568.27	625.00	15,560.00
(*c*)	7649.82	1243.54	344.29	13,560.75
(*d*)	5783.95	505.00	00	9,000.00
(*e*)	8205.20	463.25	5%	12,875.80
(*f*)	7524.50	750.60	125.00	11,650.00
(*g*)	8426.45	216.87	100.60	12,000.00
(*h*)	3320.70	739.28	None	5,000.00
(*i*)	4500.00	104.53	$2\frac{1}{2}\%$	6,247.50
(*j*)	5000.00	590.46	None	7,500.00

State the per cent of profit or loss on each of the above cases, based on net sales.

2. From the following facts determine the per cent of profit on trading over the gross cost: Inventory at beginning, $5000; purchases, $12,000; sales, $20,000; purchases returned, $560; sales returned, $225; claims allowed to customers, $42.50; in-freight, $125.60; inventory at end, $4500.

3. Total sales, $13,500; goods returned to us, $150.45; allowances to customers for shortage, $46.75; estimated loss on uncollectible accounts, $2\frac{1}{2}\%$. Cost of goods sold, $8760.50. Find the net returns from sales and the per cent of profit or loss on trading, based on gross cost.

Cost of Doing Business

The cost of doing business is an expression which defines itself. In a merchandising business it includes rent, salaries, light and heat, taxes, advertising, depreciation, and all other expenses except those that make up the gross cost of merchandise.

Depreciation is the decrease in the value of property which takes place as it wears out, goes out of use, etc.

WRITTEN

1. From the following facts determine the gross cost of mdse. sold and the cost of doing business for one year: Mdse. on hand Jan. 1, $12,000.00; purchases, $22,350.50; sales, $33,500.00; rent, $62.50 per month; equipment, $2500; salaries, $35.00 per week; in-freight, $308.00; $10,000 insurance, at 85¢ per $100 for 3 years; miscellaneous expenses, $47.50 per month; advertising, $350; depreciation on equipment, 10%; mdse. on hand Dec. 31, $9850.

2. Using the figures of problem 1 as a basis, what per cent must be added to prime cost to cover direct charges against purchases? To cover cost of doing business? To net a profit of 15% of prime cost?

3. What price must be asked for an article costing $4.50 to net a profit of 10% of sale price?

4. J. A. Carleton's store is worth $3000. The store fixtures are worth $750. He pays a clerk $15 and a boy $9 a week. His horse and wagon are valued at $400. It costs $250 a year to maintain the delivery equipment. The rate of depreciation is 4% on the building, 10% on the horse and wagon, and 8% on fixtures. All other expenses, $39.50 per month. What is his daily cost of doing business, in a year of 310 business days?

5. A man bought a farm of 190 acres at $125 per acre. The dwelling was valued at $1500, barn $1000, granary $750, tool shed $300, and other buildings $200. Depreciation on these buildings averages 4% of the value and interest on the investment is charged at 6%. Taxes are paid upon an assessment of $12,000 at 12 mills on the dollar. Fire insurance is carried on all buildings to the amount of $2500 at 30¢ per $100, and live stock insurance costs $35.00 for one year. Disregarding labor, what is the annual cost of this farm and how much per day on a basis of 313 working days per year?

6. A man has $15,000 invested at 5%. His salary is $50.00 a week. His living expenses are $30.00 a week. He has an opportunity to go into a business in which he can figure on a gross profit of 22%. He estimates that his cost of doing business will be $200.00 a month, and that his living expenses will be increased to $45.00 a week. How much business will he have to do to make the same net saving as at present?

The average cost of doing business in various lines of retail trade is stated below. These percentages are based upon the volume of sales.

	Dry Goods	Groceries	Hardware	Clothing	Furniture	Drugs	Shoes
Rent	3.1%	3.3%	2.9%	2.2%	5.5%	4.6%	3.1%
Salaries	9.6%	8.1%	11.2%	9.1%	8.3%	10.9%	11.1%
Advertising	1.5%	.7%	.9%	3.4%	2.9%	2.6%	1.5%
Heat and Light	.4%	.5%	.5%	.3%	.8%	.8%	.9%
Delivery	.9%	1.9%	.6%	.6%	1.1%	.4%	.3%
Supplies	.4%	.3%	.4%	.2%	.5%	.3%	.4%
Insurance and Taxes	1.1%	.4%	1.1%	1.1%	1.4%	1.4%	1.2%
General Expenses	4.4%	.3%	.8%	1.8%	1.8%	2.4%	4.1%
Depreciation and Shrinkage	1.4%	.6%	.5%	2.2%	1.9%	.5%	.6%
Bad Debts	.3%	.3%	.4%	.4%	1.2%	.2%	.1%
	23.1%	16.4%	19.3%	21.3%	25.4%	24.1%	23.3%

This table is compiled from *Keeping up with Rising Costs*, A. W. Shaw Co., Chicago, and is based on an investigation of more than 1500 business houses.

7. According to the above table, how much rent per month could a grocer afford to pay for a building in which he could do an annual business of $20,000?

8. A hardware store paid annual expenses amounting to $8753.47. If this store maintained the averages indicated by the above table, what was the volume of sales annually? What amount was paid for salaries? (Depr. and bad debts are not "paid ex.")

9. A furniture store sold a dresser for $45. Using the above table, how much of this price would go for advertising? How much for delivery and general expenses?

10. A drug store does an annual business of $25,000. What should be the annual cost of doing business, and the annual charge for each of the above expenses?

11. A dealer in dry goods estimates that he loses on an average $45 per month on account of bad debts. What must be the volume of his sales according to the above schedule?

Depreciation

There are several methods of figuring annual depreciation. One very simple method is to find the difference between the value at the beginning of the life of the property and the value at the end, and divide by the number of years of life. Use this method in the following problems. If no residual or junk value is stated, assume that there is none.

WRITTEN

1. If a frame building is estimated to be usable for forty years, what per cent of the original cost should be charged off each year?

2. If a frame building cost \$1500.00 and an amount equal to $2\frac{1}{2}\%$ of the cost be charged off each year, what is the average amount invested in the building for 40 years?

3. The duration of an ordinary patent is 17 years. If a firm paid \$5000 for a patent, what amount should it charge off the cost each year?

4. The life of the average horse is 14 years. He begins to be useful when he is 3 years old. If he is worth \$150.00 at the beginning of usefulness, what is the annual depreciation?

5. If the average cow gives milk for seven years, and at the beginning of the period is worth \$80.00, and at the end \$31.00 for beef, what is the annual depreciation? What is the per cent of depreciation?

6. On a farm in Kansas 16,156 bushels of corn, 70 lb. to the bushel, was weighed as it was husked. In July following, it was sold and measured 14,896 bushels. What was the per cent of shrinkage? If it sold for $56\frac{3}{4}¢$ a bushel, what did it bring?

7. A man owns a herd of 250 cows worth \$100 each at the beginning of their usefulness as milkers. At the end of seven years he sells them for beef at \$34.00. The average death rate is 1.2% per annum. Whenever a cow dies he replaces it with another costing \$100. What is his annual loss for replacement and depreciation?

8. A man paid \$2100.00 for an automobile. His state license cost him \$6.00, and his wheel tax \$10.00. His expenses for repairs for the year were \$72.40; for oil, \$5.85; for gasoline, 440 gallons at 16¢. He ran 5280 miles and his machine depreciated $28\frac{4}{7}\%$. What was the cost per mile for the season?

In each of the following problems interest on the investment is to be allowed at the rate specified in the problem.

9. A frame building that cost \$1500.00 sheltered 38 head of cattle. What amount should be charged to each animal if interest is worth 6%, the usual depreciation (3%) of building be considered, and annual repairs to building are estimated at \$15.00?

10. Find the annual cost per acre on the following machines: Walking plow, cost \$14.00, depreciation 6%, interest 5%, repairs \$1.50, acres plowed, 45. Binder, cost \$125, value last inventory \$100, depreciation 8% of cost, repairs \$5.00, labor 5 hours at 19¢, interest at 5% on last inventory; acres cut, 42.

11. 890 bushels of corn were picked in December, and could have been sold at 53¢. The corn was held until July when the shrinkage was found to be 7.8%. It sold for $58\frac{1}{2}$¢ per bushel. If interest was worth 6%, did the farmer make or lose by holding the corn?

12. The cost of a barn to shelter 40 head of cattle was \$2000.00. Interest is reckoned at 6%, insurance $\frac{1}{2}$%, depreciation 3%, repairs $1\frac{1}{2}$%, taxes $\frac{1}{2}$%. What was the annual cost of sheltering each animal?

13. A building was erected in 1888 at a cost of \$42,375. In 1923 it was condemned and torn down. What was the average annual depreciation of the building? What per cent of the original cost was the average annual depreciation?

14. A manufacturing company purchased machinery which cost \$27,639.40. For four years it made an annual charge of 12% of cost, for depreciation. After the fourth year a new invention rendered the machinery worthless except as junk, and it was sold for \$2683.95. How much greater should have been the annual depreciation charge than it was?

15. A four-year old horse is purchased by a farmer for \$240. It is estimated that a horse is past usefulness when 13 years old. What is the annual depreciation cost of the horse, and what per cent?

16. A cow is purchased when 3 years of age for \$75. She is past usefulness when 12 years of age and is fattened and sold as beef. She weighs 950 lb. and is sold at \$5.25 per cwt. What is the annual depreciation?

17. A man built a summer cottage which cost $1250. He occupied it on an average of 4 months per year and the cottage was useful for 12 years. What did the cottage cost each month of occupation?

18. A manufacturing company has the following property accounts upon which it must allow an annual depreciation charge:

	Cost	Rate of Depreciation
Buildings	$46,397.20	4%
Machinery and Tools	75,354.38	$16\frac{2}{3}$%
Drawings and Patterns	6,950.30	20%
Factory Equipment	12,382.97	$12\frac{1}{2}$%
Furniture and Fixtures	3,517.54	10%

What is its annual charge for depreciation?

Problems of a Trading Business

1. The trading accounts of a certain business showed the following: Purchases, $16,000; sales, $27,500; inventory at the beginning of business $2700, and at the close of business $3000. What was the per cent of gross trading profit on the net cost of goods sold?

Trading Statement*

	Cost		
Sales of Merchandise (for period)			*****
Inventory, at beginning (of period)	****		
Purchases (during period)	*****		
Total cost of merchandise	*****		
Inventory, at close (of period)	****		
Net cost of merchandise sold		*****	
Gross Trading Profit		*****	*****

2. The accounts of a certain business showed, on Dec. 31, sales for the year amounting to $46,387.60. The discount allowed on sales amounted to $758.39. What was the average per cent of discount on sales?

3. The Mutual Trading Co. began business with a stock of goods valued at $27,390.46; during the year its net purchases amounted to $89,392.16; its net sales during the same period amounted to

* The student should follow the form given here in solving problems involving the trading accounts. Other forms may be used if the teacher assigns them.

$147,362.29 and its closed business with an inventory of $31,562.90. What per cent on the net cost of sales was the gross trading profit?

4. The profit and loss statement of a company showed the following results at the end of the year:

Expenses			Income	
Selling Expense:			Gross Trading Profit	$10,400.00
Advertising	$ 800.50		Discounts on Purchases	280.00
Salesmen's Salaries	3680.00		Rental of Stands	480.00
		$4480.50	Interest, overdue accounts	20.00
General Expense:				
Rent	$1500.00			
Insurance	150.00			
Miscellaneous	901.60	$2551.60		
Total Expense		$		
Net Profit		$		
		$		$

The net cost of merchandise sold by this company amounted to $29,500. The gross trading profit was what per cent of this? The net profit was what per cent of it?

5. If the rent in the preceding problem is increased $50 a month, by what per cent does this increase the general expense?

6. By what per cent would the $50.00 increase in rent decrease the net profits of the company, based upon the net profits of this year?

7. The net profits of the above company were what per cent of the sales?

8. The total expenses in the above statement were what per cent of the total income? Each kind of expense in the above statement was what per cent of the sales?

9. The gross trading profit was what per cent of the total income of the company?

10. The above company was capitalized at $10,000. The net profits of the company were what per cent upon the capital?

Drill Problems

ORAL

1. A furniture dealer sold a dresser for $37.50, which was at a profit of 20% over cost. What was the cost of the dresser?

2. A set of books was sold at a loss of 15% of cost. What was the cost if they were sold for $25.50?

3. Find the cost per dozen of hats that retail at $3.00 each at a profit of 50% of cost.

4. A manufacturer sells goods at 20 and 10% from the list price and makes a profit of 50% on the cost of production. What is the cost of production of an article that is listed at $50?

5. What per cent of profit should be added to gross cost in order to allow a discount of $16\frac{2}{3}\%$ from the marked price and still gain 25% on the gross cost?

6. A publishing company sells books at 25% from the list price. The dealer who purchases these books sells them at the list price. What is the dealer's per cent of profit based on cost?

7. A hardware dealer bought certain articles at $15 per dozen less 25 and 20%. He sold the same articles at a discount of 10% from the list price. What was his per cent of profit based on cost?

8. A manufacturer listed a piece of machinery in a catalog at $75.00. He now allows discounts of 25, 20, and 10% from the list price and still makes a profit of 50% upon the cost of production. What does it cost him to produce this article?

9. A company allows a cash discount of 2% on bills paid within a certain time. The discount deducted by a customer amounts to $53.68. What is the amount of the bill?

10. A grinding machine was priced at a profit of 20% of cost. If the profit amounted to $50.90, what was the selling price of the machine?

Miscellaneous Problems

WRITTEN

1. Two horses were sold for $250 each. On one a profit of 25% of cost was made and on the other 25% of cost was lost. What was the net gain or loss and the rate of gain or loss?

2. The trading statement of a business shows gross sales amounting to $21,749.83. If the sales were made at an average profit of 30% on invoice cost, what is the invoice cost of the goods sold?

3. I had a claim against a bankrupt and received from my attorney \$776.25 in settlement of it. What was the amount of my claim if the bankrupt paid 60¢ on the dollar and my attorney charged 10% of the amount collected as his fee for making the collection?

4. At what price each must I mark goods that were purchased at \$25.00 per dozen, that I may allow a discount of 20% from marked price and still gain 20% on cost?

5. Certain goods are bought by a dealer at discounts of 20, 20, and 10% from the list price. What per cent over his cost is made by the dealer if he sells these goods at the list price?

6. It costs \$18 to manufacture a certain machine. At what price must it be listed in order to allow discounts of 20 and 10% and still make a profit of 20% upon the cost of manufacturing the machine?

7. An advertisement concerning a house for sale reads: "Rents \$162 per month. Expenses: Taxes, \$130; insurance, \$75; interest on mortgage, \$420; miscellaneous, \$150. It will net the purchaser 8% upon cost." What can be paid according to the above figures in order to realize 8% upon the cost of the equity?

8. A herder bought a band of sheep and sold them at a profit of 20% of cost. With the money received from this sale he purchased a herd of cattle which he sold at a profit of 25% of cost. His profit on the two transactions was \$793.50. Find the cost of the sheep and the selling price of the cattle.

9. A man sold a house at a profit of 20% of cost. With the money received from this sale he purchased another house, which he sold at a loss of 5% of cost. If his profit on the two transactions amounted to \$562.80, what was the cost of the first house and the selling price of the second?

ORAL (See table, opposite page.)

1. Use columns a and b to find the gain and selling price.
2. Use columns a and c to find the loss and selling price.
3. Use columns a and d to find the rate per cent of profit.
4. Use columns a and e to find the rate per cent of loss.
5. Use columns a and f to find the rate per cent of profit.
6. Use columns a and g to find the rate per cent of loss.

Oral and Written Table

	a	*b*	*c*	*d*	*e*	*f*	*g*
	Cost	Per Cent of Cost Gained	Per Cent of Cost Lost	Gain	Loss	Selling Price	Selling Price
1.	\$ 10.	10 %	10 %	\$ 2.00	\$ 2.	\$ 13.	\$ 6.00
2.	12.	$12\frac{1}{2}$%	$12\frac{1}{2}$%	3.00	4.	16.	4.00
3.	15.	20 %	25 %	6.00	5.	20.	12.00
4.	18.	40 %	30 %	6.00	9.	30.	17.00
5.	20.	$37\frac{1}{2}$%	20 %	5.00	8.	30.	5.00
6.	22.50	10 %	$33\frac{1}{3}$%	7.50	4.50	37.50	18.50
7.	25.	25 %	16 %	5.00	2.50	35.	24.00
8.	30.	$16\frac{2}{3}$%	40 %	7.50	10.	40.	27.
9.	50.	8 %	2 %	2.00	5	60.	37.50
10.	75.	16 %	12 %	12.00	24.	90.	70.
11.	80.	3 %	$1\frac{1}{2}$%	5.00	10.	90.	75.
12.	125.	6 %	4 %	5.00	25.	140.	90.
13.	175.	8 %	6 %	25.00	35.	200.	87.50
14.	200.	14 %	18 %	45.00	27.	201.	190.
15.	250.	$1\frac{1}{2}$%	1 %	10.00	5.	275.	170.
16.	300.	$62\frac{1}{2}$%	35 %	20.00	25.	315.	120.
17.	400.	$1\frac{1}{4}$%	$\frac{1}{2}$%	1.00	10.	425.	380.
18.	450.	8 %	$\frac{4}{5}$%	50.00	9.	500.	375.
19.	500.	$7\frac{1}{2}$%	$\frac{3}{4}$%	25.00	10.	875.	450.
20.	600.	2 %	$1\frac{1}{2}$%	9.00	18.	700.	590.
21.	750.	$8\frac{1}{3}$%	$\frac{2}{3}$%	150.00	15.	1000.	500.
22.	900.	17 %	21 %	100.00	150.	1200.	800.
23.	1000.	38 %	53 %	50.00	10.	1450.	937.
24.	1200.	8 %	$6\frac{2}{3}$%	36.00	144.	1300.	1150.

WRITTEN AND ORAL

1. Use columns *b* and *d* to find the cost and selling price.
2. Use columns *c* and *e* to find the cost and selling price.
3. Use columns *b* and *f* to find the cost and profit.
4. Use columns *c* and *g* to find the cost and loss.
5. Use columns *d* and *f* to find the rate per cent of gain, based on cost.
6. Use columns *e* and *g* to find the rate per cent of loss, based on cost.

TRADE PROBLEMS

Groceries

ORAL

1. Cider is purchased at 12¢ per gallon and sold at 5¢ per quart. What is the per cent of profit on cost?

2. Eggs are bought at $6.00 per crate (30 doz. each) and retailed at 10 for 25¢. What is the per cent of profit on cost?

3. What must be the cost per dozen of an article that retails at $7.50 each at a profit of 20% on cost?

4. What must be the cost per bushel of berries that retail at 10¢ per quart at a profit of 25% on cost?

5. Berries cost a peddler $1.44 per crate, 16 boxes to a crate. He sells them at 15¢ per box. What is his per cent of profit on cost?

6. A grocer buys new potatoes from a farmer at $1.20 per bushel, and retails them at 20¢ per half peck. What is his per cent of profit on cost?

WRITTEN

1. Equal quantities of coffees costing 28¢ and 20¢ are mixed. At what price per pound must the mixture be sold to gain 25% on average cost?

2. 10 lb. of 32¢ coffee is mixed with 15 lb. of 24¢ coffee (cost prices). At what price per pound must the mixture be sold to gain 25% on average cost?

3. A grocer bought 10 bbl. (30 bu.) of apples at $2.35 per bbl. and drayage cost $1.40. He sorted them and obtained 6 bbl. of first class apples and 3 bbl. of second class apples, the remainder being unsalable. Four barrels of the first grade apples were sold at 40¢ per peck. The remaining 2 bbl. of first class apples were sold at the rate of 2 apples for 5¢, the apples averaging 90 per bu. The second class apples were sold at $2.75 per bbl. Labor of sorting cost $3.85. What profit did the grocer make and what per cent was it on the gross cost including labor?

4. If the apples in the above problem had not been sorted and cleaned they would have been sold at 30¢ per peck. Did the grocer gain or lose by sorting the apples? How much?

Hardware

1. A hardware dealer bought bolts at 75¢ per gross and sold them for 1¢ each. What was his per cent of profit on cost?

2. Nails are purchased at $1.90 per keg (100 lb. to keg). What per cent of profit on cost is made by selling them at $2.75 per keg? At 4¢ per pound in small quantities?

3. Saws are bought at $11.50 per dozen. What per cent of profit on cost is made by selling them at $1.50 each?

4. Axes costing $7.50 per dozen are retailed at 85¢ each. What per cent of profit on cost does the dealer realize?

5. A dealer bought 10 M. washers (7 gross per M.*) at 24¢ per gross. He sold 40 gross at 30¢ per gross and the remainder at 10¢ per dozen. What was his per cent of profit on cost?

6. A hardware dealer bought chisels at a list price of $24 per dozen less 80, 10, and 5% and sold them at retail at 50¢ each. What was the rate of profit on cost?

7. Jackplanes were bought at a list price of $11.40 per dozen less discounts of 40 and 10%. At what price each must they be sold to gain 50% on cost?

8. A bill of hardware amounting to $229.48 net was purchased by Gillivan Bros. The freight on it amounted to $17.38. Among other items were two 20″ x 30″ cast iron enameled sinks listed at $9.50 less 30%. If the freight is distributed on this bill according to the amounts of the items, what is the gross cost of each sink?

9. The dealer agreed to set one of the above sinks in a kitchen for $12.00. If a plumber worked $2\frac{1}{2}$ hr. at 90¢ per hour, and a helper $2\frac{1}{2}$ hr. at 45¢ per hour, and material furnished amounted to 65¢, what was the dealer's profit and his rate of profit on the gross cost of the contract?

Furnishings and Clothing

ORAL

1. Hats of a given brand are bought at $21.60 per dozen. At what price must they be retailed to gain 25% on cost?

2. Ties of a given kind are bought at $4.50 per dozen. At what price must they be retailed to gain 20% on cost?

* This rough count of 7 gross to the thousand is common in the hardware business

3. Shoes of a certain make are bought at \$90 per case (3 doz.) At what price per pair must they be sold to gain 20% on cost?

4. Shirts of a certain quality are bought at \$17.40 per dozen and sold at \$2.00 each. What per cent of the selling price is profit?

WRITTEN

1. A dealer bought 3 dozen Panama hats at \$42.80 per dozen. He sold 28 hats at \$5 each and the remainder he sold at \$3.60 each. What was his per cent of profit on cost? What would it have been if he had sold the entire lot at \$5 each?

China and Glassware

1. One dozen dinner plates cost me \$17.50. At what price each must I sell them to gain 30% on cost?

2. Glass tumblers are bought at \$2.80 per gross and sold at 2 for 5¢, or 3¢ each. What rate of profit on cost is made by selling each way?

3. The following bill is received from a manufacturer:

3—100 pc. (B) Dinner Sets Style 17	@ \$5.55	\$16.65
3—100 pc. (B) Dinner Sets Style 843	@ 6.00	18.00
3—100 pc. (B) Dinner Sets Style 1095 H	@ 8.10	24.30
Cask		1.90

Freight on the above shipment was 43¢ per cwt. on 415 lb. and drayage \$1.25. What per cent must be added to the cost of each set to pay for the cask, freight, and drayage? One set of "style 17" is sold, retail, at \$8.50; what is the per cent of profit on gross cost?

4. The dealer selling the dishes in the 3d problem must meet the following expenses annually: Rent, \$50 per month; depreciation on fixtures, \$115 per year; salary for his own time, \$100 per month; one clerk as an assistant, \$12 per week; general expenses, \$340 per year; and interest at 6% on a stock of merchandise valued at \$3000. If his sales average \$150 per week, what per cent of the sales is required to do business?

5. The dealer (See problem 3) sold one set, style 843, for \$12. Adding to cost, to cover direct charges, the per cent computed in problem 3, what was the gross cost? What was the profit, if the cost of doing business was the per cent of sales figured in problem 4?

6. The following year this same dealer spent $325.80 for advertising, and his general expense increased 25%, but his sales rose to $200 per week. If all other expenses remained the same, what per cent of his sales was required to pay the cost of doing business? He then sold the last set of "Style 1095 H" at $12.00. What was his profit or loss on it?

7. Find how much of the advertising for the past year is chargable to one set of dishes, Style 1095 H, if the cost of advertising is distributed proportionately over the sales.

Furniture

ORAL

1. The material for an office desk cost $8, labor cost $10, and overhead expenses were $6. The manufacturer sold the desk for $40. What was his per cent of profit on total cost?

2. The desk (in the above problem) retailed at $60. What was the retailer's rate of profit on the invoice cost? on the selling price?

3. The material, labor, and overhead expenses in the first problem were each what per cent of the cost of production of the desk?

Miscellaneous

1. A feed store bought shorts at $28.00 per ton. Labor, bagging, and cartage cost $4.50 per ton. At what price must a 50 lb. bag be sold to gain $33\frac{1}{3}\%$ on gross cost?

2. A certain brand of 5¢ cigar costs $39.00 per M. In place of a discount the dealer receives 250 cigars per M gratis. To what discount is this equivalent, and what is the per cent of gain on cost?

3. Baldwin's advertising for the year cost $3360, which was 24% of his gross trading profit. What was his gross trading profit?

4. Pens bought at 75¢ per gross are retailed at 6 for 5¢. What is the per cent of profit on cost?

5. A certain kind of paper cost us 18¢ per lb. and we sold it at 25¢ per lb. The cost price has been increased to $19\frac{1}{2}$¢ per lb. At what price per lb. must we sell it to make the same per cent of profit on cost?

6. The steamship Arizonian, plying between Atlantic and Pacific ports of the United States, has a capacity of 6576 net tons, canal

measurement. By passing through the Panama canal this ship can save on an average 26.8 days time as compared with a voyage around South America. The cost of operating this ship is estimated at $450 per day. The canal toll is $1.20 per net ton, canal measurement. How much is saved by going through the canal?

7. If the manifests of the Arizonian showed 6374 tons cargo, what would be the charge against each ton of cargo on acccount of the expenses of passing through the canal if the canal toll were divided equally among the tons of cargo?

8. If the total value of the cargo carried through the canal by the Arizonian was $945,000.00, what would be the charge for canal toll against one ton valued at $275, if the total charge were distributed according to value?

Agriculture

1. An Ohio farm of 140 acres was purchased at $90 per acre and rented at $5 per acre. The annual expenses of the farm are as follows: Insurance on buildings, 55¢ per $100 on $2000; taxes $56.98; repairs $75. Depreciation on buildings is offset by the natural increase in the value of the land. The purchaser assumed a mortgage of $3000, bearing 6% interest. What net rate of income is made upon the money invested by the owner?

2. A man bought a farm of 60 acres at $72 per acre. There were no buildings on the land. The buyer built a house costing $950, a barn costing $450, and other buildings costing $200. He then sold the farm for $7000. The interest on his investment was offset by the crops he raised. What was his profit per acre and the per cent of profit on the total cost?

3. A man is offered 70¢ for a bushel of corn where it stands. If fed to hogs, it will produce 9 lb. of gain, and hogs are worth $12.45 per cwt. Which will be the more profitable, to feed the corn to hogs or to sell it? What will be the difference for 100 bu. of corn?

4. A tenant cut 20 acres of clover that produced 1.6 bushels of clover seed per acre. The clover seed sold at $5.40 per bushel. Previously the same year, the tenant had cut from the same land a crop of hay that was worth $14.50 per acre. If rental was $5 per

acre, the hay cost \$7.50 for mowing and \$34.75 for hauling and storing, and the expenses for the crop of clover seed amounted to \$8.10 for mowing and \$1.73 per bushel for all other expenses, what was the total profit from this field? What was the per cent of profit on the total cost of both crops?

5. A farmer has 120 acres to plow each year. He wants to decide whether to buy a gang plow for \$70.00, or two walking plows at \$18 each. He figures interest at 6%, and he estimates depreciation on machinery at 8%. Man labor costs \$60 per month of 26 working days each and board is estimated at \$24 per month. Horse labor costs 80¢ per day each and five horses will be used in either case. Two men are required for the walking plows or one man for the gang plow. If the plowing can be done in 30 working days either way what is the cost per acre for each method of plowing? How much is saved or lost by using the gang plow?

6. If butterfat makes $\frac{1}{8}$ more butter than its own weight, which is the best plan for the farmer: to sell 100 lb. of whole milk at 5¢ a quart; to sell the cream at 40¢ a quart, and use the skim milk, which is worth 20¢ a hundred pounds, for feeding purposes; or to make butter at 35¢ a pound, sell the buttermilk at 2¢ a quart, and use the skim milk?

100 lb. of the milk = 46.51 qt. = 5.75 qt. cream and 88 lb. skim milk.
1 qt. of the cream tests .66+ lb. butter fat and yields $1\frac{1}{3}$ pt. buttermilk.

7. In raising a crop of barley on a certain field of $7\frac{1}{2}$ acres the man labor was 156 hr. at 18¢ and the horse labor 366 hr. at 11¢. There were $12\frac{1}{2}$ bu. of seed used at 75¢. Twine, 16 lb. at 11¢. Threshing charge for yield of 294 bu. at 3¢ per bu. Charge for depreciation of equipment was \$6.30. Taxes, \$3.65. Land was valued at \$100 per acre and interest was charged at $5\frac{1}{2}$%. The crop at threshing was worth 76¢ a bushel and there were $6\frac{1}{2}$ tons of straw at \$2.50 per ton. Find the profit per man hour.

8. If milk is sold at \$1.80 per cwt. and it tests 4% butterfat, what is the equivalent price per pound of butterfat? What is the equivalent price of the whole milk sold by the quart if the milk weighs $8\frac{5}{8}$ lb. per gal.? Not counting the value of the skim milk, what is the equivalent price of butter if the butter received is $\frac{1}{5}$ more in weight than the butterfat?

Accounting Problems

1. An electric light company "wrote off" $3125 as depreciation on boilers. At what per cent of cost did it estimate its yearly loss from depreciation if the cost of the boilers was $39,062.50?

2. A trading corporation provided for a loss of 3% of its accounts receivable by crediting $3168.45 to a reserve for bad debts account. Find the amount of its accounts receivable.

3. A furniture store sold on an average $62.80 worth of goods daily last year. These sales were made at an average net profit of 12.3% of sales. If the store was open for business 305 days, what were the net profits?

4. The trading accounts of a business for the first year showed the following balances: Purchases of goods, $15,960.38; sales of goods, $21,365.97; inventory at closing, $3140.18; goods returned by customers, $983.50. Make a statement showing the gross trading profit of the business. What per cent was this profit of the cost of the goods sold?

5. The profit and loss statement of a business contained the following: Gross profit from trading, $4673.18; income from rentals, $750; selling expenses, $1321.60; general expenses, $2259.82. Find the net profit of the business. The expenses were what per cent of the gross trading profit?

6. If the net profits for the past year (previous problem) were 11.3% of the sales, by what per cent must sales increase to meet an advance in rent of $20 per month?

7. A man received a salary of $1800 per year. He had been paying $25.00 per month rent and 10¢ per day, 6 days per week, for carfare. His salary was increased to $160 per month. He moved out of the city, paid $30.00 per month for rent, and traveled on a commutation ticket which cost 18.5¢ per day. In proportion to his salary, did expense for rent and carfare increase or decrease and what per cent?

8. A business sold goods in one year amounting to $63,797.48. The gross trading profit was 23.7% of the sales. The expenses were 72% of the gross trading profit. The net profit was what per cent of the sales? What per cent was the net profit of the cost of merchandise sold?

9. Based upon the year's business shown by the preceding problem, what must be the amount of sales for the following year, if expenses are to be increased 55% and the same rate of profit maintained, to realize the same amount of net profit?

10. The gross income of a railroad was $2,282,900 last year. 63% of this income was from freight traffic, 30% from passenger traffic and 7% from miscellaneous sources. It is proposed to increase freight rates 5%. If this proposed increase is allowed and the same amount of traffic is handled, by what per cent will the gross earnings of the company be increased?

11. Assuming that the increase in rates referred to in the 10th problem is allowed, and that the company has a capital of $1,500,000, the increase in earnings will be equal to what per cent of the capital of the company?

12. A street railway company's gross earnings for 1927 amounted to $3,003,400 and its expenses $1,691,700. It has been operating on a basis of 3 tickets for 25¢. It is now proposed that it shall give 4 tickets for 25¢. If this change in fare goes into effect and the same amount of traffic is handled, by what per cent will it reduce the net income of the company?

13. The change in rates referred to above went into effect. If the expenses for 1928 remained the same as for 1927, they were what per cent of the gross income?

14. A furniture dealer was selling at $18.00 each brass beds that cost him $12.25 each. He sold three of these beds each day at an average expense of $3.75 daily. He cut the price to $15.00 each and advertised the bed as a "Special." He used 8 inches of advertising space each day costing $92\frac{1}{2}$¢ per inch. His sales then increased to 8 beds per day. If his expenses remained the same, by what per cent did his profit increase or decrease?

15. The trading accounts of a certain business are as follows: Inventory at the beginning of the year, $8,450.47; purchases of merchandise, $38,629.50; sales of merchandise, $51,827.56; purchases returned, $971.20; sales returned, $1,263.80; inventory at closing, $9,623.87. Make a statement showing the gross gain or loss from trading and find what per cent the trading profit is of the net cost of merchandise sold. What per cent is it of the net sales?

CHAPTER XXIII

TAXATION

A **tax** is a sum of money demanded by the Federal, state, or municipal government from some of those who live or own property under its protection.

A *poll tax* is a tax upon the person. A *property tax* is a tax upon property, real or personal. A *franchise tax* is a tax upon a privilege exercised under authority of the state. An *internal revenue tax* is a tax placed upon certain commodities produced, manufactured, or traded in within the boundaries of the United States. Liquors and tobaccos are illustrations of commodities subject to this tax. A *duty* is a tax upon goods imported or exported. There are many special forms of taxation, such as the income tax, inheritance tax, wheel tax, dog tax, road tax, licenses to do business, etc.

All taxes are *direct*, as to the person who pays in the first instance; but internal revenue taxes and duties, being taxes upon commodities, are added to the purchase price of the commodities and become *indirect* taxes upon the purchaser. For this reason, duties and excises are usually spoken of as "indirect taxes."

The assessed valuation of property is the valuation fixed upon it by the tax assessor, which valuation is used as the basis for taxation.

Sometimes personal service will be accepted instead of money in payment of taxes. Thus, farmers often pay their road tax by personal service in filling and grading the roads.

It is customary to express the tax rate as mills on $1, as dollars on $100, or as dollars on $1000. Sometimes it is expressed as a per cent, but this method is not prevalent in the various offices where the tax rates are made and the revenues distributed.

When a tax rate is expressed as mills, it means that the given number of mills is the tax on each $1 of valuation.

Written

Find the tax on the following:

1.	$10000 @ 12.62 mills	6.	$ 8200 @ $17.47 per $1000
2.	15500 @ 15.48 mills	7.	3870 @ $1.875 per $100
3.	6400 @ 20.27 mills	8.	2450 @ $20.27 per $1000
4.	8700 @ 1.637%	9.	4750 @ $21.35 per $1000
5.	10500 @ 1.485%	10.	22800 @ $1.678 per $100

The Method of Taxation

The methods of taxation followed by taxing bodies, whether the taxing body is the nation, state, county, town (township), or village, are very similar. A budget of appropriations is prepared, the total of which is the amount required for the expenditures of the ensuing year or period of years to be covered. Taxable properties (and persons) are listed. The rate of taxation is fixed by comparing the amount needed with the valuations of taxable properties (and persons).

ILLUSTRATION. The following table shows the assessed valuation, by wards, of all property in the city of Detroit, Mich. in a certain year.

ASSESSED PROPERTY VALUATIONS, DETROIT, MICH.

WARD	REAL ESTATE	PERSONAL	TOTAL
1	$61,113,350	$27,999,320	$ 89,112,670
2	65,858,760	67,924,730	133,783,490
3	12,474,410	2,990,250	15,464,660
4	19,882,170	5,358,030	25,240,200
5	10,812,940	2,057,750	12,870,690
6	10,711,760	2,116,180	12,827,940
7	9,135,430	4,902,580	14,038,010
8	12,153,800	2,722,610	14,876,410
9	10,026,830	3,538,570	13,565,400
10	10,692,630	1,832,330	12,524,960
11	7,926,900	3,787,110	11,714,010
12	11,506,220	2,975,550	14,481,770
13	10,060,880	2,889,470	12,950,350
14	11,427,800	2,348,960	13,776,760
15	13,876,420	7,756,720	21,633,140
16	10,370,940	2,243,590	12,614,530
17	34,257,150	6,193,230	40,450,380
18	13,343,680	6,055,070	19,398,750
			$491,324,120
Less personal property of Detroit United Railway exempt from city taxes but subject to state and county taxes			4,561,000
Net valuation for city taxation			$486,763,120

The Detroit city government is divided into different departments, such as the department of education, department of police, fire department, department of public works, department of parks and boulevards, library department, department of water supply, and department of public health.

The head of each department makes an itemized estimate of what it will cost to run that particular department for the coming year. Following are estimates of amounts needed.

1.	For Detroit City Sinking Fund	$ 158,950.
2.	For Annexed District Sinking Fund	14,400.
3.	For Interest Fund	398,760.56
4.	For Water Commissioners' Fund	75,000.
5.	For Fire Department Retirement and Pension Fund	34,111.79
6.	For Police Pension Fund	5,610.85
7.	For Superannuated Police Fund	30,000.
8.	For Street Opening Fund	14,451.95
9.	For Recorder's Court Fund	9,000.
10.	For Belle Isle Bridge Fund	10,300.
11.	For Public Entertainment Fund	1,500.
12.	For Public Building Fund	72,632.46
13.	For Market Fund	5,245.50
14.	For Fire Commission Fund	1,100,928.
15.	For Police Fund	1,215,353.93
16.	For Poor Commission Fund	361,676.
17.	For Public Health Fund	250,775.
18.	For Public Lighting Fund	594,276.67
19.	For Department of Public Works (General Fund)	69,525.
20.	For Public Sewer Fund	180,511.95
21.	For Garbage Fund	214,069.30
22.	For Public Library Fund	172,195.27
23.	For Board of Education Fund	1,905,389.85
24.	For General Fund	213,432.40
25.	For General Road Fund	2,229,429.
26.	For Park and Boulevard Fund	539,658.

Written

1. The city's income from special sources (fines, fees, etc.) called "Miscellaneous revenue to reduce taxation" was estimated at $4,694,930.00 for the year. How many mills were levied on each dollar of real and personal property (carried to hundredths of a mill) to cover Detroit's requirements for that year?

2. Not considering the city's income from special sources, what tax rate would cover the requirements of each of the first 5 of the 26 departments of city administration, separately considered?

3. In a certain county there is property of all kinds subject to taxation, valued at $2,358,400. The total revenue to be raised by taxation is $28,546.37. The county has 800 residents who are subject to a poll tax of $2.00 each. What is the tax rate on each one dollar of valuation.?

WRITTEN

1. In a certain city the taxable property is assessed at $133,783,400. If the rate for school purposes is 3.9144 mills on each dollar of assessed valuation, what is the amount of the school fund when collected?

2. In a county the taxable property was assessed at $25,240,200, and the tax to be raised amounted to $398,760.50. What was the tax rate in per cent, in mills on $1, in dollars on $100, and in dollars on $1000?

3. In a certain city the assessed value of all taxable property is $40,425,300. The fund for maintaining the police department requires a tax of 2.3767 mills and the public library fund requires a tax of .34465 mills. How much more is spent each year on the police department than on the public library?

4. Mr. Gibson owned a house and lot assessed at $2450. If the tax rate for the board of education fund was 3.914409 mills on each dollar of valuation, how much would Mr. Gibson contribute to this fund?

5. The city of Detroit, Mich., wants to build a new technical high school, for girls, that will cost $325,000. According to the assessed valuation of property given on a preceding page, a levy of how many mills on the dollar of valuation would be required to build this school?

6. In a county in Ohio the taxable property is assessed at $18,323,300. It is necessary to raise a fund of $27,478.70. Find the rate of taxation on each dollar of assessment.

7. A city has taxable property assessed at $263,601,020. It has been decided to build a new high school that will cost $250,000 and bonds payable in 20 years bearing interest at 5% have been

issued for that purpose. What rate per $100 of assessed valuation must be levied to pay the annual interest charge on these bonds?

The Port Commission of New Orleans, La., issued on April 1, 1914, $3,000,000, 5% gold bonds, parts of the issue to mature serially as follows:

SCHEDULE OF MATURITIES

MATURITY	AMOUNT	MATURITY	AMOUNT
1920	$33,000	1926	$45,000
1921	35,000	1927	47,000
1922	37,000	1928	49,000
1923	39,000	1929	51,000
1924	41,000	1930	53,000
1925	43,000		

8. The taxable value of property in Louisiana at the time of the above issue amounted to $581,063,253. Based on this valuation what must be the general tax levy per dollar of valuation to pay the interest charge on the entire issue of bonds?

9. In 1930 the state must pay $53,000 on these bonds. If the taxable value of all property has increased 10% by that time, what will be the rate of a general levy to meet this expenditure and pay interest due that year?

10. In a county in Iowa the levy for state purposes was .45 mill, for county purposes 3.05 mills, for school purposes 2 mills, and for township purposes 2.20 mills. What per cent of all money collected by the county treasurer was paid over to the state? (Give the result in three decimal places.)

11. The levy for state purposes in Nebraska for one year was:

General fund	.5 mills
University fund	.1 mill
Special university building fund	.75 mill
Normal school fund	.85 mill
State aid bridge fund	.2 mill

The taxable value of all property in the state was $471,933,972 for that year. Find the amount paid into each fund.

12. The total value of all taxable property in Douglas county, Nebraska, the same year, amounted to $46,769,081. How much did this county contribute to the general fund of the state according to the rate given in the 11th problem?

Income Tax

The Federal Income Tax law, originally passed on September 8, 1916, has been changed in many particulars by subsequent legislation. The treatment given here follows the 1926 legislation.

The law provides for both a "normal tax" and a "surtax." The normal tax is computed on the individual's "net" income, less certain "credits."

The net income is determined by subtracting from the "gross," or total, income certain "deductions," the most important of which are: (1) Necessary business expenses; (2) Interest on personal indebtedness; (3) Taxes on home and business holdings; (4) Business losses not covered by insurance; (5) Contributions for religious and charitable purposes; (6) property depreciation.

The most important "credits" are: (1) Dividends from domestic corporations; (2) Personal exemption—$1500 for a single person and $3500 for a head of a family or a married person living with wife or husband; (3) $400 for each dependent person supported by the taxpayer.

The rate for the normal tax is 1½% of the first $4000 of taxable income, 3% of the next $4000, and 5% of all in excess of the $8000. The amount thus obtained is then to be reduced one-fourth.

The surtax is in addition to the normal tax and is computed on the total net income above $10000. The table below shows rates and amounts of the graduated income surtax for individuals.

Amount of Net Income	Rate %	Surtax	Total Surtax on Each Amount	Amount of Net Income	Rate %	Surtax	Total Surtax on Each Amount
A	B	C	D	A	B	C	D
$10,000	..	....	....	$40,000	10	$400	$1800
14,000	1	$40	$40	44,000	11	440	2240
16,000	2	40	80	48,000	12	480	2720
18,000	3	60	140	52,000	13	520	3240
20,000	4	89	220	56,000	14	560	3800
22,000	5	100	320	60,000	15	600	4400
24,000	6	120	440	64,000	16	640	5040
28,000	7	280	720	70,000	17	1020	6060
32,000	8	320	1040	80,000	18	1800	7860
36,000	9	360	1400	100,000	19	3800	11,660

EXAMPLE. William Selden, a married man, has an annual income of $20750. His deductions amount to $4375. Estimate his income tax.

Normal Tax

Gross income	$20750.00	1½% of first $4000	$ 60.00
Deductions	4375.00	3% of second $4000	120.00
	$16375.00	5% of remainder—$4875	243.75
			$423.75
		Less 25%	105.94
		Normal Tax	$317.81

Surtax (See Table)

Net income, on which surtax is computed,	$16375.00
Tax on first $10000	$ .00
Tax on $4000 ($10000 to $14000) at 1%	40.00
Tax on $2000 ($14000 to $16000) at 2%	40.00
Tax on $375 ($16000 to $16375) at 3%	11.25
Surtax	$91.25

Total Tax, $317.81+$91.25=$419.06

1. James Saunders (unmarred) receives a salary of $7500 per year and has other income amounting to $3275. During the year his expenses, allowable as deductions, amounted to $2150. Make a statement showing the amount of his income tax.

2. John Harvey (married) owns three houses which rent for $25 per month each; four store building which rent for $60 per month each; two mortgages at 6% per annum for $12,000 and $15,000 respectively; and an interest in the partnership of Harvey and Baxter from which he receives a profit of $6500. The general deductions allowed to him are as follows: Interest paid on personal indebtedness, $450; taxes, $476.90; depreciation of property, $185.25. What amount of income tax should he pay?

3. John Davies receives a salary of $5000 per annum and has other income amounting to $2750. His wife has an income of $4500 from certain property. His general deductions amount to $585.40 and Mrs. Davies' general deductions amount to $375.80. They make a single return; how much should they pay?

CHAPTER XXIV

UNITED STATES CUSTOMS

Customs, or *duties*, are taxes upon commodities imported or exported. They are of two kinds, *ad valorem* and *specific.*

An **ad valorem** duty is a duty levied "according to value" (from *ad*, according to, and *valorem*, value). It is a certain per cent of the value of the goods in the country of origin.

Imported merchandise is appraised by the custom house officials, and the duty is based upon this appraisal of value. The value fixed is the price at which the merchandise is freely offered for sale in the country of origin, in usual wholesale quantities, on the date of shipment.

The date of shipment, in case of goods originating from an interior point, is the date of starting from that point. Thus, on goods originating in Switzerland and shipped from Antwerp, the price fixed would be the price in Switzerland at the time of leaving the point of origin.

Before imports are cleared, the importer "declares" their value to the custom house officials and pays a duty based on the declaration. The goods are then appraised and cleared if the declared value is O. K'd. by the appraisers. If not, further duties and penalties are imposed.

Specific duty is a duty levied according to quantity, as measured by the yard, pound, bushel, gallon, or other unit of measure.

The duty on a commodity may be ad valorem, or specific, or both.

A **tariff** is a list or schedule of rates of duties. The word *tariff* is also used to mean the duties themselves, hence is synonymous with *customs* and *duties.*

Tare is a deduction from the gross weight of a shipment to allow for the weight of the box, bottle, barrel, crate, or other container in which it was shipped.

Leakage and **breakage** are allowances for waste from leaking and breaking.

Goods can be cleared only at a "port of entry," where a government custom house is located. The goods imported must pass through the hands of the officials of the custom house, who assess and collect the duty. The coasts and borders of the United States are divided into districts each one of which has its port of entry.

Goods may enter the country at a point which is not a port of clearance but if so must be bonded at once (to a customs official) for clearance at a regular, port of entry.

Goods not for immediate delivery or "consumption" and upon which duty is not paid at the time of entry, may be stored in a bonded warehouse for a period of three years or less. A bonded warehouse is a warehouse the owner of which is under bonds to the government not to deliver the goods, except for export or to another bonded warehouse, until the duty is paid.

Invoices of goods imported are usually made out in the currency of the country from which they came. To figure these bills in United States money consult the tables of foreign money in the appendix.

The following table gives the rates upon some of the more common articles of imports, according to the Fordney-McCumber Tariff Act of 1922.

RATES ON SOME COMMON IMPORTS

ARTICLE	AD VALOREM	SPECIFIC
Fire Brick, plain	25%	
Lime		10¢ per 100 lb.
Marble		$1.00 per cu. ft.
Bar Iron		1½¢ per lb.
Aluminum Plates		9¢ per lb.
Aluminum Scrap or Alloys		5¢ per lb.
Pens, metallic		15¢ per gross
Zinc		2¼¢ per lb.
Maple Sugar and Syrup		4¢ per lb.
Railroad Ties, Telegraph Poles, etc.	33⅓%	
Cigars	25%	$4.50 per lb.
Horses and Mules	Valued over $150 20%	Valued to $150 $30 per head.
Butter and Substitutes		8¢ per lb.
*Hay		$4 per T.
Apples		25¢ per bu. (50 lb.)
Poultry, live		3¢ per lb.
Stockings, Hose, Half-hose, etc.	30%	
Linoleum, plain	35%	
Handkerchiefs (Linen)	45%	
Carpets	40%	
Silk Handkerchiefs	55%	
Ledger, Bond, Typewriter, etc., paper	25%	
Toys	70%	
Lead Pencils		45¢ per gross

*A custom house ton is 2240 pounds.

WRITTEN

Using the foregoing table find the duty on the following. In figuring ad valorem duty, reckon the duty on the nearest dollar of valuation. Treat 50¢ as $1.00.

1. A block of marble 3′ x 3′ 6″ x 8′.
2. A shipment of aluminum scrap weighing 8750 lb.
3. 550 gross of steel pens.
4. 980 gallons of maple syrup averaging 10.5 lb. per gallon.
5. 11,650 lb. of apples.
6. 1250 gross of lead pencils.
7. 32,480 lb. of hay.
8. 4 cases of cigars weighing $46\frac{3}{4}$ lb. each; value, $480.00.
9. 3420 lb. chickens; 2870 lb. geese, and 1747 lb. ducks.
10. 120 tubs butter, 50 lb. each.
11. $2\frac{1}{2}$ T. aluminum plates.
12. 3725 lb. of apples.
13. 17 horses valued at $160 each.
14. 1750 railroad ties at 65¢ each.
15. 25 M cigars at $21.60 per M.; weight, 520 lb.
16. 15 M fire brick at $31.20 per M.
17. 500 rm. of ledger paper at $1.24 per rm.
18. 3850 telephone poles at 90¢ each.
19. 570 tons of lime at $5.20 per T.
20. 8 tons zinc at 8¢ per lb.
21. 20 cases of cotton hose valued at £6 8s each.*
22. An importation of silk handkerchiefs that cost 437.5 francs.
23. Toys imported from France to the value of 3297 francs.
24. Carpet imported to the value of 7590 francs.
25. A man bought in Manchester 10,000 yd. cloth at 3 sh. per yd. less 3%. What import duty did he pay at New York at 35% ad valorem?

* A standard pound is $4.8665; shilling, $24\frac{1}{3}$c; franc, 19.2¢.

CHAPTER XXV

INSURANCE

Insurance is a contract by which one party, for a consideration, agrees to indemnify another party in case of loss arising from a stipulated cause, up to a specified amount.

The most common kinds of insurance are fire insurance, life insurance, and accident insurance. There are many other kinds, however, including marine insurance, plate glass insurance, live stock insurance, etc. An employer can be insured against loss on account of accidents to employees for whose safety he is responsible; a merchant can be insured against loss of profits on goods destroyed by fire or against loss of the use and occupancy of buildings or equipment. In short, insurance may be affected in almost any case where there is a risk, by the party having an insurable interest.

Fire Insurance

Fire insurance is a contract by which the insurer, for a consideration, agrees to indemnify the insured, up to an amount stipulated, against loss of property by fire.

"Loss" means *loss or damage*. "Loss by fire" also covers damage by water or by chemicals used in extinguishing the fire, or other damage done in extinguishing the fire.

The contract between insurer and insured is called a *policy* of insurance. The money paid by the insured, for the insurance, is called the *premium*.

The *rate* of insurance is usually expressed as a certain amount for each $100 of insurance, as $1.50 per hundred. It may, however, be expressed as a per cent; as, $1\frac{1}{2}\%$, meaning $1.50 per hundred.

A fire insurance policy covers only direct loss by fire, and not any indirect damages which the insured may contend that he has sustained, such as suspension of business activity. The loss must be to the property specified in the contract of insurance.

Losses will not be paid in excess of the sum insured. Losses will not be paid in excess of the interest of the insured, except by special agreement endorsed on the policy, to protect a mortgagee or other interested party or parties.

The insurance company is not liable, in case of total loss, beyond the *sound value* of the property. The sound value is the actual value on the day of the fire.

In no case is the company liable beyond what it would cost to replace or repair the property.

The company is liable for loss from fire caused by lightning (but not tornado) but not for damage from the lightning itself, except by special agreement.

If a party who has an insurable interest but who is not an owner, is to be protected, a clause extending the insurance to cover the additional interest must be embodied in the policy.

In case there are claims against persons other than the insurer, for damages on account of the fire, the insurer will exercise its right of subrogation—that is, that the insurer shall be substituted for the insured as the holder of such claims, so that the insured may not collect damages from two sources.

By special agreement the insured may be permitted to carry other insurance, to work overtime, to make alterations and repairs, to store gasoline, and to do other things for which permission is denied in the body of the policy. If other insurance is carried all insurers will bear the loss *pro rata*, as will be explained more fully later.

The "special agreements" referred to in the foregoing paragraphs are written on separate slips of paper called "riders" and attached to the policy. A rider may cover some feature not mentioned in the printed policy form, or it may be a waiver of some right of either party. The agreement on a rider supersedes any terms of the printed contract with which it conflicts.

The 80% co-insurance clause, which will be described more fully later, and other similar stipulations, if required, are attached as riders.

The foregoing particulars conform to the New York Standard Policy form, which is generally accepted, though subject in some states to statutory provisions not in effect in other states.

Rates. Rates of fire insurance are fixed by rating bodies. Sometimes these bodies are composed of representatives of the companies, as the *Chicago Board of Underwriters*, sometimes of persons appointed by the local or state government, sometimes of committees of both kinds, acting together either officially or in a mutually advisory capacity.

Rates are compiled by individuals or rating bureaus whose sole business is the measuring of fire hazards. Schedules of fire insurance rates compiled by such individuals or bureaus are often used.

The rates are separately determined for each building and depend upon physical condition, exposures, moral hazards, nature of occupancy, facilities for extinguishment of fire, and other related considerations.

WRITTEN

1. J. A. Cameron insured his house for \$3500 at 22¢; his barn for \$1000 at $27\frac{1}{2}$¢; and his garage for \$450 at 34¢. What did his insurance cost him?

2. The Otis Building Co. insured four buildings as follows: One for \$3000 at $32\frac{1}{2}$¢; one for \$5000 at 30¢; one for \$2500 at $32\frac{1}{2}$¢; and one for \$3600 at 33¢. What did this insurance cost?

3. Harris & Co. insured their building for \$10,000 for 3 years, the annual rate being 42¢, and took out a one-year policy for \$16,000 on mdse. in the same building. If the rate on merchandise was 25% higher than the rate on the building, and if the three-year rate was $2\frac{1}{2}$ times the yearly rate, what did their insurance cost them per year?

4. John Hoover insured his house for \$2400 and the contents for \$1600. Both policies were for three years, at $2\frac{1}{2}$ times the annual rate. The rate for contents was 25% greater than the rate on the house. If he paid \$33.00 for this insurance, what was the annual rate on the house?

Three-fourths Value Clause. Where provisions for protection from fire are inadequate, and when over-insurance is feared, insurance companies sometimes stipulate that the insurer shall in no event be liable for more than three-fourths of the sound value of each item of the property covered. What these items are is determined by the enumeration and valuation of each which is written in the body of the policy.

WRITTEN. In each of the following cases the three-fourths value clause was in effect. How much did the insured receive in each case?

1. Property worth \$4000 and insured for \$3000 was damaged \$3500.

2. Property worth \$4000 and insured for \$3000 was totally destroyed.

3. Property worth \$4000 and insured for \$3000 was damaged \$3000.

4. Property insured for \$3000 and worth \$4250 was found to be damaged \$500

5. Property worth $4000 and insured for $4000 was totally destroyed.

6. Property worth $3500 and insured for $4000 was damaged $3000.

7. Walton insured his barn and contents all under one policy, for $12,500 at an average rate of 48¢. The items insured, as enumerated in the policy, were: Barn, $5000; horses and cattle, $3500; wagons and harness, $2250; hay and grain, $1750. The barn caught fire and the losses were as follows: Barn, $1500; horses and cattle, $1250; wagons and harness, $2000; hay and grain, $1750. How much money did the insurance company owe Walton?

The Co-Insurance Clause. A large percentage of fire losses are only partial. If all policies were framed to permit the policy holder to collect in full for his partial loss, provided that loss were within the limits of the policy, the tendency would be for parties taking out insurance to take out only enough insurance to cover probable partial losses. In such a case, the policy holder would be insuring only a part of his whole property, but would be fully protected to all probable extent of damage.

To encourage property owners to carry enough insurance, companies offer reduced rates to apply to policies which contain a special clause called a co-insurance or contribution clause. The co-insurance clause most frequently used is the 80% co-insurance clause. It is an agreement upon the part of the insured that he will carry insurance equal to 80% of the value of the property insured. All of this 80% that is not placed with insurance companies the insured agrees to carry himself.

Thus an owner of property worth $10,000.00 agrees under the 80% co-insurance clause, that he will either carry $8000.00 worth of insurance or will himself be responsible for that part of the $8000.00 which he does not place with the insurance companies.

If this owner takes out only $6000.00 worth of insurance, he himself carries $2000.00, or one-fourth of the total of $8000.00 insurance agreed upon. In case of any loss he agrees to bear one-fourth of the loss. The company bears the remainder of the loss provided it does not exceed the face of the policy.

If this owner takes out $8000.00 insurance, or more, he is living up to his agreement fully and can collect the full amount of the loss, provided, of course, that it is not more than the amount of the policy.

WRITTEN. The policy in each of the first six problems following contains the 80% coinsurance clause. How much was due the insured in each case?

1. A property worth $30,000, insured for $20,000, was damaged $6,000.

2. A property worth $25,000, insured for $20,000, was damaged $3000.

3. Household goods worth $8000, insured for $5000, were damaged $2500.

4. Property worth $1000, insured for $800, was damaged $300.

5. A stock of goods worth $2000, insured for $1200, was damaged $500.

6. A barn worth $3000, insured for $2250, was damaged $750.

7. Barnes Bros. insured a stock of goods worth $11,000, for $9000 under a policy containing a 90% coinsurance clause. In case of a loss of $600, how much should they receive?

8. A policy containing a 90% coinsurance clause was issued for $4000 on a stock of goods worth $7500. How much should the company pay in case of a loss of $1200?

Risks Covered by Several Companies. A person may wish to insure his property in several companies, or a company may not wish to carry all the risk on a property. In such a case two or more policies, covering the same property, may be issued by different companies.

NOTE. Sometimes a single policy for a large amount will be issued by one company, which will then proceed, through its underwriting department, to reinsure the property in other companies for smaller amounts, thus dividing the risk. Such a case involves no problem, however, for the policy-holder.

When there are two or more insurers each dealing directly with the policy-holder, and a loss occurs, each company will contribute to the loss its *pro rata* share. The policy-holder cannot collect more for his loss than he would have been able to collect had all his insurance been placed with one company.

It is important that the written portions of all policies covering the same property should read exactly alike. If they are not concurrent, great confusion will result in case of adjustment of a fire loss.

WRITTEN

1. A stock of goods worth \$25,000 is insured in four companies, under policies containing the 80% coinsurance clause, for \$8000, \$5000, \$3000, and \$4000 respectively. If there is a fire loss of \$4000, how much will each company pay?

2. A stock of goods worth \$25,000 was insured for \$15,000 under policies containing the 80% coinsurance clause. A fire damaged the stock to the amount of \$1200. The policies were issued by the following companies: Aetna, \$5000; Albany, \$3000; Bankers and Merchants, \$3000; and Columbia Fire, \$4000. How much should each company contribute to the loss?

3. A building worth \$90,000 was insured for \$75,000, under policies containing the 90% coinsurance clause, in the following companies: Hartford Fire Ins. Co., \$15,000; Home Ins. Co., \$10,000; Merchants Ins. Co., \$25,000; National Ins. Co., \$25,000. Following a fire it was decided that the damage amounted to \$15,000. What amount should each company contribute?

4. A country merchant insured his stock of goods worth \$15,000 in several companies, each of which issued its policy containing the "three-fourths value clause." The policies were as follows: Eastern Fire, \$3000; German-American, \$3000; and Old Colony, \$4000. A fire destroyed goods worth \$800. How much should each company contribute?

5. A farmer insured his barn, which cost \$3000, for \$2500, under the provisions of the "three-fourths value clause." The Ohio Farmers Ins. Co. took a risk of \$1500 and the Reliance Ins. Co. took a risk of \$1000. Lightning set the barn on fire and the fire damaged it to the amount of \$2400. How much should each company contribute?

Cancellation of Insurance. A policy of insurance may be canceled by either the insured or the insurer.

The insurer must give five days notice of cancellation and must refund *pro rata* the unearned premium.

Thus, if a policy is issued for one year and the premium is paid, and the insurer should cancel the policy after 4 months, the insurer must refund the premium for the other 8 months. The *pro rata* basis would require the insurer to refund $\frac{8}{12}$ $(\frac{2}{3})$ of the premium for one year.

The insured need not give notice to the insurer in advance in canceling the policy. In this case the insurer does not refund unearned premium on a *pro rata* basis, but instead charges what is known as a "short rate" which is the rate the insured would have had to pay had he bought short time insurance at the beginning. The difference between the premium paid at first and the short rate charge, is refunded to the insured. Time is counted by exact days.

When insurance is canceled because of removal, and a new policy for the same amount taken out in the same company, short rates do not apply.

If a policy is canceled and a smaller policy issued on the same risk by the same company, short rates apply to the amount dropped but not to the amount continued.

The following is a table of short rate charges:

"Union" Short Rate Table for One-Year Policies

ALL POLICIES WRITTEN FOR A TERM OF LESS THAN ONE YEAR SHOULD BE CHARGED ACCORDING TO THE FOLLOWING TABLE

	Per Cent. of Annual Premium		Per Cent. of Annual Premium		Per Cent of Annual Prem.
1 day	2	18 days	16	95 days	42
2 days	4	19 days	16	105 days	45
3 days	5	20 days	17	108 days	46
4 days	6	25 days	19	120 days or 4 mo.	50
5 days	7	30 days	20	135 days	55
6 days	8	35 days	23	150 days or 5 mo.	60
7 days	9	40 days	26	165 days	65
8 days	9	45 days	27	180 days or 6 mo.	70
9 days	10	50 days	28	205 days	74
10 days	10	55 days	29	210 days or 7 mo.	75
11 days	11	60 days	30	215 days	76
12 days	12	65 days	33	240 days or 8 mo.	80
13 days	13	70 days	36	255 days	83
14 days	13	75 days	37	270 days or 9 mo.	85
15 days	14	80 days	38	288 days	88
16 days	14	85 days	39	300 days or 10 mo.	90
17 days	15	90 days or 3 mo.	40	345 days	98
				360 days or 12 mo.	100

Different short term rates apply if original term of insurance was three, four, or five years.

WRITTEN

1. On Aug. 18, 1925, I insured in the Royal Insurance Co. for $4500, for one year at 50¢. On Dec. 1, following, I canceled my policy and the company returned the unearned premium. How much did I receive?

2. Mr. Harris insured his goods in the Newark Fire Ins. Co., of New Jersey, on July 1, for $15,000 for 1 year at 33¢ per $100. On Oct. 29, following, he canceled his insurance. How much should be returned to Mr. Harris?

3. On April 1, Mr. Currie insured in a certain company for $3500 for one year, at an annual rate of 42¢. The company canceled the policy on Sept. 13, following, and sent him a check for the unearned premium. What was the amount sent?

4. On March 15 a policy was issued by the American National Ins. Co. of Ohio, for $9000, for one year at 38¢. The policy was canceled by the insured on July 28, following. How much premium was returned by the company?

5. An insurance company issued a policy for $15,000 for 1 year at 32¢ per $100. After 10 months, the company canceled the policy and returned the unearned premium. How much was returned?

6. On Jan. 1, I took out a policy for $3000 on my household goods, for 1 yr. at 45¢. On May 16 I canceled $1000 of this policy, and I canceled the remainder on Dec. 12. What did my insurance cost me for the year?

Open Policies. An open policy is one in which the amount of the risk is left open and changes from time to time as the property increases or decreases in total value. The changes in value are recorded in the company's Open Policy Book. Premiums are paid on the amounts shown by this policy book as having been at risk from time to time, and settlements in case of loss are made on the basis of the showing of the policy book at the time of the fire.

Whenever the total value diminishes, the amount of insurance is reduced accordingly, and the cost is figured, short rate, on the amount so canceled. Charges cannot be made except as goods are taken out. The amount canceled is always taken from that which has been longest standing.

WRITTEN

1. An open policy was issued on Feb. 1, 1927, to cover all goods while stored at a certain location. Rate, 75¢. On Feb. 1, 1928, the insurance then in force was terminated.

The list on the next page indicates the various articles stored by the insured, which are all treated as one subject of insurance. What was the total cost of the insurance?

IN		OUT	
DATE	AMOUNT	DATE	AMOUNT
Feb. 1	$10000	May 7	$10000
Mar. 15	15000	Sept. 11	8000
Mar. 20	17500	Oct. 11	17500
Apr. 19	12000	Dec. 15	5000
		Dec. 15	6000

Miscellaneous Problems

1. Smith insured a building worth $10,000, for $7000 under a policy containing an 80% coinsurance clause. The building was totally destroyed. What was the amount of the fire indemnity paid by the insurance company?

2. In problem 1, Brown held a $5000 mortgage at 6% against the property dated 6 mo. 15 da. before the fire and Smith's policy was so assigned as to protect Brown. The insurance company's draft for the amount of the indemnity was made payable jointly to Smith and Brown. To how much of it was each entitled?

3. White and Black each owned a building worth $5000. Each building was insured for $3000 under a policy containing an 80% coinsurance clause. White's policy contained a lightning clause but Black's did not. Both buildings were struck by lightning and set afire. The damage was $1000 by lightning and $1000 by fire in each case. How much did each receive from the company?

4. A man insures his furniture for $3000 for 1 year in a building where the annual rate on the contents is 60¢. After 5 mo. he moves the furniture to a building where the annual rate is 45¢, and insures for a 3-year term at $2\frac{1}{2}$ times the annual rate in a different company. How much less does his insurance cost him the second year than the first?

5. A man insures his stock of goods for $12,500 for 1 year in a building where the annual rate is 58¢. After 7 mo. he moves 40% of his stock to a building where the rate is 45¢ and 60% of it to a building where the rate is 35¢. He keeps the same amount of insurance, in the same company, taking two annual policies. By what per cent does he reduce his insurance cost?

6. On Jan. 1, Gray stores \$50,000 worth of cloth and insures it for \$40,000. He plans to keep it insured at $\frac{4}{5}$ of its value to the nearest thousand dollars. The rate is 38¢. Short rates apply to the part canceled. On Feb. 16 he uses \$10,000 worth of cloth; on Mar. 1, \$5000 worth, on May 15, \$20,000 worth; and the remainder on Sept. 28. What was his cost for insurance for the entire time?

7. On Jan. 1, I stored 60,000 lb. of paper worth 8¢ a pound. On Feb. 1, I used 10,000 lb., and the value of the paper had advanced 25%. On Mar. 1, I used 5000 lb., and the value of the paper had advanced 10%. On Apr. 1, I used 20,000 lb. Paper was then worth 9¢. On May 1, I used 15,000 lb. Paper was then worth $7\frac{1}{2}$¢. Balance was used on June 1. The insurance rate was 37¢. If I carried insurance to 80% of the value of the stock, on an open policy, what did my insurance cost me for this year? (Count time by months, in this problem.)

8. The Acme Furniture Company insured its lumber yards for \$100,000, for 1 year at 75¢; its buildings for \$70,000, for 3 yr. at $2\frac{1}{2}$ times the annual rate of 48¢; its machinery at \$40,000, for 3 yr., at $2\frac{1}{2}$ times the annual rate of 63¢; its unfinished and finished stock at \$60,000, for 1 year at 75¢. What was its cost for insurance for one year?

Life Insurance

Life Insurance is a contract by which the insurer, for a consideration, undertakes to pay a given sum upon the death of a person named therein (the insured) or after the lapse of a certain number of years, the insured being still living.

The consideration is usually a series of annual payments in advance, but an advance payment may be made for any number of years, or the entire consideration may be a single payment in advance.

The **beneficiary** is the person to whom the insurance is payable, as designated in the policy.

There are many kinds of insurance. The principal kinds are (1) *ordinary life*, also called *straight life*, or *life*, (2) *limited payment life*, and (3) *endowment*.

Ordinary Life. Under an ordinary life policy payments of premium are made annually in advance and the face of the policy is payable at death only.

The Northwestern Mutual Life Insurance Company

No. 900545

Age 31

In Consideration of the payment of

Twentythree and 95/100 Dollars,

the receipt whereof is hereby acknowledged, and of the annual payment of a like sum to the said Company, on or before the 7th day of November in every year during the lifetime of

Oscar L. Thomas

of CHICAGO, ILLINOIS (hereinafter called the Insured), promises, upon receipt of due proof of the death of the Insured, to pay at its Home Office unto his wife, Eva M. Thomas

Beneficiary

the sum of One Thousand Dollars, less any unpaid premium or premiums for the then current policy year and any other indebtedness on account of this Policy; provided, however, that if there be no Beneficiary or Contingent Beneficiary surviving the Insured, such payment unless otherwise directed by the Insured and endorsed by the Company on this Policy shall be made to the executors, administrators or assigns of the said Insured.

Subject to the Rights of any Assignee and With or Without Reserving the Right of Revocation, the Insured, (1) may designate a Beneficiary or Beneficiaries if none be named in this Policy, or in the event of the death of any person designated; (2) and may designate a Contingent Beneficiary or Beneficiaries whose interest shall be as expressed in, or by endorsement of the Company on, this Policy; (3) and may change any Beneficiary or Contingent Beneficiary not irrevocably designated. If there be more than one Beneficiary the interest of any deceased Beneficiary shall pass to the survivor or survivors unless otherwise directed by the Insured and endorsed by the Company on this Policy. No designation, direction, revocation or change shall be effective unless duly made in writing, and filed at the Home Office of the Company (accompanied by the Policy for suitable endorsement) prior to or at the time this Policy shall become payable.

No Assignment of this Policy shall be binding upon the Company until it be filed with the Company at its Home Office. The Company assumes no responsibility as to the validity of any assignment, and satisfactory proof of assignee's interest must be produced on making claim.

This Policy is issued and accepted by the parties in interest subject to the provisions stated on the second and third pages hereof which are a part of this contract.

In Witness Whereof, THE NORTHWESTERN MUTUAL LIFE INSURANCE COMPANY, of Milwaukee, Wisconsin, has by its President and Secretary executed this contract, this seventh day of November, one thousand nine hundred and eleven.

Asst'l Secretary.

Geo. C. Markham
President.

WHOLE LIFE POLICY.
PREMIUMS PAYABLE FOR LIFE.
DIVIDENDS PAYABLE ANNUALLY.
V. L.

Limited Payment Life. Under this form of policy payments of premium are made annually for a stipulated number of years only; as 10, 20, or 30. Twenty-payment life is probably the most common of the limited payment policies. The face of the policy is payable at death, whether before or after the expiration of the specified period.

The term *whole life policy* includes both *ordinary life* and *limited payment* life.

Endowment. Under this form of policy the method of paying premiums is the same as under the limited payment policy, payments being completed in a given number of years. Ten- and twenty-year endowment policies are both common. The face of the policy, however, under this plan, is payable at the end of the period specified in the contract, if the insured is then living. If the insured dies before the end of the specified period, the face of the policy is payable at once, and premium payments cease.

Term Insurance is that form of life insurance whereby a life is insured for a given term of years or until the insured reaches a given age. The payments are usually made annually. It has no value beyond the period of time for which the premium is paid. In this respect it is similar to fire insurance.

ANNUAL PREMIUM OF $1000 OF INSURANCE

Age	Ordinary Life	10 Payment Life	20 Payment Life	10 Year Endowment	20 Year Endowment	Age
21	$18.40	$46.30	$28.25	$101.78	$48.08	21
22	18.80	47.00	28.69	102.01	48.17	22
23	19.23	47.73	29.15	102.24	48.25	23
24	19.67	48.47	29.63	102.49	48.35	24
25	20.14	49.24	30.12	102.73	48.46	25
26	20.63	50.04	30.63	102.85	48.58	26
27	21.15	50.87	31.16	102.93	48.69	27
28	21.69	51.72	31.71	103.03	48.83	28
29	22.26	52.61	32.28	103.14	48.96	29
30	22.85	53.52	32.87	103.25	49.12	30
31	23.48	54.46	33.49	103.37	49.28	31
32	24.14	55.44	34.13	103.50	49.46	32
33	24.84	56.45	34.80	103.65	49.66	33
34	25.58	57.50	35.49	103.79	49.87	34
35	26.35	58.58	36.22	103.95	50.11	35

Assessment Insurance is a form of life insurance under which the survivors of a mutual association are assessed to pay the claims for deaths of members, and the expenses of the organization.

There are many other forms of life insurance and many different kinds of policies issued under each.

Premiums. The amount paid as annual premium on a life insurance policy depends upon the age at which the policy was taken out and the kind of insurance bought. On the preceding page is a table of annual premiums issued by one of the leading companies of the United States. The heading *Age* means the age of the insured at the time the policy was taken out. Compare the premiums on policies taken out at 23 years of age, for the different kinds of policy.

WRITTEN. Find the annual premium on each of the following policies according to the table of premiums given.

1. Ordinary life, $2,000, age 21.
2. Ordinary life, $5,000, age 25.
3. 10 payment life, $3,500, age 23.
4. 20 payment life, $5,000, age 27.
5. Ordinary life, $10,000, age 26.
6. 10 yr. endowment, $2,500, age 30.
7. 20 payment life, $6,000, age 29.
8. 20 yr. endowment, $8000, age 22.
9. 10 payment life, $20,000, age 35.
10. Straight life, $12,000, age 33.
11. 20 yr. endowment, $50,000 age 31.
12. 10 payment life, $10,000, age 32.
13. 20 payment life, $25,000, age 24.
14. 10 yr. endowment, $15,000, age 34.
15. Whole life, $8,500, age 28.
16. 10 yr. endowment, $20,000, age 35.

17. A young man takes out an ordinary life policy for $2000 when 24 years of age and pays the premium according to the table given. Making no allowance for interest or annual distributions of the company, how much will he have paid the company when he is 35 years of age, including the premium for his 35th year?

Theory of Life Insurance. All persons do not live to the same age. There is an average length of life, however, and there is an average death rate at each age. Compilers of insurance statistics and tables, called *actuaries*, prepare tables of mortality and tables of expectation of life at various ages, and upon these statistics they figure the premiums which the company should charge for insurance. The companies, of course, expect to lose in individual cases where the insured dies "before his time," but they collect additional premiums from those who live longer than the average.

Reserves. Under the forms of insurance shown in the table of premiums, the insurer sets aside a part of each premium. This reserve is invested and allowed to accumulate to meet claims.

If the insured ceases to pay his premiums, he can receive from the insurer that part of its reserve which comes from his payments. The amount which he would thus receive is called the *cash surrender value* of his policy. Instead of the money, the insured at his option, may accept from the company whatever amount of fully paid up insurance the money will buy, or he may arrange that the full face of his policy will remain in force for whatever period of time the money will pay for.

Loan Value. A policy-holder can borrow on his policy. The amount which he can borrow, called the loan value of the policy, is in some companies as great as the cash surrender value.

In most states there is some degree of regulation of life insurance companies by law, particularly in regard to the reserves to be created. The laws of some states require that statements concerning the cash surrender value, the loan value, the amount of paid up insurance its reserve will purchase, and the extension of the entire policy its reserve will purchase, be included in the policy itself, and be a part of the contract of insurance.

Dividends. Some companies make their policy-holders participators in their earnings. In such a case, the policy-holder, when he receives notice of premium due, also receives notice of the amount of his share of the previous year's earnings. This dividend may be (1) withdrawn in cash, (2) applied to the payment of premiums, (3) used to purchase paid-up insurance, (4) left at interest.

From the foregoing it is clear that money invested in life insurance cannot be entirely forfeited even by failure to continue premium payments. The following table of non-forfeiture provisions shows the three alternatives which the policy-holder is offered by one well-known company in case of non-payment of premiums: (1) the time for which the policy will be extended, (2) the amount of paid-up insurance which he can get, (3) the cash surrender or loan value. Note that in this case the cash surrender value and the loan value are the same; this is not always true. The values shown are supposed to be based on the reserve. The table is not complete.

WRITTEN. (Use table of lapsed policy values and table of premiums given in this book.)

1. At the age of 25, a young man took a 20-payment life policy for $2500. He made five payments and discontinued. For how

VALUES OF A LAPSED POLICY FOR $1000

	End of What Year	Insurance Issued at Age 23 Years: Extension	Paid up	Cash or Loan	Insurance Issued at Age 25 Years: Extension	Paid up	Cash or Loan	Insurance Issued at Age 30 Years: Extension	Paid up	Cash or Loan	End of What Year
Ordinary Life	3rd	3:82*	$ 68	$24.61	3:159	$ 71	$26.61	3:358	$ 80	$32.45	3rd
	4th	4:146	91	33.34	4:249	95	36.04	5:144	106	43.92	4th
	5th	5:228	113	42.33	5:355	119	45.76	6:300	133	55.73	5th
	6th	6:326	136	51.61	7:111	142	55.77	8:93	159	67.90	6th
	7th	8:73	156	61.17	8:240	166	66.09	9:240	185	80.41	7th
	8th	9:193	181	71.02	10:6	189	76.72	10:363	211	93.28	8th
	9th	10:315	204	81.17	11:134	212	87.67	12:81	236	106.50	9th
	10th	12:70	226	91.64	12:245	236	98.94	13:115	261	120.10	10th
20-Pay. Life	3rd	7:135	147	53.15	7:194	148	55.31	7:307	151	61.29	3rd
	4th	10:98	196	72.13	10:167	198	75.06	10:265	201	83.14	4th
	5th	13:134	246	91.78	13:195	248	95.49	13:222	252	105.74	5th
	6th	16:216	296	112.13	16:241	297	116.64	16:115	302	129.13	6th
	7th	19:286	345	133.19	19:238	347	138.54	18:263	352	153.30	7th
	8th	22:264	395	154.99	22:121	397	161.20	20:293	403	178.32	8th
	9th	25:109	445	177.57	24:238	448	184.66	22:213	453	204.18	9th
	10th	27:186	495	200.95	26:233	498	208.95	24:41	503	230.94	10th
20-Year Endowment	3rd	16:209	171	107.19	16:11	170	107.19	14:184	170	107.24	3rd
	4th	16:	226	145.59	16:	226	145.58	16:	225	145.62	4th
	5th	15:	280	185.41	15:	280	185.39	15:	279	185.40	5th
	6th	14:	334	226.71	14:	334	226.67	14:	333	226.66	6th
	7th	13:	387	269.54	13:	387	269.49	13:	386	269.43	7th
	8th	12:	439	313.98	12:	439	313.91	12:	438	313.79	8th
	9th	11:	491	360.08	11:	490	359.98	11:	489	359.79	9th
	10th	10:	541	407.91	10:	541	407.79	10:	540	407.51	10th

* 3:82 means 3 years and 82 days.

long would the company have been willing to extend his insurance? For what amount would paid up insurance be issued?

2. If the young man in the first problem had taken in cash the surrender value of his policy, how much less cash would he have received than the amount he paid in?

3. What would be your answers to problems 1 and 2 had the policy been an ordinary life policy? a 20-year endowment policy?

4. At the age of 33, Jones borrowed all he could on the following policies. How much did he borrow?

$2000.00 ordinary life, taken at the age of 23.
$3000.00 20-payment life, taken at the age of 25.
$2500.00 20-year endowment, taken at the age of 30.
$1000.00 20-payment life, taken at the age of 23.
$1500.00 ordinary life, taken at the age of 25.

5. At the age of 23, Jones took a 20-payment life policy for $5000, in a company in which the loan value was 95% of the amount of loan value shown by the table. Nine years later, he borrowed all he could, and the following year was compelled to surrender his policy. How much cash did he receive at the end of the tenth year, all the premiums being paid?

6. The annual "term rate" of the company whose rates are given in this book is $10.95, at age 25. If a man at 25 takes a policy of term insurance for $5000, and another man of the same age takes an ordinary life policy, how much will each have paid to the insurance company in eight payments, if no allowance is made for interest? Considering the cash surrender value of the ordinary life policy, which policy has cost the more, and how much?

7. On May 7, 1928, I borrowed on an ordinary life policy for $2000 taken out on Dec. 5, 1919, at the age of 23, its entire cash surrender value. The company charged for interest at 5%. When my next premium fell due I paid it, together with the loan and interest on the loan. How much did I pay?

8. At 22, I took out a 10-yr. endowment policy for $3000. Annual dividends averaged $5.61 a year for each thousand dollars of the policy. At the end of the ten years did I receive more or less than I had paid in, and how much?

9. A man 32 years old had his life insured for $3000, ordinary life policy. He died after making thirteen payments. How much more did his beneficiary receive than he had paid in?

10. A life insurance company had ledger assets valued at $64,036,850. The interest received during the year averaged 4.63% on the ledger assets of the company. How much income does the company receive from interest?

11. A life insurance company received total premiums amounting to $9,091,420.23 in one year. The total amount of insurance

outstanding was $274,450,000. What was the average amount received by the company per $1000 of insurance carried?

12. The insurance company in problem 11 had expenses in one year amounting to $1,853,539.29. What was the expense per $1000 of insurance outstanding?

13. An insurance company paid, during one year, $2,664,410 for death claims. The death claims paid were 55.44% of the expected death loss estimated by the company. What amount of death claims had the company provided for?

14. If the company in problem 13 was carrying half a billion dollars in insurance, and all the saving on death claims was returned to the policy-holders in dividends, what amount would be returned to the holder of a thousand-dollar policy?

15. The total assets of an insurance company amount to $65,437,929.15. The reserve of the same company amounts to $57,545,644.65. What per cent of the assets are held in the reserve?

CHAPTER XXVI

COSTS

Costs, Manufacturing Business

Costs, in a manufacturing business, include *Manufacturing Cost, Shipping Cost, Selling Cost,* and *General Expense.* (See Fig. 1.)

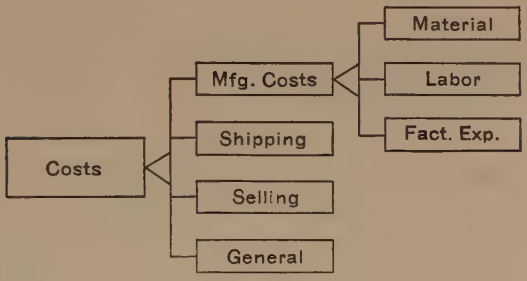


Fig. 1.

Manufacturing Costs include *Materials, Labor,* and *Factory Expense.* (See Fig. 1.) Each one of these includes direct costs and indirect costs. (See Fig. 2.)

Direct Costs (see Fig. 2) are the materials, productive labor, and direct expense consumed in manufacture, and chargeable specifically to some unit of manufacture.

Indirect Cost (or overhead) is the cost of materials, labor, and expense not so chargeable.

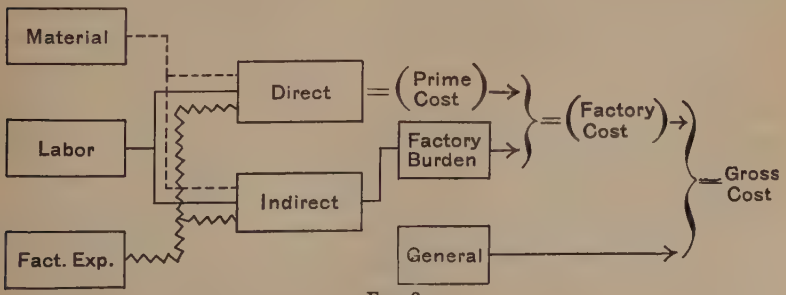


Fig. 2.

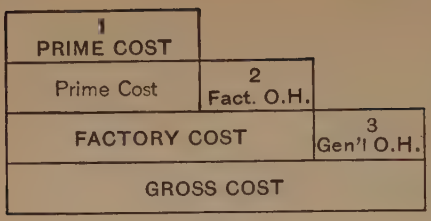


Fig. 3.

The Prime Cost of any unit of manufacture is the cost of materials and labor charged directly to it. The *Factory Cost*, or production cost, is the prime cost plus factory overhead. The *Gross Cost* is the factory cost plus a burden for general overhead expense. (See Figs. 2 and 3.)

Fixed Factory Charges, including such items as light and heat, and power, can be charged directly to the departments, in whole or in part, each department being charged with its fair proportion and the balance being charged to General Expense.

Written

1. The materials used in an article of manufacture cost \$43.56; labor, $3\frac{1}{2}$ hours at 55¢; factory expense, both direct and indirect, 7¢. What was the total manufacturing cost? (See Fig. 1.)

2. It cost \$1.47 to ship the article mentioned in problem 1, and \$2.15 to sell it. General overhead chargeable to the article, $12\frac{1}{2}\%$ of its manufacturing cost. What was its gross cost? (See Fig. 1.)

3. A factory spent in a month \$12,000 for materials, \$7000 for factory payroll, and \$235.60 for sundry factory expenses. Of this, there were charged directly to jobs \$11,500 for material, \$6450 for labor, and \$120 for factory expense. The remainder was factory burden. What was the burden? (See Fig. 2.)

4. If direct charges and indirect charges were divided as in problem 3, and the general expense was \$750, what was the total prime cost? factory cost? gross cost? (See Fig. 2.)

5. In problems 3 and 4, what per cent was added to prime cost for factory burden? What per cent was added to factory cost for general overhead?

6. The prime cost of manufacturing an article was \$24.50, to which 15% was added for factory burden. What was the factory cost? (See Fig. 3.)

7. The prime cost of an article was \$42.76, to which 20% was added for factory burden. Factory cost was increased 10% to cover general overhead. What was the gross cost? (See Fig. 3.)

Distribution of Overhead

Overhead expenses must be distributed in some fair proportion among the units of manufacture. In the following pages four well-known methods are briefly treated.

Direct Labor Method

Under this method the burden is distributed over labor costs. The rate of burden is the ratio of the overhead charge to the total of direct labor.

EXAMPLE. Find the factory cost of an article if the materials used cost \$40 and the direct labor \$55, factory overhead for the month being \$800 and total direct labor \$4000.

\$800 ÷ \$4000 = 20%, rate of burden
\$55 × .20 = \$11. , burden on one article
\$40 + \$55 + \$11 = \$106, factory cost

WRITTEN

1. A certain factory has been in operation for one year and during that time has paid \$15,000 for productive labor and \$3750 for factory overhead. What is its rate of factory burden to be added to direct labor cost?

2. If the material for a certain job cost \$8.50 and labor \$4.60 what would be the production cost if the work was done in the factory in problem 1? What price must be charged to make a profit of 40% on factory cost?

3. The Crescent Mfg. Co.'s labor pay roll for March was \$3750; and its factory overhead, \$975.00. The cost card for a certain job showed material used, \$82.94; and labor, 32 hr. at 40¢, 42 hr. at 50¢, and 18 hr. at $66\frac{2}{3}$¢. What was the factory cost of this job?

4. A shop order was issued to produce a certain number of articles. The material purchased for this order amounted to \$397.50. The rate for distributing factory overhead was to be determined from the previous month which showed productive labor \$10,380.40 and factory overhead \$4257.28. According to the time cards the following labor was consumed by this order: $147\frac{1}{2}$ hr. @ 40¢, $147\frac{1}{2}$ hr. @ $62\frac{1}{4}$¢, 18 hr. @ $57\frac{1}{2}$¢, and 4 hr. @ 50¢.

Material valued at $23.70 was returned to the stores at the completion of the work. Find the factory cost.

5. The Bartlett Gear Co. received an order for a number of gears. The material on hand to begin the job amounted to $137.85. During the period of manufacture $359.47 was expended for more material for this job. The cost sheet showed labor on this job as follows: 214 hr. at $37\frac{1}{2}$¢, $221\frac{1}{2}$ hr. at 55¢, 184 hr. at $62\frac{1}{2}$¢, 59 hr. at $56\frac{1}{4}$¢. Material valued at $113.87 was returned to the stores. Direct labor total was $11,492.57 and factory overhead $4139.94. Find the factory cost.

Direct Labor Hour Method

Under this method, the overhead expense is distributed among the different jobs in proportion to the time spent on each job in direct labor.

EXAMPLE. The number of hours of direct labor in a factory for one month was 5000. The factory overhead for the same period amounted to $500. A job required $35 worth of raw material, and direct labor as follows: 28 hr. at 50¢, 10 hr. at 65¢, and 4 hr. at 75¢. What was the factory cost of the job?

$500 ÷ 5000 = 10¢, amt. of burden for every hour of labor.

28 hr. at 50¢ =	$14.00	
10 hr. at 65¢ =	6.50	
4 hr. at 75¢ =	3.00	
42	$23.50	**direct labor**
42 hr. at 10¢ =	4.20	burden
	35.00	material
	$62.70	**factory cost**

WRITTEN

time sheets of a certain factory showed that productive employed 3500 hours in one month and that total overhe[illegible] the same period **amounted to $315.** What was the overhead burden that month for each hour of labor? The material and labor for a job cost as follows: Material, $7.43; labor, 35 hr. at $33\frac{1}{3}$¢ and 11 hr. at $66\frac{2}{3}$¢. What was the gross cost of production?

2. The time sheets of the Leader Mfg. Co. for September showed the following hours for each man employed at productive labor:

*Note that here factory and general overhead are combined.

SUMMARY FOR SEPTEMBER, 1919

Employee No.	1	2	3	4	5	8	9	10	11	12	14	15	16	17	18	19	21	22	23	24	25	26	28	29	30
100	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
101	8	8	8	8	8	8	8	0	8	8	8	8	8	8	8	8	8	7	8	8	8	8	8	8	8
102	8	8	8	8	0	8	8	8	8	8	8	8	8	8	8	8	8	8	6	8	8	8	8	8	8
103	8	8	8	8	8	5	8	8	8	8	8	8	8	8	8	8	8	8	4	8	8	8	8	8	8
105	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
108	8	8	6	8	8	4	8	8	8	7	5	8	8	8	8	8	0	4	8	8	8	8	8	8	8
109	6	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
110	8	8	0	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
111	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
112	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
113	8	7	4	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
114	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8

The total overhead for September amounted to $381.54. The material for a certain piece of work cost $11.83 and the following time was spent on it: Workman No. 105, $18\frac{1}{2}$ hr. at 50¢; No. 111, 5 hr. at 75¢; No. 114, 13 hr. at 60¢. The work has been delivered to the customer and is to be billed at a profit of 20% on gross cost of production. What is the amount of the bill?

3. During July the Portland Tool Co. placed its entire plant at the disposal of the Acme Motor Co. Materials on hand July 1, $237.45. Purchases during July, $357.83. Productive labor charge for the month, $312.98. Overhead for July, $138.57. Unused material July 31, $85. The work was billed at a profit of 12% on gross cost. What was the amount of the bill against the Acme Motor Co.?

4. The Winston-Spencer Machine Shop did some experimental work for a customer at cost. They purchased special material amounting to $21.35 and used from stock material amounting to $38.27. The labor record showed 31 hr. at $33\frac{1}{3}$¢, 45 hr. at $72\frac{1}{2}$¢, and 38 hr. at $55\frac{5}{9}$¢ spent on this work. The labor record of the entire shop showed 8470 hours of productive labor. The overhead was $532.65. There was special material unused, valued at $8.75, at the completion of the work. For how much should the bill be rendered?

5. Miller Bros. Tool Co., a general job shop, agreed to machine 2000 castings, thread each, and drill two holes in each, at 8¢ per casting. All material was to be furnished by the customer. The labor record showed the following time on this job: $67\frac{1}{2}$ hr. at 40¢; 78 hr. at $38\frac{4}{9}$¢; 74 hr. at 50¢. The records for the entire shop showed that the number of hours of productive labor for the month was 3058 and there was a departmental overhead charge of $398.70. What was the cost of the job, the average factory cost per casting, the profit over factory cost on the entire job, and the per cent of profit over factory cost?

Prime Cost Method

Under this method, overhead charges are distributed to the different jobs or articles in the ratio which the prime costs of the different jobs or articles bear to the total prime costs.

EXAMPLE. During one month, a factory paid $2000 for productive labor and used material that cost $1500. The overhead expense for that month amounted to $875. The material for one desk cost $18 and the direct labor on it was 17 hr. at 75¢ per hour. What was the gross cost of production of the desk?

$2000 + $1500 = $3500 = total prime cost.
$875 ÷ $3500 = 25%, rate of burden.

$18.00 = cost of material for the desk.
$12.75 = cost of productive labor on the desk.

$30.75 = prime cost of the desk.
$ 7.69 = overhead charges (25% of $30.75).

$38.44 = gross cost of production.

WRITTEN

1. The material used in one month by the Ideal Manufacturing Company cost $6392.85; amount paid for productive labor was $3570, of which $422.80 was for work done the preceding month (accrued productive labor); and overhead amounted to $932.90. The cost sheets for a certain job showed that requisitions had been honored for material amounting to $39.87, and that labor on the job was as follows: 71 hours at $33\frac{1}{3}$¢; 68 hours at 60¢; and $41\frac{1}{2}$ hours at 80¢. Unused material valued at $8.40 was returned to the stores. Find the gross cost of the job.

2. Some experimental work was done in the above shop on a "time and material plus 10%" basis. That is, the customer was to pay for material and for time spent on the work and to this was to be added 10% for profit. According to the rate of burden for overhead established in the above problem, did the factory gain or lose, and how much if material cost $11.80 and labor $17.35?

3. The following is a cost sheet of the Monarch Mfg. Co.

COST SHEET **SHOP ORDER #1637**

Requisitions for Material			Labor			Labor			Overhead
Date	No.	Amount	Hr.	Rate	Cost	Hr.	Rate	Cost	
1920									
July 10	742	$110.85	46	33½					
15	807	19.43	59	40					
			51	50					
			63	62½					
Materials Returned to Stores		$----------							

Summary		Remarks
Material	----------	
Labor	----------	
Overhead	----------	
Production Cost	----------	

Costs for the entire shop for the month were: Materials, $3287.27; productive labor, $6274.93; and overhead, $1145.37. Make a form similar to the above and show on it the cost of production of shop order No. 1637.

4. The Carter Tool Co. uses a tag system to follow the different jobs through the shop. On the next page is a representation of the tag from a certain job that has just been completed.

The material used in this shop during the month amounted to $3728.45, the productive labor was $1431.72, and departmental overhead amounted to $650.94. What is the factory cost of order No. 14 — 732 if the material used amounted to $81.65?

It is billed to the customer at a profit of 60% on factory cost. What is the amount of the bill?

CARTER TOOL CO.

SHOP ORDER #14--732

Workman's Number	Process	Hours	Rate
78	Grinding	110	33⅓c
81	Drilling Punching	87 1/2	50c
83	Threading Milling	81	40c
87	Work with hand tools	44	88 4/9c
90	Inspection	5	60c

DATE COMPLETED Aug. 31, 1921

FOREMAN .....

Machine Rate Method

Under this method a rate per hour of use is fixed upon for each machine, this rate being large enough to include all of the overhead charges.

To determine this rate, many elements must be considered; as, (a) Space occupied; (b) value; (c) rate of depreciation from wear and tear; (d) amount spent for repairs; (e) power used; (f) light and heat used; (g) oil and other supplies consumed; (h) indirect labor required; (i) supervision required; (j) time unused; and so on. These elements must be considered for each machine, and the cost items for each machine considered in their relation to the total cost of similar items for all machines.

WRITTEN

1. Miller and Long operate a machine shop containing four machines exactly alike, and a work bench, each occupying the same space. Their overhead charges for one year were: Rent, $480; insurance, $80; depreciation, $240; repairs, $100; power, light, and heat, $275; taxes, $50; general expense, $750. On rent and on general expense the work bench bears its fifth of the total burden,

but on the other items the cost of operating the work bench is only $\frac{1}{3}$ of the cost of operating a single machine. What is the cost per hour to operate one machine, and to maintain the work bench, if all are used full time 280 days per year and 8 hours per day?

2. Miller and Long did work which required material valued at \$11.38, 18 hours labor at 60¢, and 31 hours labor at $66\frac{2}{3}$¢. Of the 49 hours labor, 42 hours were spent at the machines and 7 hours at the bench. What was the factory cost of the work? If billed at prime cost plus 50%, what was the profit?

3. In one department of a factory were 20 sewing machines, all alike. The rate of overhead burden for March was based upon the overhead charges for the preceding year, which were:

Foreman and other unproductive labor	\$1380
Its share of depreciation of building	210
Depreciation of its machinery	240
Its share of light and heat bill	100
Power used in department	600
Taxes and insurance (prorated)	73
Office overhead (prorated)	360

The factory operates on an 8-hour day basis 288 days each year. Materials on hand Mar. 1, \$295.50; purchased during March, \$3002.30; on hand Mar. 31, \$185.80. Each machine was operated 26 full days. Labor payroll, \$1289.60. What was the value of the partly finished product passed on to the next department?

4. During April, five of the machines mentioned in problem 3 were idle and the power charge was reduced proportionately. Other charges remained the same. What was the new rate of burden, if the remaining machines were operated 26 full days?

5. On May 1, five new machines were added, making a total of 25 machines. All machines were operated 26 full days. The changes made increased proportionately the charges for depreciation of machinery, power, and taxes and insurance, but did not affect other overhead expenses. What was the new rate of burden?

Departmental Costs

The Gross Cost of each unit of manufacture is ascertained by dividing the general expenses among the departments, then

dividing each departmental overhead, including its share of general expense, among the units of manufacture in the department, and adding this total overhead to the prime cost per unit.

WRITTEN

1. The expenses of a manufacturing business during 1918 were:

Manufacturing	$105,000.00	Selling	$17,665.00
Shipping	8,750.00	General	12,960.00

An article cost $12.75 to manufacture. If overhead is distributed in proportion to manufacturing cost, what must be added to cover shipping expense? To cover selling expense? To cover general expense? What must be the price to net 10% profit on total cost?

2. The business referred to in problem 1 has four departments, in which the sales for 1918 were as follows:

A	$31,628.00	C	$56,470.41
B	44,729.10	D	25,984.99

If the selling expenses were charged to the different departments in proportion to sales, how much was charged to each?

3. If the shipping expenses in problem 1 were charged to the four departments in proportion to sales, how much was charged to each?

4. If the general expenses in problem 1 were charged to the four departments in proportion to sales, how much was charged to each?

5. Taking into consideration the distribution of costs in problems 2, 3, and 4, what was the gross cost of an article costing $12.75 to manufacture, in Dept. A, if mfg. costs in Dept. A were $19,604.15?

6. The direct manufacturing costs in the four departments mentioned in problem 2 were:

A	$19,687.50	C	$32,812.50
B	29,523.46	D	22,976.54

How much burden must be borne by each department, if the three items of overhead be distributed in the ratio which the direct manufacturing costs bear to each other? What is the rate in each case (carried to hundredths of 1%)?

7. What must be the selling price of an article costing $12.75, if manufacturing cost and overhead are as stated in problem 1, to net 10% profit on total cost?

8. What must be the selling price of an article costing $12.75, if manufacturing cost and overhead are as suggested in problem 1, to net 10% profit on sale price?

9. The estimated expenses of a factory for 1928 and the distribution of these into three accounts were as follows:

	Total	Distribution		
		Shipping	Selling	General
Building Expense	$ 1,200.00	$ 600.00	.....	$ 600.00
Labor	1,500.00	1,500.00	.....	.....
Salaries	13,200.00	.....	$9,600.00	3,600.00
Officers' Salaries	4,800.00	.....	.....	4,800.00
Commissions	2,500.00	.....	2,500.00	.....
Advertising	1,500.00	.....	1,500.00	.....
Insurance	192.00	12.00	.....	180.00
Taxes	215.00	15.00	.....	200.00
Depreciation	200.00	100.00	.....	100.00
Repairs	120.00	50.00	.....	70.00
Delivery Expense	2,300.00	.....	.....	2,300.00
Discount on Sales	1,200.00	.....	.....	1,200.00
Reserve for Bad Debts	750.00	.....	.....	750.00
Miscellaneous Expense	325.00	75.00	100.00	150.00
	$30,002.00	$2,352.00	$13,700.00	$13,950.00

If the estimated production cost of completed work is $120,000.00, how much should be added to the production cost of Job No. 218 ($562.50) to cover shipping expense? selling expense? general expense?

What is the gross cost of Job 218?

10. The general overhead expenses in a factory during 1917 and the distribution of these among the three departments were as shown by the table at the top of the next page.

Using the sames rate of distribution for each item prepare a table showing the distribution for 1928, if the general overhead expenses were: Indirect Labor, $8400.00; Building Expense, $4250.00; Power, $9120.00; Insurance, $530.00; Taxes, $500.00; Depreciation, $2735.00; Repairs, $5625.00; General Factory Expense, $5150.00; Miscellaneous Supplies, $910.00; Miscellaneous Expense, $1950.00.

	Total	Dept. A	Dept. B	Dept. C
Indirect Labor	$7,300.00	$2,500.00	$3,000.00	$1,800.00
Building Expense	3,900.00	1,200.00	1,500.00	1,200.00
Power	8,560.00	2,800.00	3,960.00	1,800.00
Insurance	420.00	144.00	192.00	84.00
Taxes	525.00	180.00	240.00	105.00
Depreciation	2,625.00	900.00	1,200.00	525.00
Repairs	5,400.00	1,500.00	2,700.00	1,200.00
General Factory Expense	4,950.00	1,650.00	2,100.00	1,200.00
Miscellaneous Supplies	790.00	240.00	360.00	190.00
Miscellaneous Expense	1,818.00	720.00	900.00	198.00
	$36,288.00	$11,834.00	$16,152.00	$8,302.00

Problems from an Engraving Business

Assume an engraving business having an art department, an etching department, and a foundry. Expenses for Jan., 1928, were:

	Prime Costs		Departmental Overhead Chgd. in Jan.
	Materials	Labor	
Art Department	$ 97.35	$ 707.50	$111.52
Etching Department	535.05	1092.88	282.25
Foundry	485.26	1327.83	446.54

To be Distributed: Office Salaries, $750.00; Rent, $420.00; Light and Heat, $173.50; Shipping Expense, $165.00; General Expense, $875.26.

(The problems following are based on the foregoing figures.)

1. *Distribution of Rent.* The art department occupied 400 sq. ft.; the etching department, 600 sq. ft.; the foundry, 800 sq. ft.; and the general offices, halls, and other unproductive space 600 sq. ft. How much rental was charged to each department, and how much to general expense, on the basis of space occupied?

2. *Distribution of Light and Heat.* This charge was distributed on the basis of past experience as follows: 40% to the foundry, 30% to the etching dept., 15% to the art dept., and 15% to general expense. How much was charged to each?

3. *Distribution of Shipping Expense.* This item was distributed in proportion to sales, which were as follows: Art dept., $1750.00; etching dept., $3000.00; foundry, $3500.00. How much was distributed to each department?

4. *Distribution of general overhead.* Office Salaries, General Expense, and the balances of Rent and Light & Heat not charged directly to the departments, were distributed on the basis of total departmental charges after all other distributions were made. How much was charged to each department?

5. The direct costs entered on several job tickets of the engraving business mentioned in the preceding problems and the prices received for the jobs are as follows:

Job No.	Dept.	Materials	Labor	Dept. O. H.	Genl. O. H.	Prices Received
243	Art	$ 0.45	$ 2.25	.....	.....	$ 5.50
244	Etching	5.00	12.50	.....	.....	35.00
245	Foundry	8.75	7.50	.....	.....	34.50
246	Etching	7.50	18.75	.....	.....	52.25
247	Foundry	122.50	75.00	.....	.....	365.00

Fill the spaces for overhead, basing the rates on prime costs.

6. In problem 5, what was the prime cost, the factory cost, the gross cost, and the profit on each job?

7. In Job No. 247, what was the rate of burden for departmental overhead? For general overhead? What was the rate of profit on prime cost? on factory cost? on gross cost? on sale price?

Problems in Farm Cost

1. A kerosene 15-horsepower tractor engine pulling 5 plows will plow 16 acres in a day of ten hours. For the engine the farmer figures per day: Wages, two men, $3.25 each; kerosene, 30 gallons at 10¢; oil, 50¢; depreciation of machine, $\frac{1}{5}\%$ of cost; depreciation on tools, 50¢; interest, 4% per annum. A man and two horses will plow 3 acres in the same time. Man labor is figured at 31.47 cents per hour; horse labor at 15.36 cents each per hour; interest and depreciation on tools is estimated at 8¢ per day. What is the acre cost of plowing in each manner if the tractor costs $1750.00?

2.

	Field A, 30 A	Field B, 40 A	Field C, 50 A
1918	Corn	Clover	Oats
1919	Oats	Corn	Timothy and Clover
1920	Clover	Oats	Corn

A farmer has three fields, A, B, and C, cropped as above. In the winter of 1927-1928, field A was coated with manure at the rate of 8 tons per acre, cost of hauling 55¢ per ton; and 300 lb. rock phosphate per acre at $8.00 per ton, hauling $1.70 per ton. The value of manure was $1.25 per ton at the barn. Field B was coated at the rate of 9 tons of manure per acre; hauling, 50¢ a ton; and Field C with 200 lb. nitrate of soda per acre, at $50 per ton; hauling, $1.65 per ton.

Assume that 50% of the value of the manure is exhausted the first year, 25% the second, 15% the third, and 10% the fourth; that 20% of the value of the rock phosphate becomes available each year; and that the nitrate of soda is exhausted with one crop. What should be charged to each field for fertilizer for 1918, 1919, and 1920?

3. In the month of August, the labor sheet of Smith, a farm hand, showed work on the following: Cattle, 31 hours; horses, 15 hours; hogs, 30 hours; repairs to buildings, 12 hours; field A, 60 hours; field B, 42 hours; field C, 38 hours; field D, 92 hours. His wages were $32 and his board was estimated at $15.50. Determine his cost per hour. What was the labor charge for each field for the month?

Review Problems

1. To make a piano, I used materials costing $42.63 and paid workmen $25.60. What was the prime cost of the piano?

2. The outlay of Department A of the Acme Manufacturing Co. during January was $2475.60 for material and $2325.79 for direct labor. Indirect labor amounted to $460.60. The cost of indirect labor was what per cent of the cost of direct labor? Of prime cost?

3. In a factory the cost of direct labor was $8000 and the indirect expenses for the same period were $7000. If the labor on an

article was 5 hours at 62¢ and the cost of material was 60¢, what was the gross cost of the article after distributing the indirect expenses according to the cost of direct labor?

4. In a factory the number of working hours of direct labor was 28,000 and the indirect expense was $9000. If labor on a certain article was 6 hours at 60¢ and the material 75¢, what was the cost of the article after distributing the indirect expense according to the number of hours of direct labor?

5. A factory pays its 24 men by what is known as the Gantt system, otherwise known as the "Task work with bonus" plan. Each workman receives a regular wage of $4.65 a day but if he does a certain amount of work he receives a bonus of 25%. Five men get nothing but the regular wage; 19 men are entitled to the bonus. What is the amount of the week's payroll?

6. Cost of materials used in Department B of the Blank Manufacturing Co. during January, $3260.00; direct labor, $2472.69; indirect labor, $640.25. If the cost of indirect labor was charged against the units of product in amounts proportionate to the amounts of direct labor charged against them, how much did this add to the cost of an article against which was charged 12 hours' direct labor at 45¢ an hour and 20 hours' direct labor at $17\frac{1}{2}$¢?

7. The report of Department C of the Jones Manufacturing Co. for January showed: Materials used, $2950.62; direct labor, $1456.29; indirect labor, $624.72; other departmental overhead, $114.27. If the two items of overhead were charged against the units of product in proportion to prime cost, how much did this add to the cost of an article against which $12.50 had been charged for materials and direct labor?

8. Departmental overhead charges in Department D for March, 1918, were $562.50. Prorating this among its three units of manufacture in proportion to prime cost, find the departmental cost, per unit, of each of the articles manufactured in that department:

	Materials	Direct labor	Dept. O. H.	Dept. Cost per Unit
1474 hubs.....	$372.46	$129.64	.....	.....
1526 sets of spokes.	726.43	220.00	.....	.....
1392 rims.....	493.26	270.35	.....	.....

9. The Eckhart Manufacturing Co.'s manufacturing statement for April, 1928, showed the following prime costs and departmental overhead charges. Find the per cent to be added for departmental overhead charges in each department, if prorated on the basis of departmental prime costs.

	Prime Costs		Department Overhead		
	Materials	Direct labor	Indirect labor	Misc. Dept. Exp.	Rate
Dept. A	$3246.29	$1234.66	$340.27	$110.60	.....
Dept. B	2750.65	1526.72	249.63	92.73	.....
Dept. C	3127.50	1839.26	275.44	54.69	.....
Dept. D	1826.40	927.48	264.38	75.86	.....

10. The general overhead charges shown in the manufacturing statement of Keith and Williams for May, 1928, amount to $1560.60. These are distributed as charges against the three departments of the business in the proportion which the three total departmental costs bear to each other. The total departmental costs are: Department A, $4235.60; Department B, $2962.75; Department C, $3276.29. What per cent of the general overhead charges must be borne by each of the three departments?

11. The departmental costs, per unit, for iron tires, manufactured in Department C of Keith and Williams' factory (see the preceding problem), amount to $1.62 per set of four. Adding to the departmental cost its burden for general overhead charges, find the total production cost for one set of tires.

12. During May, 1927, costs of material, costs of labor, and departmental overhead charges for the four departments of my business were as shown by the following table. Find the per cent that must be added in each department for departmental overhead charges, if distributed in proportion to prime costs, and the departmental cost per unit. Then find the per cent that must be added to the departmental cost per unit to give the total production cost per unit, if the total general overhead charge, $762.50, was distributed in proportion to total departmental costs. Also find total cost per unit for each department.

Dept.	Units M'f'd	Materials	Labor	Departmental Overhead			General Overhead	
				Amount	%	Dept. cost per unit	%	Per unit
A	964	$456.29	$247.23	$ 57.50	.....	.....	.....	.....
B	875	324.84	429.60	63.25	.....	.....	.....	.....
C	920	675.20	324.65	125.60	.....	.....	.....	.....
D	892	126.95	732.50	100.00	.....	.....	.....	.....

13. Styler and Co. maintain a book printing department, a color printing department, and a binding department, designated respectively departments A, B, and C. Find the production cost, per unit, of 5124 copies of *Smith's History of New Zealand*, from the following data, if the outputs of Depts. A and B are delivered to Dept. C at cost plus $17\frac{1}{2}\%$ for overhead: Dept. A: Cost of paper, $202.50; ink, $18.75; printing, workmen's time, 16 hr. @ $1.25, 20 hr. @ $1.10, and 20 hr. @ 60¢; machine time, 18 hr. @ $4.00. Dept. B: Paper, $48.50; ink, $15.20; printing, $104.60. The records of Dept. C show the following items in addition to the charges from Depts. A and B: Materials used in binding, stamping and wrapping, $450.60; machine time, 14 hr. @ $4.00; workmen's time, 44 hr. @ $1.05, $57\frac{1}{2}$ hr. @ 50¢. Add $11\frac{1}{9}\%$ for overhead in Dept. C.

14. A factory spends $100,000 a year to manufacture its product, $40,000 a year to promote it, and $20,000 for expenses of running the business. If the manufacturing cost of a single unit of its product is $26\frac{1}{2}$¢, what must it ask for this article, retail, if it gives 20% discount to dealers, and wishes to make a net profit of $12\frac{1}{2}\%$ of total cost, including promotion and expense of doing business, on its sales to dealers?

15. The retail price of a certain school book is 65¢, from which a discount of 20% is given to dealers. It costs $17\frac{3}{8}$¢ per book to manufacture this book, to which must be added the cost of promotion and current expenses. If the publishers annual expenditures are $50,000 for manufacturing, $18,000 for promotion, and $12,000 for other current expenses, what must be figured as the gross cost of each book if overhead is distributed in proportion to manufacturing cost?

CHAPTER XXVII

VOCATIONAL PROBLEMS

Shipping and Transportation

In shipping merchandise, the question which most frequently presents itself is whether it is cheapest to ship by freight, express, or parcel post.

Each method of shipment has its advantages and disadvantages from the standpoints of speed, security, and other points relating to the service given by the carrier, and sometimes one of these will be the principal factor in deciding how to ship.

Parcel Post. The parcel post has been treated at the end of Chapter III. The student should review *Postal Rates* at this time.

Freight. Commodities are classified as First Class, Second Class, Third Class, etc., and different rates given for each. Different rates are given for carload lots (C.L.) and for less than carload lots (L.C.L.). Each railroad publishes lists, called tariffs, of rates from each of the cities on its line to every other city. To determine in advance what the freight charges will be, it is necessary to know, first, to what class the goods shipped belong; second, the rate on that class from shipping point to destination. Except for coal, and other commodities shipped by the ton, the rate is quoted per 100 lb. Less than 100 lb. is charged as 100 lb.

In problem 1 following, all merchandise is first class, L.C.L., and the rate is stated for each package.

Express. There are 295 different scales of express rates in use in the United States, each designated by a number. To find what the express charges will be, the shipper must know: 1st, whether his goods are first, second, or third class; 2d, what scale of rates applies between the point of shipment and the destination (determined by consulting a Directory of Express Stations); and 3d, what is the rate, on this scale, for packages of the weight and class of his shipment. The express rate is also affected by the valuation. A shipper can sometimes get a reduced rate by agreeing to a low valuation in case of loss.

In the problem following, all packages are first class. The weight and scale number are given, and the student will consult the table herewith to find the rate.

SCHEDULE OF EXPRESS RATES BY SCALE NUMBERS
(First Class)

Pounds	Scale Numbers							
	2	8	10	13	22	26	38	70
1	.21	.21	.21	.21	.22	.22	.23	.24
2	.21	.21	.22	.22	.23	.23	.24	.28
5	.22	.23	.24	.25	.27	.28	.31	.39
10	.24	.27	.28	.29	.34	.36	.42	.58
15	.26	.30	.32	.34	.41	.44	.53	.77
20	.28	.34	.36	.39	.48	.52	.64	.96
25	.30	.37	.40	.44	.55	.60	.75	1.15
50	.40	.55	.60	.67	.90	1.00	1.30	2.10
100	.60	.90	1.00	1.15	1.60	1.80	2.40	4.00

WRITTEN

1. A Chicago merchant had to ship the following packages:

10 lb. to Indianapolis, Ind.
15 lb. to Milwaukee, Wis.
160 lb. to Ashtabula, Ohio
215 lb. to Buffalo, N. Y.
50 lb. to Toledo, Ohio
25 lb. to New York, N. Y.
20 lb. to Milwaukee, Wis.
1 lb. to Indianapolis, Ind.
20 lb. to Toledo, Ohio
2 lb. to Buffalo, N. Y.

Freight rates: To Indianapolis, 33.1¢; to Milwaukee, 30¢; to Ashtabula, 46.2¢; to Buffalo, 47.3¢; to Toledo, 38.9¢; to New York, 90¢. Expense of boxing, 60¢ each for the two large packages, 27¢ each for the others. Hauling to freight depot, 10¢ per box.

Express rates: The scale applying for shipments to Indianapolis, is 10; to Milwaukee, 8; to Toledo, 13; to Ashtabula, 22; to Buffalo, 26; to New York, 38. The cost of materials for wrapping is $4\frac{1}{2}$c a package, and the two large shipments would each require 3 packages. Figure no hauling charge.

Parcel Post: Indianapolis is in the 2d zone from Chicago; Milwaukee in the 2d; Toledo in the 3d; Ashtabula in the 4th; Buffalo in the 4th; and New York in the 5th. Expense of wrapping same as for express packages. Hauling to post office, 15¢ a hundred pounds.

Prepare a table showing the cost of each package by each way of shipping. If the merchant shipped cheapest way, how was each

package shipped, and what was his total expense for boxing or wrapping, shipping, and hauling?

2. A shipping clerk bought a reel of No. 6 Jute rope weighing 50 lb. and containing 5044 yd. of rope, at 20¢ a pound; a ream of kraft wrapping stock weighing 50 lb., at $8.75 per cwt.; and a ream bundle of rag lining stock weighing 150 lb., at $2.75 per cwt. He uses 1 sheet kraft, 3 sheets rag, and 16 ft. rope to wrap one package. He receives a salary of $40.00 a week and his assistant receives $25.00 a week. A stenographer whose salary is $22.00 for a week of 44 hours devotes 3 hours a week to addressing labels. Incidental expenses of shipping room, $5.00 a week. Including interest at 6% on an investment of $300.00, what is the shipping room expense per package if 550 packages, all like the foregoing, are shipped in one week?

3. The freight rate on books from Chicago to Nashville, Tenn., is 98¢ a hundred; to Jacksonville, Fla., $1.76 a hundred. If a box worth 60¢ holds 300 books and weighs 500 lb. packed for shipment, what is the boxing and shipping expense per book, to Nashville? to Jacksonville?

4. The freight rate on 475 bu. wheat shipped from Billings, Mont., to Chicago, was $2.63 a hundred. Drayage was $17.50. What was the transportation cost per bu.? (See table of statutory weights.) If the wheat cost $1.75 per bu. at Billings, what was the gross cost at Chicago?

5. I have 17,000 lb. of household goods to ship from St. Louis to Denver, Colo. The minimum weight of a carload of this commodity is 20,000 lb. If the carload rate is 1.07\frac{1}{2}$ per 100 lb. and the L. C. L. rate is $2.20 per 100 lb., which is cheaper, to take the whole car or ship L. C. L., and how much?

6. Five men combined their shipments from St. Louis to Topeka and thus secured a carload rate of 65¢. A shipped 3500 lb.; B, 12,000 lb.; C, 11,500 lb.; D, 6750 lb.; E, 15,000 lb. How much did each shipper save, if the L. C. L. rate was 97¢?

7. Gallagher and Asher make up cars from Chicago to other cities, securing the C. L. rate and charging their customers a 15% advance thereon. If the rate from Chicago to New York is 67¢ C. L., and $1.01 L. C. L., how much can I save by shipping 6000 lb. in a Gallagher and Asher car?

Written Test.

The following examination in arithmetic was given for many years by a well-known railroad, to applicants for employment. Time yourself on this work. Remember that the work must be neatly done and every result must be correct. There is no such thing as "95% accuracy" in the railroad business. Your results will be either right or wrong.

A. Find the amount due for transportation of given number of pounds at rate, and place the sum in proper column opposite each item. Make footings of weight and revenue.

Weight	Rate Per 100 lb.	Amount	
75,416	$0.33		
49,275	.47		
14,762	1.19		
16,843	4.36		
35,994	2.19½		
27,514	3.47		
38,710	2.76		
92,997	3.04		
17,133	6.73		
86,542	1.11		
34,222	.14		
87,475	.72		
76,227	.26¾		
42,999	.07		

B.

49407 lb. coal @ 67 per gross ton ..... $.....

1,747.01 gross (long) tons coal @ $1.14 per net (short) ton ..... $.....

Total ..... $

C. Find the amounts of revenue due the Central Railroad and the Reading Railroad in proportion to the number of miles for each road. Make footings of total amount and of each company's proportion.

Amount	Miles		Proportion	
	Central	Reading	Central	Reading
$4,848.48	46	185	.....	.....
1,238.77	177	120	.....	.....
3,375.26	216	510	.....	.....
2,789.04	300	111	.....	.....
5,004.10	409	837	.....	.....

D

453 miles = ... % of total mileage

819 miles = ... % of total mileage

222 miles = ... % of total mileage

....

Time .....

Signature .....

Address .....

Paper

The sizes of paper principally used are:

Cap	14″ x 17″	Double Cap	17″ x 28″
Demi	16″ x 21″	Double Demi	21″ x 32″
Folio	17″ x 22″	Double Folio	22″ x 34″
Royal	19″ x 24″	Double Royal	24″ x 38″

Book papers: 24″ x 36″, 28″ x 42″, 25″ x 38″, 32″ x 44″, etc.

A ream of paper, except for some fancy papers, is 500 sheets. The weight is indicated with the size as in the expression: 32 x 44 —100, which means paper 32 inches wide, 44 inches long, and weighing 100 lb. to the ream. The abbreviation M means 1000.

WRITTEN.

1. Ordinary typewriting stock measures $8\frac{1}{2}$ x 11 and cuts out of folio. How many reams of folio are required for 3 M sheets of typewriter size?

Suggestion.—Find how many small sheets cut out of one large sheet by cancellation, thus: $\frac{17 \times 22}{8\frac{1}{2} \times 11} = 4$.

2. How many reams of $8 \times 10\frac{1}{2}$ writing paper will cut out of $4\frac{1}{4}$ reams demi?

3. I manufactured 1 M pads of $8 \times 10\frac{1}{2}$ penmanship paper, 80 sheets to the pad. What did these cost me per pad, if I bought double demi writing stock 40 lb. to the ream at 15¢, had it ruled at a cost of $1.50 per ream before cutting, and paid $\frac{3}{5}$¢ per tab for cutting and tabbing?

4. John H. Stehman manufactured 1000 blank ledgers containing 32 pages each. The size of the books when closed was $8\frac{1}{2}$ x 14. For inside stock he used 32 lb. double cap writing stock at $14\frac{1}{2}$¢ a pound. For covers he used 28 x 34—110 cover stock at $10\frac{1}{4}$¢ a pound. He paid $12.50 for ruling, and $7.25 for printing and binding. Adding to stock $\frac{1}{10}$ for waste, how much did the books cost?

5. How many sheets $6\frac{1}{3}$ x $12\frac{1}{2}$ can be cut out of a sheet 25 x 38?

6. What is the best size to use for an 8-page circular measuring $3\frac{1}{6}$ x 6 when folded?

7. How many pounds of paper shall I buy to make 5 M 256-page books, if one sheet of 32 x 44—100 will make 64 pages. Allow $\frac{1}{10}$ extra for waste.

8. What weight in a 40 x 60 sheet is equivalent to 95 lb. in a 32 x 44 sheet?

9. For 6 years Jones & Co. had been using 44 x 64—210 book paper costing $6\frac{1}{2}¢$ a pound, when it was discovered that by careful handling $42\frac{1}{2}$ x $62\frac{1}{2}$ stock could be used. They used 512 reams a year. How much did they save by using the smaller size stock in an equivalent weight? How much had they lost in the six years?

10. A wholesaler buys 10 rm. of bond paper (500 count) 17 x 22—20 at 9.5¢ per lb. He cuts it into letter heads $8\frac{1}{2}$ x 11 and sells it, 500 sheets in a box, for $16.20 per doxen boxes. If cutting the stock costs $2.50 and the boxes cost $2\frac{1}{2}¢$ each, what is his profit or loss?

11. How much 28 x 42 paper stock will be needed for 15 M. 16-page circulars $5\frac{1}{4}''$ x 7''? If the paper weighs 80 lb. to the ream, and the price is $9.80 a hundred, what will it cost?

12. What weight is equivalent to 20 lb. folio (to the nearest $\frac{1}{2}$ lb.) in royal? in demi? in double cap? Give also the equivalents of 24 lb. folio in the same three sizes.

Smoke Inspection Problems

In large and well-kept cities, it is the business of the smoke inspection department to rid the city of heavy black smoke. This it accomplishes largely through the education of the smoke makers. Instead of being satisfied with arresting and fining violators of anti-smoke ordinances, the inspectors call upon them and show them that they can save money by the adoption of devices which will provide for burning all of the coal, eliminating the waste which otherwise goes up in smoke.

WRITTEN.

1. A power plant burned 8 tons of Illinois mine run, costing $6.45 a ton, every 24 hours. Thirteen per cent of the coal was ash. The smoke department recommended improvements costing $597.35, after adopting which the plant consumed $5\frac{1}{2}$ tons of Illinois Screenings, at $2.95, every 24 hours. The per cent of ash is approximately the same in both kinds of coal. If hauling ashes cost $2.00 a ton, in how many days would the cost of the improvements be saved? What would be the saving over the cost of the improvements, in one year of 365 days? What per cent did the

owner realize the first year on the cost of his investment in improvements?

2. An apartment building was heated with a common fire box boiler which consumed 108 tons of anthracite coal, costing \$15.50 a ton, in one season. A coal agent recommended semi-bituminous (Pocahontas) coal, guaranteeing that 135 tons at \$13.25 would heat the building for one season. The smoke department recommended that the old boiler be sold for its junk value (\$43.50) and a modern down-draft boiler be purchased at a cost of \$534, this boiler to consume 160 tons of Illinois coal per season, at \$6.10. Pocahontas coal is $6\frac{1}{2}$ per cent ash, and Illinois coal 11 per cent ash. Cost of hauling ashes, \$4.00 a ton. Assuming that both recommendations were correct as to the facts stated, which plan would be the more economical, and how much, in three seasons?

3. The plant of the Harms Mfg. Company, which consumes $4\frac{1}{2}$ tons of Illinois washed screenings costing \$6.45 a ton, was visited by the Smoke Inspector, who informed the owner that on account of his plant making too much smoke it would be necessary for him either to reconstruct his furnace or burn Pocahontas coal. Should he decide to burn Pocahontas coal he would burn $3\frac{3}{4}$ tons of coal a day at a cost of \$13.25 a ton. Should he decide to reconstruct his furnace the cost of reconstruction would be \$81.25. With the new furnace he could operate his plant on 4 tons of Illinois unwashed screenings a day at a cost of \$2.95 a ton. Per cent of ash and cost of hauling ashes same as in problem 2. Which would be the more economical plan to follow and what would be the saving over the present plan in one year of 300 days?

4. A power plant consumed 32 tons of Illinois screenings in 24 hours at a cost of \$2.95 a ton. This plant was visited by the smoke inspector who recommended reconstruction to cost as follows:

Smoke-stack	\$2173.50
New breeching	396.74
Dampers and damper plate	243.75
Stokers and furnaces	2947.24

After making the above changes the plant was operated on $25\frac{3}{4}$ tons of the same fuel at the same cost per ton. The per cent of ash was reduced from $14\frac{1}{2}$ to $11\frac{1}{4}$. Cost of hauling ashes, \$4.00 a ton. How long did it take to save the cost of the reconstruction?

5. The Acme Company's plant contained a small water heater in which $\frac{1}{4}$ ton of Pocahontas coal was burned each day, which coal cost \$13.25 a ton. There was also burned in this heater 235 pounds of waste paper each day. The owner was advised that on account of burning the paper in the heater he was not getting the full value of his coal and was further advised to secure a paper press which would cost \$12.75. The waste paper could be sold for 35¢ per 100 pounds and the baling cost 10¢ a ton for the wire used. After securing the paper press the heater was operated on 450 lb. of coal a day. The per cent of ash was reduced from $14\frac{1}{2}$ to $11\frac{1}{4}$. Cost of hauling ashes, \$4.00 a ton. How many days would it take to pay for the outfit and what would be the net profit in one year of 300 days?

U. S. Food and Fuel Administration

The following problems are suggested by data furnished by the U. S. Food and Fuel Administration Bureaus. They relate to the conservation and price regulation of household necessities.

WRITTEN

1. In 1915, a coal dealer paid \$1.90 at the mine for Wayne County, Iowa, Mine Run coal and sold it at \$2.75 retail. In 1917 the U. S. Fuel Administration Bureau fixed the price of this coal at \$3.15 at the mine, the retailer to sell it at a margin of gross profit 30% greater than the gross profit in 1915. What was the retail price un 1917?

2. If 15,000,000 householders use 20% of the coal consumed annually in the U. S., and 250,000 firemen use 60% of it in power plants and railroads, how many tons should the average fireman save to be equivalent to a saving of 1 bu. of 80 lb. by the average janitor?

3. In September, 1917, the consumption of sugar per capita was 21 lb. in France and 90 lb. in the U. S. During October the U. S. government permitted the exportation to France of 100,000 tons of sugar in addition to its regular exportation. Other conditions in both countries remaining the same, how much did each person in the U. S. have to cut down his sugar consumption and how much did the per capita consumption in France increase, if the

population of this country was 110,000,000 and that of France was 40,000,000, and the additional supply was for 3 months?

4. The consumption of sugar per capita in France was what per cent of the consumption per capita in this country before October 1, 1917? After October 1?

5. In 1917, the sugar refiners of the U. S. agreed to fix a selling price for their product which would give them the same average profit as was made by them during the five years before the European War.

Find what margin of advance over pre-war price the refiners asked per 100 lb., based upon the following facts:

Sugar loses 7% in refining, and the advance in cost of raw sugar was 2¢ per pound; advances in cost of bags, jute, cotton, and other materials increased the cost of refining 17¢ per hundred; advances in cost of coal and labor increased the manufacturing cost .13¢ per pound; interest, insurance, etc., on increased investment added .019¢ per pound; the pre-war profit was 8.41 mills per pound.

6. If the gross cost to the refiner was $7\frac{1}{4}$¢ per pound of refined sugar, during the period covered by the above agreement, and the sugar was distributed to retailers through jobbers whose expenses were $\frac{1}{2}$¢ per pound and who netted a profit of 15% on gross cost, what did the ultimate consumer have to pay to give the grocer a profit of 10% gross on his cost?

7. Average annual exports of butter for the three years before the war were worth $1,072,896. During the fiscal year ending June 30, 1917, they were worth $8,749,170 and our European Allies received 78.1% of them as against 16.26% received by the same countries before the war. By what per cent did our butter trade with the Allies increase?

8. Statistics compiled from several leading cities of the U. S. show that during August, 1917, 6.6% less garbage was collected than during August of 1916. There was a decrease of 29.2% in the amount of grease contained in the garbage, and also a decrease of 15% in the richness of the grease. If these statistics indicate conditions that were general, by what per cent was waste from this source cut down throughout the country?

Farm Problems

1. A farmer has two 40-acre fields. One is 80 rd. square and the other is 40 rd. wide and 160 rd. long. If the turns are not counted how far will a team travel in plowing each field with a plow turning 16 in. of land?

2. If it takes $\frac{3}{4}$ minute to each turn how much longer will it take to plow one field than the other, going the long way?

3. Two thirty-acre fields of oats are 100 rods and 20 rods respectively from the barnyard where the oats are to be stacked. If there are three loads to the acre, how many more miles will be traveled in hauling the oats from one field than from the other?

4. If the team average 3 miles an hour in the above problem how many more hours of horse and man labor are used in the hauling from the first field than from the second?

5. A field is 40 rd. wide and 80 rd. long. If a man can make 36 trips across this field and back, the long way, in one day with a plow turning a furrow 16 in. wide, how many days will be required to plow the field? How many acres can the man plow in one day?

6. A harrow is 10 ft. wide. If a team can pull it 15 miles in a day how many acres will it cover in that day?

7. A tile is 12 in. long. What will the tile cost, at \$23.50 per thousand, to lay 160 rods of ditch?

8. A field is 40 rd. by 80 rd. If rows of corn are planted the long way, and 3 ft. 8 in. apart, how many rows will there be? If the hills of corn are 3′ 6″ apart in the rows, how many hills will there be in each row?

9. Shipments of 738 eight-gallon cans of milk were made in one month. The railroad company charged \$125.46 for hauling. What was the cost per quart for freight?

10. A man buys seed corn for \$4.00 per bushel. If 5 bushels will plant 40 acres, what is the cost for one acre? How many pints are used for one acre?

11. A fruit grower set out trees in a square 40-acre field, planting as many trees in the field as possible without getting any tree closer to any other tree than 50 ft. He set the trees in rows, 50 ft. apart in the rows. The trees of each odd row he set 25 ft. away from points in the row directly opposite the trees in the even rows. How far apart were the rows? How many trees were set in the field?

Miscellaneous Trading and Industrial Problems

1. A feed store bought 10 tons of shorts from a mill at \$16.80 per T. The labor for hauling and storing amounted to \$8.75. At what price per cwt. must it be sold to make a profit of $\frac{1}{5}$¢ per lb.?

2. A grocer bought a barrel of vinegar containing $30\frac{3}{4}$ gal., for \$5.60. He estimates a waste and leakage of $\frac{1}{2}$ pt. per gal. What will be his profit on this barrel if the vinegar is sold at 7¢ per quart?

3. A coal dealer bought 20 T. of coal at \$6.40 per ton. He retailed it at 35¢ per bu. of 80 lb. What was his gross profit per ton?

4. A foundry bought pig iron at \$10.50 per ton and from this iron made castings weighing $2\frac{1}{2}$ lb. each at an expense of \$55.60 for labor. If there is a loss of $\frac{1}{10}$ in melting and casting, and the castings are sold at 15¢ each, what is the gross profit?

5. The United States half dollar contains 173.61 grains of pure silver. In a certain year the state of Georgia produced 50 lb. (Troy) of silver. How many half dollars could be coined from this amount of metal?

6. If silver has a commercial value of 54¢ per oz. how much greater is the coin value of the silver in the preceding problem than the commercial value?

7. A grocer bought 5 bu. cherries at \$2.15 per bu. and retailed them at 12¢ per quart. By mistake he used a liquid quart measure instead of a dry quart measure. What was his profit? How much would his profit have been had he used the proper measure?

8. A vending machine company collected in one week 35,620 one cent pieces. The one cent pieces of the United States weigh 48 grains (Troy) each, and are $\frac{19}{20}$ copper. If copper is worth 14¢ per lb. (avoirdupois) what is the difference between the coin value and the commercial value of the copper contained in these one-cent pieces?

CHAPTER XXVIII

COMPOUND INTEREST

Compound interest is interest computed at stated intervals upon principal and unpaid interest.

The interest periods, or periods of time at the end of which interest is computed, are usually annual, semi-annual, or quarterly, though other periods may be used.

Under a strict construction of the term "compound interest," it is assumed that interest accruals are credited at regular intervals, but that no interest is paid until the end of the time for which the obligation draws compound interest. The compound interest tables are all compiled upon this basis. Payments of part of the interest might be made, leaving a balance of unpaid interest to itself draw interest, but problems involving such transactions are not strictly within the range of compound interest as generally understood.

Compound interest is generally paid by savings banks and other institutions accepting deposits of a reasonably permanent character.

There is no law against the *payment* of compound interest by the party who owes, but generally speaking the law will not lend its aid to the *collection* of compound interest. In most states penalties are imposed for attempting to collect it.

EXAMPLE. What will $500 amount to in 2 yr. 4 mo., at 6% interest compounded semi-annually?

Principal	$500.00
Int. for 6 mo. at 6%	15.00
	$515.00
Int. for 2d 6 mo. at 6%	15.45
	$530.45
Int. for 3d 6 mo. at 6%	15.91
	$546.36
Int. for 4th 6 mo. at 6%	16.39
	$562.75
Int. for last 4 mo. at 6%	11.26
Amt. at end of 2 yr. 4 mo.	$574.01

The interest is the difference between the amount and the original principal. In this case $574.01 − $500. = $74.01, compound interest.

WRITTEN. Find the amounts in the following problems without the use of the table:

1. $150 at 4% compounded quarterly for 2 yr.

2. $300 at 5% compounded semi-annually for 1 yr. 6 mo.

3. $500 at 5% compounded annually from Jan. 1, 1916, to July 1, 1919.

4. $3000 at 8% compounded semi-annually from Mar. 16, 1918, to Dec. 5, 1921.

5. George deposited $200 in a savings bank on Jan. 1, 1918, at 4% interest compounded semi-annually. How much did he withdraw if he took it all out on July 1, 1919?

6. What will $1500 amount to in two years with interest at 6% compounded quarterly?

How to Use the Compound Interest Table

First find the amount of $1.00 for the given number of periods, at the rate of interest for the period. Then multiply by the number of dollars in the principal.

If the interest is compounded annually, the number of periods (at left) is the number of years in the time, and the rate (at top) is the annual rate of interest.

If the interest is compounded semi-annually, the number of periods (at left) is double the number of years in the time, and the rate (at top) is half the annual rate of interest.

If the interest is compounded quarterly, the number of periods (at left) is four times the number of years in the time, and the rate (at top) is one fourth the annual rate of interest.

If the number of periods exceeds 50, find the amount for 50 periods and with this as a new principal find the amount for the remainder of the time.

Problems in Use of the Table

1. Find the interest on $570 at 8% per annum compounded quarterly for 6 years.

2. What will be the amount of $2000 at the end of 10 years if interest is compounded semi-annually at 5% per annum?

3. Find the interest on $10,000 at 4% compounded annually for 4 yr. 2 mo. 15 da.

Compound Interest Table

Showing the amount of $1.00 at compound interest at various rates, for any number of interest periods up to 50.

Period	Rate for Period						
	1%	1½%	2%	2½%	3%	3½%	4%
1	1.0100 000	1.0150 000	1.0200 0000	1.0250 0000	1.0300 0000	1.0350 0000	1.0400 0000
2	1.0201 000	1.0302 250	1.0404 0000	1.0506 2500	1.0609 0000	1.0712 2500	1.0816 0000
3	1.0303 010	1.0456 784	1.0612 0800	1.0768 9062	1.0927 2700	1.1087 1787	1.1248 6400
4	1.0406 040	1.0613 636	1.0824 3216	1.1038 1289	1.1255 0881	1.1475 2300	1.1698 5856
5	1.0510 101	1.0772 840	1.1040 8080	1.1314 0821	1.1592 7407	1.1876 8631	1.2166 5290
6	1.0615 202	1.0934 433	1.1261 6242	1.1596 9342	1.1940 5230	1.2292 5533	1.2653 1902
7	1.0721 354	1.1098 450	1.1486 8567	1.1886 8575	1.2298 7387	1.2722 7926	1.3159 3178
8	1.0828 567	1.1264 926	1.1716 5938	1.2184 0290	1.2667 7008	1.3168 0904	1.3685 6905
9	1.0936 853	1.1433 900	1.1950 9257	1.2488 6297	1.3047 7318	1.3628 9735	1.4233 1181
10	1.1046 221	1.1605 408	1.2189 9442	1.2800 8454	1.3439 1638	1.4105 9876	1.4802 4428
11	1.1156 683	1.1779 489	1.2433 7431	1.3120 8666	1.3842 3387	1.4599 6972	1.5394 5406
12	1.1268 250	1.1956 182	1.2682 4179	1.3448 8882	1.4257 6089	1.5110 6866	1.6010 3222
13	1.1380 933	1.2135 524	1.2936 0663	1.3785 1104	1.4685 3371	1.5639 5606	1.6650 7351
14	1.1494 742	1.2317 557	1.3194 7876	1.4129 7382	1.5125 8972	1.6186 9452	1.7316 7645
15	1.1609 690	1.2502 321	1.3458 6834	1.4482 9817	1.5579 6742	1.6753 4883	1.8009 4351
16	1.1725 786	1.2689 855	1.3727 8570	1.4845 0562	1.6047 0644	1.7339 8601	1.8729 8125
17	1.1843 044	1.2880 203	1.4002 4142	1.5216 1826	1.6528 4763	1.7946 7555	1.9479 0050
18	1.1961 475	1.3073 406	1.4282 4625	1.5596 5872	1.7024 3306	1.8574 8920	2.0258 1652
19	1.2081 090	1.3269 507	1.4568 1117	1.5986 5019	1.7535 0605	1.9225 0132	2.1068 4918
20	1.2201 900	1.3468 550	1.4859 4740	1.6386 1644	1.8061 1123	1.9897 8886	2.1911 2314
21	1.2323 919	1.3670 578	1.5156 6634	1.6795 8185	1.8602 9457	2.0594 3147	2.2787 6807
22	1.2447 159	1.3875 637	1.5459 7967	1.7215 7140	1.9161 0341	2.1315 1158	2.3699 1879
23	1.2571 630	1.4083 772	1.5768 9926	1.7646 1068	1.9735 8651	2.2061 1448	2.4647 1555
24	1.2697 346	1.4295 028	1.6084 3725	1.8087 2595	2.0327 9411	2.2833 2849	2.5633 0417
25	1.2824 320	1.4509 454	1.6406 0599	1.8539 4410	2.0937 7793	2.3632 4498	2.6658 3633
26	1.2952 563	1.4727 095	1.6734 1811	1.9002 9270	2.1565 9127	2.4459 5856	2.7724 6979
27	1.3082 089	1.4948 002	1.7068 8648	1.9478 0002	2.2212 8901	2.5315 6711	2.8833 6858
28	1.3212 910	1.5172 222	1.7410 2421	1.9964 9502	2.2879 2768	2.6201 7196	2.9987 0332
29	1.3345 039	1 5399 805	1.7758 4469	2.0464 0739	2.3565 6551	2.7118 7798	3.1186 5145
30	1.3478 490	1.5630 802	1.8113 6158	2.0975 6758	2.4272 6247	2.8067 9370	3.2433 9751
31	1.3613 274	1.5865 264	1.8475 8882	2.1500 0677	2.5000 8035	2.9050 3148	3.3731 3341
32	1.3749 407	1.6103 243	1.8845 4059	2.2037 5694	2.5750 8276	3.0067 0759	3.5080 5875
33	1.3886 901	1.6344 792	1.9222 3140	2.2588 5086	2.6523 3524	3.1119 4235	3.6483 8110
34	1.4025 770	1.6589 964	1.9606 7603	2.3153 2213	2.7319 0530	3.2208 6033	3.7943 1634
35	1.4166 028	1.6838 813	1.9998 8955	2.3732 0519	2.8138 6245	3.3335 9045	3.9460 8899
36	1.4307 688	1.7091 395	2.0398 8734	2.4325 3532	2.8982 7833	3.4502 6611	4.1039 3255
37	1.4450 765	1.7347 766	2.0806 8509	2.4933 4870	2.9852 2668	3.5710 2543	4.2680 8986
38	1.4595 272	1.7607 983	2.1222 9879	2.5556 8242	3.0747 8348	3.6960 1132	4.4388 1345
39	1.4741 225	1.7872 103	2.1647 4477	2.6195 7448	3.1670 2698	3.8253 7171	4.6163 6599
40	1.4888 637	1.8140 184	2.2080 3967	2.6850 6384	3.2620 3779	3.9592 5972	4.8010 2063
41	1.5037 524	1.8412 278	2.2522 0046	2.7521 9043	3.3598 9893	4.0978 3381	4.9930 6145
42	1.5187 899	1.8688 471	2.2972 4447	2.8209 9520	3.4606 9589	4.2412 5799	5.1927 8391
43	1.5339 778	1.8968 798	2.3431 8936	2.8915 2008	3.5645 1677	4.3897 0202	5.4004 9527
44	1.5493 176	1.9253 330	2.3900 5315	2.9638 0808	3.6714 5227	4.5433 4160	5.6165 1508
45	1.5648 107	1.9542 130	2.4378 5421	3.0379 0328	3.7815 9584	4.7023 5855	5.8411 7568
46	1.5804 589	1.9835 262	2.4866 1129	3.1138 5086	3.8950 4372	4.8669 4110	6.0748 2271
47	1.5962 634	2.0132 791	2.5363 4351	3.1916 9713	4.0118 9503	5.0372 8404	6.3178 1562
48	1.6122 261	2.0434 783	2.5870 7039	3.2714 8956	4.1322 5188	5.2135 8898	6.5705 2824
49	1.6283 483	2.0741 305	2.6388 1179	3.3532 7680	4.2562 1944	5.3960 6459	6.8333 4937
50	1.6446 318	2.1052 424	2.6915 8803	3.4371 0872	4.3839 0602	5.5849 2686	7.1066 8335

Compound Interest Table

Showing the amount of $1.00 at compound interest at various rates, for any number of interest periods up to 50.

Period	Rate for Period						
	4½%	5%	6%	7%	8%	9%	10%
1	1.0450 0000	1.0500 000	1.0600 000	1.0700 000	1.0800 000	1.0900 000	1.1000 000
2	1.0920 2500	1.1025 000	1.1236 000	1.1449 000	1.1664 000	1.1881 000	1.2100 000
3	1.1411 6612	1.1576 250	1.1910 160	1.2250 430	1.2597 120	1.2950 290	1.3310 000
4	1.1925 1860	1.2155 063	1.2624 770	1.3107 960	1.3604 890	1.4115 816	1.4641 000
5	1.2461 8194	1.2762 816	1.3382 256	1.4025 517	1.4693 281	1.5386 240	1.6105 100
6	1.3022 6012	1.3400 956	1.4185 191	1.5007 304	1.5668 743	1.6771 001	1.7715 610
7	1.3608 6183	1.4071 004	1.5036 303	1.6057 815	1.7138 243	1.8280 391	1.9487 171
8	1.4221 0061	1.4774 554	1.5938 481	1.7181 862	1.8509 302	1.9925 626	2.1435 888
9	1.4860 9514	1.5513 282	1.6894 790	1.8384 592	1.9990 046	2.1718 933	2.3579 477
10	1.5529 6942	1.6288 946	1.7908 477	1.9671 514	2.1589 250	2.3673 637	2.5937 425
11	1.6228 5305	1.7103 394	1.8982 986	2.1048 520	2.3316 390	2.5804 264	2.8531 167
12	1.6958 8143	1.7958 563	2.0121 965	2.2521 916	2.5181 701	2.8126 648	3.1384 284
13	1.7721 9610	1.8856 491	2.1329 283	2.4098 450	2.7196 237	3.0658 046	3.4522 712
14	1.8519 4492	1.9799 316	2.2609 040	2.5785 342	2.9371 936	3.3417 270	3.7974 983
15	1.9352 8244	2.0789 282	2.3965 582	2.7590 315	3.1721 691	3.6424 825	4.1772 482
16	2.0223 7015	2.1828 746	2.5403 517	2.9521 638	3.4259 426	3.9703 059	4.5949 730
17	2.1133 7681	2.2920 183	2.6927 728	3.1588 152	3.7000 181	4.3276 334	5.0544 703
18	2.2084 7877	2.4066 192	2.8543 392	3.3799 323	3.9960 195	4.7171 204	5.5599 173
19	2.3078 6031	2.5269 502	3.0255 995	3.6165 275	4.3157 011	5.1416 613	6.1159 390
20	2.4117 1402	2.6532 977	3.2071 355	3.8696 845	4.6609 571	5.6044 108	6.7275 000
21	2.5202 4116	2.7859 626	3.3995 636	4.1405 624	5.0338 337	6.1088 077	7.4002 499
22	2.6336 5201	2.9252 607	3.6035 374	4.4304 017	5.4365 404	6.6586 004	8.1402 749
23	2.7521 6635	3.0715 238	3.8197 497	4.7405 299	5.8714 637	7.2578 745	8.9543 024
24	2.8760 1383	3.2250 999	4.0489 346	5.0723 670	6.3411 807	7.9110 832	9.8497 327
25	3.0054 3446	3.3863 549	4.2918 707	5.4274 326	6.8484 752	8.6230 807	10.8347 059
26	3.1406 7901	3.5556 727	4.5493 830	5.8073 529	7.3963 532	9.3991 579	11.9181 765
27	3.2820 0956	3.7334 563	4.8223 459	6.2138 676	7.9880 615	10.2450 821	13.1099 942
28	3.4296 9999	3.9201 291	5.1116 867	6.6488 384	8.6271 064	11.1671 395	14.4209 936
29	3.5840 3649	4.1161 356	5.4183 879	7.1142 571	9.3172 749	12.1721 821	15.8630 930
30	3.7453 1813	4.3219 424	5.7434 912	7.6122 550	10.0626 569	13.2676 785	17.4494 023
31	3.9138 5745	4.5380 395	6.0881 006	8.1451 129	10.8676 694	14.4617 695	19.1943 425
32	4.0899 8104	4.7649 415	6.4533 867	8.7152 708	11.7370 830	15.7633 288	21.1137 768
33	4.2740 3018	5.0031 886	6.8405 899	9.3253 398	12.6760 496	17.1820 284	23.2251 544
34	4.4663 6154	5.2533 480	7.2510 253	9.9781 135	13.6901 336	18.7284 109	25.5476 699
35	4.6673 4781	5.5160 154	7.6860 868	10.6765 815	14.7853 443	20.4139 679	28.1024 369
36	4.8773 7846	5.7918 161	8.1472 520	11.4239 422	15.9681 718	22.2512 250	30.9126 805
37	5.0968 6049	6.0814 069	8.6360 871	12.2236 181	17.2456 256	24.2538 353	34.0039 486
38	5.3262 1921	6.3854 773	9.1542 524	13.0792 714	18.6252 756	26.4366 805	37.4043 434
39	5.5658 9908	6.7047 512	9.7035 075	13.9948 204	20.1152 977	28.8159 817	41.1447 778
40	5.8163 6454	7.0399 887	10.2857 179	14.9744 578	21.7245 215	31.4094 200	45.2592 556
41	6.0781 0094	7.3919 882	10.9028 610	16.0226 699	23.4624 832	34.2362 679	49.7851 811
42	6.3516 1548	7.7615 876	11.5570 327	17.1442 568	25.3394 819	37.3175 320	54.7636 992
43	6.6374 3818	8.1496 669	12.2504 546	18.3443 548	27.3666 404	40.6761 098	60.2400 692
44	6.9361 2290	8.5571 503	12.9854 819	19.6284 596	29.5559 717	44.3369 597	66.2640 761
45	7.2482 4843	8.9850 078	13.7646 108	21.0024 518	31.9204 494	48.3272 861	72.8904 837
46	7.5744 1961	9.4342 582	14.5904 875	22.4726 234	34.4740 853	52.6767 419	80.1795 321
47	7.9152 6849	9.9059 711	15.4659 167	24.0457 070	37.2320 122	57.4176 486	88.1974 853
48	8.2714 5557	10.4012 697	16.3938 717	25.7289 065	40.2105 731	62.5852 370	97.0172 338
49	8.6436 7107	10.9213 331	17.3775 040	27.5299 300	43.4274 190	68.2179 083	106.7189 572
50	9.0326 3627	11.4673 998	18.4201 543	29.4570 251	46.9016 125	74.3575 201	117.3908 529

4. What will be the amount of \$1 at 6% compounded annually for 50 years?

5. \$12,950 was put at 6% compound interest in 1902 and was not paid until 1926. What was the amount of the claim in 1926 if interest was compounded annually?

6. What sum must be deposited now so that in 10 years, with interest at 4% compounded annually, the deposit will amount to \$5000?

7. What is the present worth of \$6000 due in 3 years, if money is worth 5% compounded semi-annually?

The compound interest table may be used for calculations not involving sums of money at interest.

8. During the past year the average number of passengers carried daily by a street car line was 15,874. If this average increases 10% per annum, how many passengers must this line be capable of carrying 20 years from now?

9. Each day 7500 persons are landed at a railroad terminal. There has been an annual increase of 5% in traffic at this station, for the last ten years. If this annual increase is continued for the next 25 years, how many passengers daily must the terminal be capable of accommodating at the end of this period?

Systematic Investment

The tables on the following pages show the total amount accruing from periodical deposits of \$1.00, at compound interest, in any number of periods not exceeding 20, at the rate for the period specified in the table.

WRITTEN

1. When Henry Brown was 5 years old his father put \$25 in a bank for him at 4% compounded annually, and each year thereafter on Henry's birthday deposited a like sum of money. How much money should Henry have when he is 21 years of age?

2. A young man wishes to engage in business for himself when he is 30 years of age. To do this will require \$2000. If he is now 17 years old, and can get 4% interest on his money, compounded semi-annually, how much should he deposit semi-annually, to have the desired amount?

Compound Interest Amount Table

Showing the amount, at compound interest, of periodical investments of $1.00.

At End of What Period	Rate for the Period					
	2%	2½%	3%	3½%	4%	4½%
1.....	1.020000	1.025000	1.030000	1.035000	1.040000	1.045000
2.....	2.060400	2.075625	2.090900	2.106225	2.121600	2.137025
3.....	3.121608	3.152515	3.183627	3.214942	3.246464	3.278191
4.....	4.204040	4.256328	4.309135	4.362465	4.416322	4.470709
5.....	5.308120	5.387736	5.468409	5.550152	5.632975	5.716891
6.....	6.434283	6.547430	6.662462	6.779407	6.898294	7.019151
7.....	7.582969	7.736115	7.892336	8.051686	8.214226	8.380013
8.....	8.754628	8.954518	9.159106	9.368495	9.582795	9.802114
9.....	9.949721	10.203381	10.463879	10.731393	11.006107	11.288209
10.....	11.168715	11.483466	11.807795	12.141991	12.486351	12.841178
11.....	12.412089	12.795552	13.192029	13.601961	14.025805	14.464031
12.....	13.680331	14.140441	14.617790	15.113030	15.626837	16.159913
13.....	14.973938	15.518952	16.086324	16.676986	17.291911	17.932109
14.....	16.293416	16.931926	17.598913	18.295680	19.023587	19.784054
15.....	17.639285	18.380224	19.156881	19.971029	20.824531	21.719336
16.....	19.012070	19.864730	20.761587	21.705015	22.697512	23.741706
17.....	20.412312	21.386348	22.414435	23.499691	24.645412	25.855083
18.....	21.840558	22.946007	24.116868	25.357180	26.671229	28.063562
19.....	23.297369	24.544657	25.870374	27.279681	28.778078	30.371422
20.....	24.783317	26.183273	27.676485	29.269470	30.969201	32.783136

3. A business man paid for fire insurance $85 a year for 2 years payable each year in advance. What would these payments have amounted to if they had been invested at 4%, compounded annually?

4. A man insured his life for $2000, at a cost of $22.78 per thousand, and made 10 annual payments. Allowing interest at 4%, compounded annually, what sum did the company receive from him, computed to the end of the tenth year?

Review Problems

1. At the age of 30 Brown took out two policies of insurance, each for $2000, and paid the annual premiums according to the table shown in the chapter on insurance. One policy was a 20-payment life and the other was a 20-year endowment. The

COMPOUND INTEREST AMOUNT TABLE.—*Continued*

Showing the amount, at compound interest, of perodical investments of $1.00.

AT END OF WHAT PERIOD	RATE FOR THE PERIOD					
	5%	6%	7%	8%	9%	10%
1.....	1.050000	1.060000	1.070000	1.080000	1.090000	1.100000
2.....	2.152500	2.183600	2.214900	2.246400	2.278100	2.310000
3.....	3.310125	3.374616	3.439943	3.506112	3.573129	3.641000
4.....	4.525631	4.637093	4.750739	4.866601	4.984710	5.105100
5.....	5.801912	5.975318	6.153290	6.335929	6.523334	6.715610
6.....	7.142008	7.393837	7.654021	7.902803	8.200434	8.487171
7.....	8.549108	8.897468	9.259802	9.616627	10.028473	10.435888
8.....	10.026564	10.491316	10.977988	11.467557	12.021036	12.579476
9.....	11.577892	12.180795	12.816448	13.466562	14.192929	14.937424
10.....	13.206787	13.971642	14.783599	15.625487	16.560293	17.531167
11.....	14.917126	15.869941	16.888451	17.957126	19.140719	20.384283
12.....	16.712982	17.882137	19.140963	20.475296	21.953384	23.522712
13.....	18.598631	20.015066	21.550488	23.194920	25.019189	26.974983
14.....	20.578563	22.275970	24.129022	26.132113	28.360916	30.772481
15.....	22.657491	24.672528	26.888053	29.304283	32.003398	34.949729
16.....	24.840366	27.212880	29.840217	32.730225	35.973704	39.544702
17.....	27.132384	29.905652	32.999032	36.430243	40.301338	44.599173
18.....	29.539003	32.759992	36.378965	40.426263	45.018458	50.159090
19.....	32.065954	35.785591	39.995492	44.741964	50.160119	56.275029
20.....	34.719251	38.992727	43.865177	49.402921	55.764530	63.002529

premiums on both policies were paid regularly for 10 years. Including interest compounded annually at 3%, what sum had the company realized at the end of the 10th year?

2. Brown surrendered his insurance at the end of the 10th year. How much did he receive from the company? See table in the chapter on insurance.

3. What per cent of the money received by the company was returned to Brown on each of the policies mentioned above?

4. How much did Brown's insurance ($4000 for 10 years) cost him, if money was worth 4% compounded annually?

5. At 35 years of age, a man took out a 10-year endowment policy for $5000 and paid an annual premium as per the table in the chapter on insurance. What would his premiums have amounted to in 10 years at $4\frac{1}{2}\%$ interest, compounded annually?

6. At the age of 30 a man took out a 20-payment life policy for $3000, paying the premium according to the table on page 323.

At the end of the 10th year he died and the amount of the policy was paid to his family. How much more did his family receive than he had paid the company, allowing compound interest at 5% compounded annually?

7. A young man, age 21, took out a 20-yr. endowment policy for $2000, paying the premium annually according to the table on page 323. It was his intention, when the policy matured, to use the money to go into business for himself. If he had made an annual deposit in a bank instead of taking insurance, would he have had more money, or less, and how much, at the end of 20 years, allowing interest at 8% compounded annually?

8. A house has been insured in one company for 18 years. The policy has been for the same amount each year, $2400 at 25¢ per $100 per year. What would these payments have amounted to at 6%, interest compounded annually?

9. An insurance company insured a house for $3000 for 12 years, at 74¢ per $100 for 3 years, payable in advance. On the last day of the 12th year, the company had to pay a $300 fire loss on this risk. Allowing interest on premium receipts at 4% compounded annually, and assuming that the company's expenses of doing business average 45% of the premiums and interest received, how much has the company lost on this risk?

10. George Gardner owns 5 properties. He has kept them insured and paid premiums on each as follows:

	Amount of Insurance	Average Annual Premium	Years Insured
No. 1	$2500	24¢	12
No. 2	2800	25¢	8
No. 3	4500	20¢	8
No. 4	3000	20¢	7
No. 5	7800	25¢	7

Including compound interest at 5%, what has the insurance on these properties cost Gardner?

11. A corporation engaged in manufacturing explosives had 4 factories and 6 warehouses. These factories and warehouses were located at various cities in the United States and had a total value, together with machinery and raw material, of $650,000. As the

business was extra-hazardous, the insurance companies asked a premium of $3.50 per $100.

Instead of insuring, the corporation decided to carry its own insurance, by setting aside each year from its net profits a sum of money equal to the amount of premium asked by the insurance company. This insurance fund was invested in securities and the interest re-invested each year. If the company realized 4% upon the investments, how much did the investments amount to at the end of 6 years?

12. On the day that the 5th appropriation to this fund had been made (11th problem), one of the warehouses, valued at $50,000 was totally destroyed. Was the insurance fund in excess of the loss, or less than the loss, and how much?

CHAPTER XXIX

PARTIAL PAYMENTS

A **partial payment** is a payment of a part of a note, account, or other obligation.

When a partial payment of a written obligation is made, the memorandum of the payment should also be in writing, usually on the back of the paper evidencing the original debt. Thus, the following might be indorsed on the back of a note by the creditor:

Recd. $500.00 on this note
Apr. 2, 1919. J. H. Williams.

The debtor should see that such indorsement is made in his presence at the time he makes the payment.

When a payment is made on an interest-bearing obligation the question arises whether it is to be applied to the payment of interest or of principal. In the absence of agreement between the parties as to how a payment shall be applied, use the following rule:

United States Rule

The Supreme Court of the United States has held that when a partial payment is made upon an interest-bearing debt, the payment must be applied first to discharge the accrued interest. What remains of the payment must be applied upon the principal if the payment is larger than the accrued interest at the time the payment is made. If the payment is smaller than the accrued interest, the previous principal, and that only, continues to draw interest.

The object of this rule is to prevent compound interest.

This rule, commonly called the United States Rule, is now the law in a majority of the states, having been adopted either by statute or by court decision.

EXAMPLE. A note dated March 1, 1927, for $1800, with interest at 6%, had the following payments indorsed upon it: Sept. 27, 1927, $500; Mar. 15, 1928, $25; June 1, 1928, $700. What amount should be paid to discharge the note on Sept. 1, 1928?

Principal Mar. 1, 1927		$1800.00
Interest on $1800 at 6%, Mar. 1 to Sept. 27 (6 mo. 26 da.)		61.80
Amount of obligation Sept. 27, 1927		$1861.80
Payment on Sept. 27, 1927		500.00
Balance of obligation after 1st payment		$1361.80
[Int. on $1361.80 at 6% Sept. 27, 1927, to Mar. 15, 1928 (5 mo. 18 da.), was $38.13, but the payment of $25.00 did not cover this.]		
Int. on $1361.80 at 6% Sept. 27, 1927, to June 1, 1928 (8 mo. 4 da.)		55.38
Amount of obligation on June 1, 1928		$1417.18
Payment Mar. 15, 1928—$25.00		
Payment June 1, 1928 —700.00		$ 725.00
Balance of obligation after 3d payment		$ 692.18
Interest on $692.18 at 6% June 1, 1928, to Sept. 1, 1928 (3 mo.)		$ 10.38
Amount due on Sept. 1, 1928		$ 702.56

Sometimes when a partial payment is made the old note is destroyed and a new note issued for the balance then due. This is a simple method, but is objectionable for two reasons: First, because all transactions relating to the obligation are not evidenced on the same paper; second, because whenever the payment is less than the accrued interest the purpose of the U. S. rule (to prevent compounding) is defeated.

Mortgage notes usually bear coupons for interest due. Each coupon bears its own due date, and each interest item bears simple interest from its date.

When by the terms of the loan interest items are payable on specified dates, and bear simple interest if unpaid, it is customary to allow the borrower interest on any payments made before any interest date.

The rules for figuring partial payments apply to partial payments on non-interest-bearing notes after maturity, as well as to interest-bearing notes at any time.

Written

1. A note for $1200 dated May 1, 1927, with interest at 6%, had the following payments indorsed on it: Oct. 12, 1927, $200; Apr. 21, 1928, $300; Aug. 12, 1928, $500. What was the balance due on Oct. 1, 1928?

2. A note was made on Sept. 10, 1927, for $950 with interest at 6%. The following payments were made upon it: Apr. 7, 1928, $10.00; May 22, 1928, $250; Aug. 12, 1928, $400; Nov. 8, 1928, $100. What was the balance due on Dec. 1, 1928?

$3000.00 CHICAGO, ILL., *June 13, 19*

One year AFTER DATE *I* PROMISE TO PAY TO

Samuel Peck, OR ORDER,

Three Thousand and no/100 DOLLARS.

VALUE RECEIVED, WITH INTEREST AT *6%*.

Thomas Singer.

3. Singer paid as follows: Dec. 1, 1927, $800; Oct. 1, 1928, $500. Find the balance due on Dec. 15, 1928.

4. On Mar. 15, 1927, a note for $2000 was issued for 4 mo. without interest. On July 1, 1927, a payment of $800 was made. No further payments were made until after the note was due. $500 was paid on Oct. 20, 1927; $200 on Dec. 5, 1927; and $200 on Mar. 24, 1928. The maker paid the balance on June 1, 1928. How much did he pay, if interest was at 6%?

5. A three year note was issued on Jan. 12, 1926, for $2500 at 7% payable annually. The maker had the privilege of paying any amount he wished before maturity. He paid $500 on July 16, 1926; $800 on Dec. 22, 1926; $400 on June 27, 1927; and $600 on Feb. 23, 1928. How much was due at maturity?

6. Find how much was due at maturity on a note for $5000 dated Apr. 10, 1926, and payable in 2 years at 5% payable annually, if the following payments were made: Aug. 17, 1926, $1500; Mar. 11, 1927, $800; July 27, 1927, $1150; Feb. 20, 1928, $1437.83.

7. On Dec. 18, 1926, William Morgan signed a note for $1850, payable in three months, without interest. On Mar. 1, 1927, he paid $200; Aug. 23, 1927, $800; Oct. 14, 1927, $300. How much did he owe on Jan. 1, 1928, interest at 5%?

8. A note for $2000 at 6% dated Feb. 10, 1927, payable in 2 years, had the following payments made upon it: Sept. 22, 1927, $350; May 12, 1928, $45; Dec. 29, 1928, $500. No more payments were made until the day of maturity, at which time a payment of $200 was made and the old note cancelled. In its place a new note

was made for six months, and interest for six months at 6% on the balance due was included in the face of the new note. What was the amount of the new note?

9. On Sept. 1, 1927, Rufus Statler signed a note for $1200 payable in 6 mo. with interest at 6%. The note was not paid at maturity, but on Apr. 1, 1928, Mr. Statler paid $350 to be applied on interest and principal. A new note was made for the balance. For what amount was the new note drawn?

Installments

Houses, lots, farms, and other properties are frequently paid for by installments, or partial payments, after an initial payment has been made.

In purchasing property on this plan the balance of the principal, after the first payment, is usually interest-bearing.

EXAMPLE. A cottage is sold on contract Jan. 1, 1925, for $1800 upon a payment of $300 cash and $15.00 per month, the payment each month to cover the interest at 6% and the balance to be applied upon the principal. If the interest is calculated upon the balance of the principal remaining unpaid each month, how much will be due upon the principal at the end of 3 years?

	First Year			Second Year			Third Year		
Amount	Date	Prin.	Int.	Date	Prin.	Int.	Date	Prin.	Int.
$1800	Feb. 1	$7.50	$7.50	Feb. 1	$7.96	$7.04	Feb. 1	$8.45	$6.55
300 Pmt.	Mar. 1	7.54	7.46	Mar. 1	8.00	7.00	Mar. 1	8.50	6.50
$1500 Bal.	Apr. 1	7.58	7.42	Apr. 1	8.04	6.96	Apr. 1	8.54	6.46
92.52	May 1	7.61	7.39	May 1	8.08	6.92	May 1	8.58	6.42
$1407.48	June 1	7.65	7.35	June 1	8.12	6.88	June 1	8.62	6.38
98.21	July 1	7.69	7.31	July 1	8.16	6.84	July 1	8.67	6.33
$1309.27	Aug. 1	7.73	7.27	Aug. 1	8.20	6.80	Aug. 1	8.71	6.29
104.28	Sept. 1	7.77	7.23	Sept. 1	8.25	6.75	Sept. 1	8.75	6.25
$1204.99	Oct. 1	7.81	7.19	Oct. 1	8.29	6.71	Oct. 1	8.80	6.20
	Nov. 1	7.84	7.16	Nov. 1	8.33	6.67	Nov. 1	8.84	6.16
	Dec. 1	7.88	7.12	Dec. 1	8.37	6.63	Dec. 1	8.89	6.11
	Jan. 1	7.92	7.08	Jan. 1	8.41	6.59	Jan. 1	8.93	6.07
		$92.52	87.48		$98.21	81.79		$104.28	75.72

Note the monthly decrease in interest, and the corresponding increase in the amount applied upon the principal. In practice the monthly or quarterly decrease in the interest (in the illustration, 4¢) is established and the credits to principal each month are calculated upon this constant decrease.

WRITTEN

1. A house was purchased for $4000, the buyer paying $800 cash at the time of purchase. He was to pay $30 per month also, the payment to be applied on interest and principal. Interest at 6% was calculated upon the balance remaining due each month. How much was paid on the principal, by these monthly payments, at the end of the first year?

2. A real estate company sells building lots for $800, requiring a cash payment of 10% of the purchase price. The balance is to draw interest at 5% and the purchaser is to pay $50 every three months. The payment is to be applied first upon interest due on the balance at the end of the preceding quarter, and the remainder upon the principal. How much will be paid on the principal at the end of the first year?

3. A man bought a house for $3800, paying $1000 cash at the time of purchase and agreeing to pay $30 per month to be applied upon interest and principal. The interest at 6% was to be calculated upon the balance remaining due at the end of each month. His taxes were 20.45 mills on the dollar upon a valuation of $2500; water tax $12 annually; and insurance on $2800 at 60¢ per $100 for 3 yr. Assuming there were no repairs for the first year, how much did he save the first year by buying the house instead of paying $30 per month rent?

4. A farmer bought 160 A. of land at $27.50 per acre, paying $900 cash at the time of purchase and agreeing to make semi-annual payments of $200 each, to be applied upon interest and principal until the debt was paid. The interest was calculated at 6% upon the balance remaining due after each payment was made. Make a statement showing balance due after the debtor had made seven payments.

5. A man bought a farm for $8000, paying $4000 cash at the time of purchase and agreeing to make semi-annual payments of $500 each, the payments to be applied upon interest at 6% and principal. After making six such payments he borrowed at 5% on a mortgage for 3 years sufficient money to pay up the entire debt on the farm. The interest on the mortgage was payable annually and the principal was payable in three years. He then paid $500 semi-annually into an investment that paid 4% interest

compounded semi-annually. Did he save on his interest payments by changing from one plan to the other? How much?

6. Mr. Robinson is considering the purchase of one of two houses on which the following terms are offered: First, the price of the house is \$3800 with a cash payment of \$800 and monthly payments of \$30 to be applied on interest and principal, interest at 7% to be calculated on the monthly balance. Second, the price of the house is \$4000 with a cash payment of \$1000 and monthly payments of \$30 to apply on interest and principal, interest to be calculated at 5% on the balance due at the end of each preceding quarter. Which plan will give Mr. Robinson the greater percentage of equity at the end of the first year's payments, based upon the balance due on the property after deducting the initial payment?

7. I held a note dated Jan. 24, 1927, for \$1185.45 payable in 6 mo. without interest. The maker paid \$200 on July 24, 1927, and on Sept. 1, 1927, paid \$300 more to be applied on interest at 6% and on principal. On Nov. 15, 1927, he paid all the indebtedness but \$300 and made a new note for that sum payable in 3 months, interest at 7% included in the face of the note. What amount did he pay on Nov. 15 and what was the amount of the new note?

CHAPTER XXX

BANKS AND BANKING

Banks of Deposit

A national bank located in New York, Chicago, or St. Louis ("central reserve" cities) must maintain a reserve equal to 18% of its demand deposits plus 5% of its time deposits. A national bank located in a "reserve" city must maintain a reserve equal to 15% of its demand deposits plus 5% of its time deposits. A "country" national bank (a national bank not in a central reserve city or a reserve city) must maintain a reserve equal to 12% of its demand deposits plus 5% of its time deposits.

Reserves must be as follows:

	In Bank's Vaults	In Its Federal Reserve Bank	In Either Place
National Bank in Central Reserve City	$\frac{6}{18}$	$\frac{7}{18}$	$\frac{5}{18}$
National Bank in Reserve City	$\frac{5}{15}$	$\frac{6}{15}$	$\frac{4}{15}$
Country National Bank	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{3}{12}$

Written

Assuming that in each case the bank has the exact amount of its required reserve and no more, and that it keeps as much in its own vaults as the law permits, how much reserve has each of the following banks, and how distributed?

1. A national bank in St. Louis having demand deposits of $2,500,000 and time deposits of $750,000.

2. A national bank in a reserve city, having deposits as follows: Demand, $1,200,000; time, $456,000.

3. A national bank in New York having deposits as follows: Demand, $18,500,000; time, $4,250,000.

4. A country national bank having demand deposits of $650,000 and time deposits of $920,000.

5. A national bank in a reserve city, having demand deposits of \$3,500,000 and time deposits of \$1,115,000.

6. A bank owns \$50,000 worth of registered U. S. bonds on which it receives $2\frac{1}{2}\%$ interest. These bonds are on deposit with the U. S. Treasurer to secure circulation and the Treasurer has issued \$50,000 in National Bank Notes in exchange for them. Of these notes, \$40,000 are in circulation and subject to an annual tax of $\frac{1}{2}\%$, and are drawing interest at an average rate of 4% from persons to whom they have been loaned. What is the bank's profit on its original investment of \$50,000 in the bonds?

7. A national bank must set aside 10% of its earnings each year until it has a surplus equal to 20% of its capital. A national bank had a capital of \$500,000 and a surplus of \$84,500. Its earnings for three years thereafter were: 1st yr., \$15,000; 2d yr., \$17,500; 3d yr., \$16,750. A dividend equal to 2% of the amount of the capital stock was declared the 2d year, and a dividend of 3% the 3d year. All profits after surplus and dividend requirements were met went into the undivided profits account. What amount went into the undivided profits account each year?

Banking Problems

1. Bates has \$18,362.50 on deposit. Not needing to use it all, on Jan. 14 he exchanges his check for \$15,500 for a certificate of deposit bearing interest at 3% if left for 3 mo. or longer. On May 1 he deposits the certificate and interest in his checking account. How much profit has he made by the plan?

2. Ames has a business that needs all his funds for about five months of each year, but during seven months \$12,500 lies idle in his checking account. Which is better for him and how much — to take a certificate of deposit bearing 3% for his surplus and leave it for seven months, or to invest the amount in a good 5% bond at par and during the five months borrow for his needs at 6%, using the bonds for collateral?

3. The Government Tax on bank notes issued by a national bank is $\frac{1}{4}\%$ on notes based on bonds bearing 2% interest, and $\frac{1}{2}\%$ on notes based on bonds bearing more than 2%. A bank had \$145,000 circulation based on 2% bonds and \$55,000 based on bonds bearing higher rates. What was its total circulation tax?

4. A bank is capitalized at $200,000. Its loans and discounts amount to $315,000 and it owns bonds and securities to the amount of $24,000. On loans and discounts this bank averages 3% net earnings, and on its bonds and securities $2\frac{1}{2}\%$ net. The income from these investments is what per cent of the capital stock of the bank?

5. The following is a condensed statement of bank's condition:

Resources		Liabilities	
Loans and Securities	$24,555,000	Capital Stock	$ 1,500,000
Checks, Cash, and Clearing House Items	4,581,000	Surplus	1,500,000
Due from Banks and Bankers	5,059,000	Undivided Profits	1,409,000
Bank Building and Ground	1,250,000	Deposits	30,936,000
		Reserve for unearned interest and taxes	100,000
	$35,445,000		$35,445,000

(*a*) What is the approximate ratio between capital stock and deposits?

(*b*) The loans and securities are what per cent of the deposits? (Give the result to the nearest tenth of one per cent.)

(*c*) What is the approximate ratio of Deposits to the total of Capital Stock, Surplus, and Undivided Profits?

6. If 80% of the deposits in the previous problem were loaned at 6% interest, what gross rate would this income be upon the capital stock of the bank?

Savings Banks

The following rules governing interest on deposits are followed by most savings banks, with slight modifications in different localities. Interest periods are usually from Jan. 1 to July 1 and from July 1 to Jan. 1.

Rules Governing Interest on Savings Deposits

1. On deposits made during the first ten days of January and July, and during the first five days of any other month, interest will be allowed from the first day of the month in which such deposit is made. On deposits made after the above named days of each month, interest will be allowed from the first day of the month following the deposit.

2. Interest will not be allowed on the average balances, nor for parts of months, nor for parts of dollars, nor on sums withdrawn between January 1 and June 30, or between July 1 and December 31. In computing interest, withdrawals between these periods will be deducted from the first deposits.

Note that it is possible under these rules to have a large amount in the bank at all times and still be entitled to no interest.

EXAMPLE. On Jan. 6, 1927, H. D. Avery deposited $100 in a savings bank at 4% interest, compounded semi-annually. On Mar. 7, 1927, he deposited $150.00. On Aug. 5, 1927, he deposited $75. On Oct. 26, 1927, he withdrew $45.50. On Feb. 8, 1928, he deposited $70.25. He withdrew the balance due him June 1, 1928. How much did he withdraw?

DATE	DEPOSITS		INTEREST		WITHDRAWALS		BALANCE	
1927								
Jan. 6	100						100	
Mar. 7	150						250	
July 1			3	50			253	50
Aug. 5	75						328	50
Oct. 26					45	50	283	00
1928								
Jan. 1			5	41			288	41
Feb. 8	70	25					358	66
June 1					358	66		

EXPLANATION. On July 1, 1927, Avery is credited with interest on $100.00 for 6 mo., and on $150 for 3 mo. (not including March). On Jan. 1, 1928, he is credited with interest on $253.50 − $45.50 for 6 mo. (Note that the $45.50 is deducted from the July 1 balance rather than from the deposit made Aug. 5), and on $75 for 5 mo. The balance being withdrawn on June 1, no further interest is credited.

Had Avery allowed his savings to remain on deposit one more month he would have been entitled to interest on $288 (no interest on the 41¢) for 6 mo. and on $70 for 4 mo.

WRITTEN

1. Had Avery (see illustrative example) made his second deposit two days earlier, his first withdrawal five days later, his fourth deposit three days earlier, and left his balance of January 1 in

the bank until July 1, 1928, what balance could he have withdrawn then?

2. H. D. Smith's savings account showed the following deposits and withdrawals: Deposits: July 1, 1926, $150.50; Aug. 6, $70.42; Oct. 4, $25; Mar. 3, 1927, $42.50; Sept. 6, $55.75. Withdrawals, Dec. 15, 1926, $20.00; July 2, 1927, $10.00. What was the balance to his credit Jan. 1, 1928, if interest was at 4%?

3. Ernest Maier made deposits as follows: Jan. 8, $100.00; Feb. 15, $50.00; May 5, $120.00; June 10, $100.00. On June 20, he drew out $150.00. What had he to his credit July 1, interest being credited semi-annually at 3 per cent?

4. Harry N. Brown started a savings account on Mar. 18, 1927, with a deposit of $56.75. The bank paid 3% compound interest, credited semi-annually. On April 6, Brown deposited $42.35. On Aug. 4 he withdrew $20. On Sept. 7 he deposited $48.60. What balance was due him on Jan. 1, 1928?

5. A young man deposited $17.50 on the first day of every month for two years, in a savings bank paying 4% compound interest credited semi-annually. He withdrew $75 on July 16 of the first year and $32.25 on Feb. 12 of the second year. What balance was due him at the end of the second year?

6. On June 25, 1927, M. B., of Covington, Ky., borrowed $150 at 6% on a Union Central life insurance policy for $1000. On Aug. 24 he repaid $30; on Oct. 24, $30; on Dec. 19, $30; on Mar. 15, 1928, $20; on Apr. 15, $20; and on May 10, $20. With each payment he included interest from June 25, 1927, on the amount repaid. How much did interest cost him?

7. Solve problem 6 by the U. S. Rule, using amounts actually paid on the dates given. Was the interest paid less or more than would have been paid by the U. S. Rule and how much?

8. On May 14, 1927, Mr. B, of St. Elmo, Tenn., borrowed $286 on his policy No. 164,253 for $2000 in the Union Central, at 6%. He made five payments of $50 each, one on July 28, one on Aug. 24, one on Sept. 29, one on Nov. 5 and one on Dec. 4. On Mar. 4, 1928, he paid $36.00. With each payment he included interest on the amount repaid. What did interest cost him?

9. What would have been your result for the preceding problem had the United States Rule been used?

Clearing House Problems

Banks located in the same town usually meet daily, through their representatives, for the exchange of checks received by each upon the others. The organization is called the Clearing House. The exchanging of checks is called "Clearing."

Although the original and principal purpose of the Clearing House is the clearing of checks, this is by no means the sole purpose of the organization. The members of the Clearing House usually agree on certain policies, exchange information, and pass rules governing the policies of member banks, especially in their relations to each other.

In the course of its business, a bank receives from its depositors or cashes for its customers many checks upon other banks which are members of the same clearing house.

Each bank takes to the Clearing House, at a certain time each day, all checks held by it against member banks. These are all turned in to the Clearing House for credit. The Clearing House officers then proceed to charge each bank with all checks against it turned in by the other banks. Finally, the banks against which there are more charges than credits pay to the Clearing House officers the excess of the charges, and the Clearing House officers proceed to pay out the total so received to the banks whose credit exceed the charges.

WRITTEN

1. Four banks meet to clear checks. Each brings checks against the other as shown by the following table:

	Checks against First National	Checks against Second National	Checks against Third National	Checks against Fourth National
First National Bank		5346.48	4457.39	3710.26
Second National Bank	8324.35		7694.48	5126.34
Third National Bank	7483.09	6512.70		4819.23
Fourth National Bank	3428.36	2947.32	2764.32	

What was the total turned in by each bank?
What was the total of debits against each bank?
Which banks were charged a balance, and how much?
Which banks received a balance, and how much?

2. The following table shows the detail of the clearings of the six members of the Judsonville Clearing House for one day:

	Checks against National City Bank	Checks against Metrop. State Bank	Checks against Union Bank	Checks against American State Bank	Checks against City State Bank	Checks against Lincoln State Bank
National City Bank		5324.83	8264.35	5744.93	2610.37	1073.62
Metropolitan State Bank	6382.34		2156.07	4672.32	1026.18	3489.43
Union Bank	2891.14	4268.98		3624.12	3847.90	1945.25
American State Bank	4215.40	1962.45	5126.07		9184.26	6251.98
City State Bank	8632.23	1570.12	6234.36	4592.16		2573.50
Lincoln State Bank	4592.16	3495.19	3789.48	2961.24	7458.20	

What was the result, for each bank, of the day's clearings?

CHAPTER XXXI

EXCHANGE

Exchange is a general term applied to various methods of making remittance from one place, city, or country to another without the transfer of money itself.

Domestic or inland exchange is exchange between points within this country. **Foreign** exchange is exchange between this country and another. A foreign draft is called a "bill of exchange."

The mediums of domestic exchange principally used are checks, postal money orders, express money orders, and bank drafts.

The fee for the collection of a check or for the issuance of a money order or bank draft, is also called *exchange*.

Check. Banks will generally receive for deposit without charge checks on other banks with which they have reasonably direct connections, but exchange fees are usually charged on checks drawn on out-of-town banks, especially when this necessitates special correspondence.

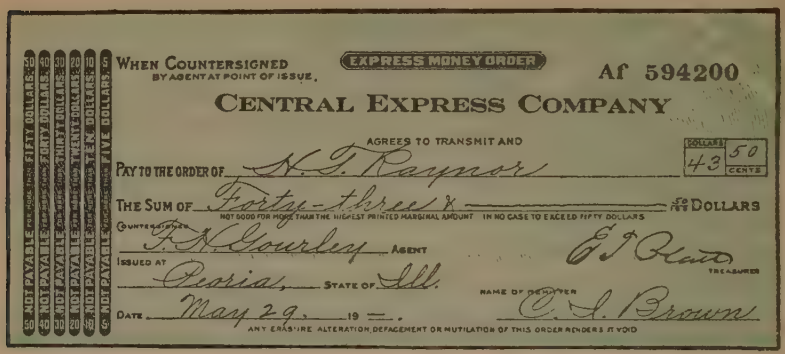
Postal Money Order. The remitter pays to the postmaster the sum of money which he wishes to remit, plus an exchange fee, and receives in return an order upon the postmaster at the point to which the remittance is to be sent, for the amount of money to be remitted. He sends this order to the person to whom he is making remittance, who cashes it at his city.

RATES OF EXCHANGE ON POSTAL MONEY ORDERS

For $2.50 or less	5¢	$20.01 to $40.00	15¢
2.51 to $5.00	7¢	40.01 to 60.00	18¢
5.01 to 10.00	10¢	60.01 to 80.00	20¢
10.01 to 20.00	12¢	80.01 to 100.00	22¢

Express Money Order. This is an order issued by one office of an express company upon another office of the same company. The procedure followed in remitting by express money order is similar to that followed in remitting by postal money order. The

exchange fees are the same as for postal money orders. No single order will be issued for more than $50. Any number of endorsements are allowed.


WHEN COUNTERSIGNED BY AGENT AT POINT OF ISSUE.
EXPRESS MONEY ORDER
Af 594200
CENTRAL EXPRESS COMPANY
AGREES TO TRANSMIT AND
PAY TO THE ORDER OF H. G. Raynor
DOLLARS 43 CENTS 50
THE SUM OF Forty-three
NOT GOOD FOR MORE THAN THE HIGHEST PRINTED MARGINAL AMOUNT IN NO CASE TO EXCEED FIFTY DOLLARS
DOLLARS
COUNTERSIGNED F. H. Gourley AGENT
ISSUED AT Peoria STATE OF Ill.
TREASURER
NAME OF REMITTER C. L. Brown
DATE May 29, 19 —.
ANY ERASURE ALTERATION, DEFACEMENT OR MUTILATION OF THIS ORDER RENDERS IT VOID
NOT PAYABLE FOR MORE THAN FIFTY DOLLARS
NOT PAYABLE FOR MORE THAN FORTY DOLLARS
NOT PAYABLE FOR MORE THAN THIRTY DOLLARS
NOT PAYABLE FOR MORE THAN TWENTY DOLLARS
NOT PAYABLE FOR MORE THAN TEN DOLLARS
NOT PAYABLE FOR MORE THAN FIVE DOLLARS

EXPRESS MONEY ORDER

Remittance by Telegraph. The procedure in remitting by telegraph is the same as that in remitting by postal money order except that the remitter does not receive the order and send it. The telegraph company wires instructions to its manager at the point to which the money is to go, or to some bank there, and the money is paid at once to the party designated.

Bank Draft. This is a draft by one bank upon another. The procedure followed in remitting is similar to that followed in remitting by postal money order. The bank from which the draft is purchased may or may not charge a fee, depending upon its relations with the buyer of the draft.

Money orders and bank drafts are no different from personal checks, as far as the buyer and payee are concerned. But from the standpoint of drawer and drawee there is this distinction: A check is always supposed to be drawn against a balance sufficient to cover it, but this is not true of a money order or a draft, these being paid upon demand, by pre-arrangement between the drawer and the drawee bank, adjustment between the banks being made later.

Domestic Exchange. Sometimes transfers of money from one important financial center to another are at par (100%), sometimes at a premium, and sometimes at a discount. Rates of exchange in such transfers are regulated by the law of supply and demand.

Thus, heavy drafts on New York banks by Chicago banks (without corresponding heavy draft against Chicago by New York) will cause a shortage

of funds on New York at the disposal of Chicago, and rates on New York drafts sold in Chicago will rise, while rates on Chicago drafts sold in New York will decline.

The following rates were quoted in the *Chicago Evening Post*, Oct. 15, 1917. (Quotations are on $1000.)

DOMESTIC EXCHANGE. Exchanges on New York quoted today as follows: Chicago and St. Louis, 10¢ discount; Minneapolis, San Francisco, 10¢ premium; St. Paul, 25¢ premium; Omaha, Detroit, Buffalo, $1.00 premium; Montreal, $1.25 discount. Other leading cities, par.

This means that on the day quoted, Chicago bankers would sell $1000 drafts on New York for $999.90; Buffalo bankers would sell them for $1001.00; etc.

WRITTEN

1. On Oct. 15, 1917, what was the cost of a draft on New York for $2500, issued at St. Paul?

2. A merchant in Montreal sent $3000 to a merchant in New York, by bank draft on Oct. 15, 1917. What did his draft cost him?

3. Find the cost of four postal money orders, for $15.00, $21.35, $62.50, and $84.58 respectively.

4. A Buffalo merchant sent $1500.00 to New York by bank draft on Oct. 15, 1917. Did the merchant pay more or less than he would have had to pay if he had made his remittance by express money order? How much?

5. A Chicago merchant desiring to remit $150,000 to Montreal found that he would have to pay a premium of 50¢ per $1000.00, but that that he could remit to New York at a discount of 15¢, and have his New York agent remit to Montreal at a premium of 10¢. How much did he save by the latter plan?

Foreign Exchange. The mediums principally used in foreign exchange are International Postal Money Orders, Letters of Credit, Travelers' Checks, Cable Transfers, and Bills of Exchange.

International Postal Money Orders are orders similar in nature to the domestic postal money orders, but payable in certain foreign countries where, by treaty with our country, they will be honored.

Letters of Credit. A letter of credit is a circular letter issued by a bank in this country, enabling its holder to secure currency at

various times and places, up to the total amount specified in it. It is addressed to a number of foreign banks with which the bank of issue has business connections, and instructs them to pay to the holder of it, on demand, such amounts as he may require, up to a specified total. Each partial payment is endorsed on the letter. A commission of 1% is charged by the bank of issue, and each payment is made at the rate of exchange on the city of issue at the time and place of such payments.

Travelers' checks are checks issued by express companies in fixed denominations of $10, $20, $50, $100, and $200. Each check is detached and surrendered as it is cashed. A commission of $\frac{1}{2}$% is charged by the issuing company, with a minimum charge of 30¢. To insure against uncertainty as to the amounts that will be received in exchange, the express companies usually guarantee the amounts of the exchanges, in the currencies of the various countries where they will be honored if desired.

Cable transfers are accomplished in much the same way as telegraphic transfers of domestic exchange. The cost of sending money by cable includes the cost of the message and the exchange fee. The money is paid at the exchange rate current at the time, in the denominations of the country where payment is made.

Bills of Exchange. A bill of exchange is a draft drawn in one country and payable in another. It may be either a sight bill or a time bill.

A bill of exchange is usually drawn in duplicate, one copy being called the "first of exchange" and the other the "second of exchange." The two copies are sent by separate routes, each being worded to show that it is payable only on condition that the other still remains unpaid. This plan is to lessen the risk of loss or delay *en route*.

A **documentary bill of exchange** is one which has some document attached to it, such as a bill of lading, invoice, governmental inspection certificate, or other document necessary to the drawee and possession of which he can get only by payment of the bill. The documentary bill of exchange is similar in this respect to a draft with bill of lading attached. A bill that has no document attached is called a "clean" bill of exchange.

Bills of exchange are usually made payable in the currency of the country in which they are payable.

Par of Exchange. The monetary standard in transactions between people in different countries is gold. The par of exchange is the value of the standard gold coin of one country expressed in terms of the standard gold coin of the other.

Thus the pound sterling is worth \$4.8665 par; the franc, \$.192; etc.

Transfers of money may be at par, above par, or below par. Rates of exchange are regulated by the law of supply and demand, as was explained in the section on domestic exchange.

The following quotations on foreign exchange are from an issue of the *Wall Street Journal:*

	Cables	Demand	90 Days
Sterling	\$4.8765	\$4.874	\$4.8435
Francs	$5.16\frac{5}{8}-\frac{1}{32}$	$5.16\frac{1}{4}+\frac{1}{32}$	5.20
Marks	$.95\frac{5}{16}+\frac{1}{32}$	$.95\frac{1}{4}+\frac{1}{32}$	$.94\frac{5}{8}$

The quotation "$5.16\frac{5}{8}-\frac{1}{32}$" means that a special discount of $\frac{1}{32}\%$ is offered; "$+\frac{1}{32}$" means that a special premium of $\frac{1}{32}\%$ is charged.

Pounds sterling are quoted in dollars per English pound. Francs are quoted in francs per U. S. dollar. Marks are quoted in U. S. dollars per 4 marks.

Example. Find the cost of a draft for 3000 marks at $\$.95-\frac{1}{32}\%$.

$3000\times\$.95\div 4=\$712.50$.
$\frac{1}{32}\%$ of $\$712.50=\$.22$, the discount.
$\$712.50-\$.22=\$712.28$, cost of the draft.

Example. What will be the cost of a draft on London for £250 75s. 6d., if exchange is quoted at \$4.86 and a commission charge of 1% is made?

£250 7s. 6d. = £250.375.
$250.375\times\$4.86=\$1216.82$.
$\$1216.82+\$12.17=\$1228.99$, cost of the draft.

Example. What will be the cost of a letter of credit for 5000 francs at $5.18\frac{1}{2}$, if there is a special premium of $\frac{1}{32}\%$ and the commission is 1%?

5000 fr. $\div 5.185=964.32$, the number of dollars.
$\frac{1}{32}\%$ of $\$964.32=\$.30$, the special premium.
$\$964.32+.30=\$964.62$. 1% of $\$964.62=\$9.65$.
$\$964.62+\$9.65=\$974.27$, cost of the letter.

WRITTEN. Find the cost of the following, at the rates quoted above:

1. A demand bill for £150.

2. A cable for 15,000 francs, commission $\frac{1}{4}\%$, and 13 words at 31¢.

3. A cable for 1000 marks, commission $\frac{1}{2}\%$, and 14 words at 38¢.

4. A demand bill for 10,000 francs.

5. A demand bill for 8250 marks.

6. A 90-day bill for 12,000 francs.

7. A cable for 6290 francs, commission $\frac{1}{4}\%$, and 15 words at 25¢.

8. A cable for £235 8s., commission $\frac{1}{2}\%$, and cablegram $1.25.

9. A 90-day bill for 7500 marks.

10. A cable for 8240 marks, commission $\frac{1}{4}\%$, and 14 words at 34¢.

11. A demand bill for 6280 francs.

12. A 90-day bill for £1200 16s.

13. What will be the total paid for travelers' checks amounting to $500?

14. A traveler purchased a letter of credit from Brown Brothers of London, Eng., for £250 at $4.87, the commission charge being 1%. What was the total cost?

15. An importer in New York owed a merchant in London £438 9s. For what amount should the importer draw his check to buy exchange for the amount of this bill at $4.87?

16. The Russian ruble at par is worth 51.5 cents. In 1916, I bought for $301.50 a Russian $5\frac{1}{2}\%$ ten-year bond for 1000 rubles, payable in Russia. If this bond were held until maturity and exchange were then normal, what would be the per cent of return not counting the annual interest?

The Amount of Exchange in Foreign Money

EXAMPLE. What amount of exchange on London can be bought for $3000 at $4.86?

$3000 \div \$4.86 = 617.284$, the number of £ in $3000.
$.284 \times 20 = 5.68$, the number of s. in £ .284.
$.68 \times 12 = 8$d., the number of d. in .68s.

EXAMPLE. What amount of exchange on Paris can be bought for \$1000 at $5.16\frac{5}{8}-\frac{1}{32}$?

$\$1000 \times 5.16\frac{5}{8} = 5166.25$, the number of francs in \$1000 at $5.16\frac{5}{8}$.

$\frac{1}{32}\%$ of 5166.25 francs $= 1.61$ francs.

5166.25 francs $+$ 1.61 francs $=$ 5167.86 francs, the amount of exchange that can be purchased for \$1000.

EXAMPLE. How many marks can be purchased for \$1500 at $95\frac{1}{4}$?

$\$1500 \div \$.95\frac{1}{4} = 1574.803$, the number of 4-marks.

$1574.803 \times 4 = 6299.21$, the number of marks.

WRITTEN. What amount of exchange can be purchased for the following amounts at the prices quoted?

POUNDS STERLING

1. \$1000 at $\$4.86\frac{3}{4}$.
2. \$2500 at $\$4.87\frac{1}{2}$.
3. \$5000 at \$4.87.
4. \$3000 at \$4.866.
5. \$16400 at \$4.855.
6. \$1200 at $\$4.86\frac{3}{8}$.

FRANCS

7. \$500 at $5.16\frac{1}{2}$.
8. \$1200 at $5.18\frac{3}{8}$.
9. \$10450 at $5.21-\frac{1}{32}$.
10. \$3750 at $5.18\frac{3}{8}$.
11. \$3200 at $5.22\frac{1}{2}+\frac{1}{32}$.

MARKS

12. \$2000 at $94\frac{1}{2}$.
13. \$12380 at $95\frac{1}{4}$.
14. \$6500 at $94\frac{1}{2}-\frac{1}{32}$.
15. \$1680 at $94\frac{3}{8}+\frac{1}{32}$.
16. \$375 at $95\frac{1}{4}-\frac{1}{32}$.

17. How large a draft on London can be purchased for \$800 with exchange at $\$4.89\frac{3}{4}$?

18. A man wishes to remit \$1500 to Paris. How large a bill can be purchased with exchange at $5.18\frac{1}{2}-\frac{1}{32}$?

CHAPTER XXXII

STOCKS

A **corporation** is an organization formed under authority of the state, endowed with a continuous legal existence irrespective of that of the individuals composing it at any time, and authorized to transact business as a single individual.

The **capital stock** of a corporation is the amount authorized to be paid in to its treasury for the conduct of its business and the benefit of its creditors.

Capital stock must be distinguished from capital. The capital stock is a fixed amount prescribed by the charter. The capital is the amount of net assets on hand at any time. The first business transaction of the corporation may occasion a profit (or a loss), and thereupon the capital will be more (or less) than the capital stock.

Capital Stock $100,000.00

1,000 Shares of $100.00 each

The Acme Manufacturing Company

Stock Certificate No. 2

This Certifies that H. M. Miller

is entitled to 200 shares of One Hundred Dollars each, in the capital stock of the Acme Manufacturing Company, fully paid and non-assessable, transferable only on the books of the Corporation by the holder hereof in person or by attorney upon surrender of this certificate properly endorsed.

In Witness whereof, the said corporation has caused this certificate to be signed by its duly authorized officers, and to be sealed with the Seal of the Corporation, at Chicago, Ill. this 27th day of May A. D. 19—

SEAL

C. J. Barber
Secretary and Treasurer.

J. E. Colby
President.

A Certificate of Stock

Shares. The capital stock of a corporation is divided into shares, usually of $100 each. These shares, represented by certificates of stock, are sold to individuals. The owners of shares are known as shareholders, and as such participate in the profits of the corporation.

The extent of a shareholder's participation depends upon the number and kind of shares which he holds.

Common and Preferred Stock. There are many different kinds of stock issued. The kinds usually issued are common stock and preferred stock. Preferred stock is stock which is sold under an agreement that the holders will be given preference over holders of common stock in the distribution of profits, or of assets, or both. Common stock is ordinary stock which carries with it no such preference.

"6% preferred" stock means stock that is issued under an agreement that $6.00 a share will be paid to its holders before holders of common stock shall receive any dividends. *Cumulative preferred* stock is stock sold under an agreement that if at any dividend period the dividend at the agreed rate cannot be paid, the holder shall continue to be entitled to it until it can be paid. *Participating preferred* stock is stock that does not so participate. It frequently occurs, when profits have been large, that greater dividends are paid on common stock than on non-participating preferred stock.

A **dividend** is an amount of money paid by a corporation to its stockholders, each stockholder receiving the part to which he is entitled by the number of shares which he holds.

Dividends are usually either annual, semi-annual, or quarterly. The directors of the corporation decide how much in dividends shall be paid, and when. Dividends must be paid out of profits. It is unlawful to impair the capital of a company by the payment of dividends.

When a sale of stock is made, the dividend goes to the purchaser unless the sale is made "ex-dividend." This term means "without dividend," the dividend being reserved by the seller.

The **par value** of stock is its face value.

The **market value** of stock is the price at which it can be bought.

The **book value** of a share of stock is that part of the net assets of the corporation, as shown by its books, which corresponds to the part of the capital stock which one share represents.

Since the capital, surplus, and undivided profits represent the net assets of a corporation, the quickest method of finding book value is to divide the total of these three accounts by the number of shares of stock.

The **price** of stock may be quoted in dollars per share, as a per cent, or in cents on the dollar.

Thus, $95.50 per share, $95\frac{1}{2}\%$, or $95\frac{1}{2}$ (meaning $95\frac{1}{2}$ cents on the dollar) all mean the same thing, in case of stock at $100 par.

WRITTEN

1. A corporation is capitalized at $100,000. Its directors declare a dividend of $3\frac{1}{2}\%$. (This means it will pay to shareholders out of profits, an amount equal to $3\frac{1}{2}\%$ of its capital.) To how much is Smith entitled, if he owns 50 shares?

NOTE. Unless otherwise specified, par value of $100 per share is meant.

2. A corporation capitalized at $250,000 of common stock and $100,000 of 6% preferred stock, had earnings of $12,250 available for dividend. What dividend could be declared on common stock?

3. A corporation capitalized at $300,000 of common stock and $200,000 of $5\frac{1}{2}\%$ non-participating preferred, declared a dividend of $22,000. Brown owned 50 shares of preferred stock and 30 shares of common. What amount of dividend did he get?

4. Jones paid $95 per share for stock that yielded him, on an average, 4%. To what interest on his money was this equivalent?

5. To what rate of interest is the earning equivalent in each of the following cases, averaged for the three years?

	No. of Shares	Par Value	Price Paid	Earning 1st Year	Earning 2d Year	Earning 3d Year
(*a*)	50	$100.00	$5450.00	5%	6%	$5.50 per share
(*b*)	2	100.00	195.00	$4\frac{1}{2}\%$	$4\frac{1}{2}\%$	4.50 per share
(*c*)	100	100.00	11500.00	6%	5%	$450.00
(*d*)	75	50.00	3000.00	4%	5%	$3.00 per share

6. How much would you be willing to pay for a share of stock, par value $100, if you wanted 5% on your investment and thought that the stock would yield 5% dividend? If you thought the stock would yield 4%? 6%? 3%? $5\frac{1}{2}\%$?

7. A man with $50,000 to invest was offered stock A, paying 5% dividends, at $95 per share; stock B, paying $4\frac{1}{2}\%$ dividends, at $86 a share, and stock C, paying 7% dividends, at $112.50. If the stocks were equally trustworthy, which did he buy, how many shares did he buy, and what was the unexpended balance?

8. A stock on which an annual dividend of 7% had been declared payable on Jan. 1 was sold on Dec. 1, "ex-dividend,"

for $90.00. What could the purchaser have afforded to pay had he been entitled to the dividend?

9. The net assets of a corporation capitalized at $150,000 were $225,500. What was the book value of its stock?

10. A corporation was capitalized at a million dollars. In twenty years it had assets of $20,000,000 and liabilities of $4,550,000. What was the book value of its stock then?

A **stock broker** is one who acts as an agent for others in buying and selling stocks. He charges for this service a fee, usually $12\frac{1}{2}¢$ per share ($\frac{1}{8}$ of 1% of the par value of stock at $100 a share).

Listed Stocks. The stock exchange publishes a list of stocks which may be bought and sold on the floor of the exchange. To get a stock listed, those promoting it must meet certain requirements of the stock exchange as to publication of profits, etc.

Unlisted Stocks. These are stocks which are not listed as above. They may be dealt in by members of the stock exchange, but not on the floor. A separate list of these is published under the head of *unlisted stocks*. As they may not be dealt in on the floor of the stock exchange, they have come to be called street or "curb" stocks.

WRITTEN. The broker's fee is $12\frac{1}{2}¢$ per share in each case. The prices quoted are in dollars per share.

1. Find the total cost of the following stocks purchased through a broker:

150 shares American Cotton Oil at $39\frac{1}{8}$.
70 shares Baldwin Locomotive at 244.
200 shares Canadian Pacific at 204.
125 shares Corn Products, comm., at $67\frac{5}{8}$.

2. Find the total returns from the following sales made through a broker:

350 shares American Smelters, pf., at $182\frac{1}{2}$.
125 shares Brooklyn M. Tran., at $56\frac{7}{8}$.
50 shares C. M. & St. P., at $17\frac{7}{8}$.
40 shares Penna. R. R., at $62\frac{1}{2}$.
110 shares Studebaker, pf., at $125\frac{1}{2}$.

3. Find the total profit or loss on stock purchased and sold through a broker at the following prices:

125 shares Anaconda, bot $58\frac{1}{4}$, sold $57\frac{3}{4}$.
75 shares Ches. & Ohio, bot $193\frac{1}{8}$, sold 191.
120 shares Inter. Harv., bot $235\frac{1}{2}$, sold 237.

4. An investor sent \$10,000 to a broker to be invested in Western Union Telegraph Co. stock. The stock was bought at 169. How many shares did the broker buy, and what was the unexpended balance?

5. Lee, Higginson and Co. receive \$10,500.00 from a customer who instructs them to purchase for his account 250 shares of Tennessee Copper at 11 and to invest the remainder in Am. Tobacco at $164\frac{5}{8}$. How many shares of Am. Tobacco were purchased, and what was the unexpended balance?

6. I sent Wm. A. Read and Co. 250 shares Penna. R. R., instructing them to sell it and buy Am. Cotton Oil. The Penna. R. R. was sold at $63\frac{1}{2}$, and the Am. Cotton Oil was bought at $39\frac{1}{8}$. How many shares of the latter did I receive, and what was the unexpended balance?

7. A semi-annual dividend of $2\frac{1}{2}\%$ on stock bought through a broker at $101\frac{1}{8}$ is equivalent to what annual rate of interest?

NOTE. Since a mid-year dividend could be loaned out at interest for the year, semi-annual and quarterly dividends are better than annual dividends. The difference is so slight, however, that for practical purposes it may be ignored. Treat this as a 5% annual dividend.

8. To what rate of interest is a quarterly dividend of 75¢ per share equal, on stock bought through a broker at $62\frac{1}{2}$?

9. To what rate of interest is a semi-annual dividend of $2\frac{1}{4}\%$ equal, on stock bought through a broker at \$88.50 per share?

10. A trustee for an estate invested the net assets of the trust in the following stocks, purchasing through a broker:

150 sh. Am. Locomotive, at 110, semi-ann. div. \$3.65.
190 sh. Gen. Electric, at $130\frac{1}{2}$, annual div., 3.1%.
1852 sh. Great Nor., pf., at 95, semi-ann. div., 2.6.
830 sh. Studebaker, pf., at $125\frac{1}{2}$ quarterly div., \$1.40.

If dividends were received as indicated, how much more or less did the investments yield than would have been received from an investment of the entire estate at 5%?

The Investment Necessary to Produce a Certain Income

EXAMPLE. How much must be invested in the stock of the Pullman Co., at $83\frac{1}{4}$, if this stock pays a quarterly dividend of 2%, in order to have an annual income of \$2000, usual brokerage?

1.2% = the quarterly rate of dividend. 4.8% = the annual rate of dividend. \$4.80 = the dividend on 1 share (\$100 par value). \$2000 ÷ \$4.80 = 416, shares of stock. $416 \times 83\frac{3}{8}$ $(83\frac{1}{4} + \frac{1}{8}) = \$34{,}684.00$, sum to be invested.

WRITTEN

1. How large an investment in American Railway Express stock, paying 1.2% quarterly, and selling at 125, brokerage $\frac{1}{8}$, would be required to provide an annual income of $1500?

2. How many shares of National Biscuit Co. stock must be purchased and what would they cost at $172\frac{1}{2}$, to provide an annual income of $3500, if this stock pays a semi-annual dividend of $1\frac{3}{4}$%?

Figure no brokerage unless problem is stated so as to require it.

3. A man owned 750 shares of American Shipbldg. Co. stock which paid a quarterly dividend of 1.9%. It was provided in his will that enough of this stock be set aside to provide an annual income of $2400 for maintaining a local library and that the remainder go to a hospital. If this stock were worth $105\frac{1}{2}$ in the market, what was the value of the stock bequeathed to the hospital?

4. A man directed, in his will, that enough 5% stock be set aside in the hands of a trustee to give his daughter an annual income of $1600. The executors transferred to the trustee sufficient National Biscuit Co. preferred stock to satisfy the bequest. How many shares did they transfer, and what was its value at $140\frac{1}{4}$?

Miscellaneous Problems

1. An investor bought stock at $62\frac{7}{8}$ plus brokerage. The stock yielded $1\frac{1}{4}$% quarterly. To what annual rate of interest on investment was this equivalent?

2. An investor desired to realize 6% interest. What price per share must he pay, buying through a broker, for stock yielding $2\frac{1}{4}$% quarterly?

3. An investor bought 150 shares Missouri-Pacific preferred stock at $107\frac{1}{2}$, brokerage $\frac{1}{8}$, and received dividends at the rate of 4.5% annually. What rate did he receive upon his investment?

4. If the Quaker Oats Co. stock pays regularly quarterly dividends of $1\frac{1}{2}$%, what can an investor pay per share to realize 5% upon his investment? (No brokerage.)

5. The Corn Products Refining Co. preferred stock at one time sold at $67\frac{5}{8}$, and paid a quarterly dividend of $.7\frac{1}{4}$ of 1%. What rate upon the investment did this stock yield? (No brokerage.)

6. Find the annual rate of income upon the total investment in the following stocks, bought through brokers:

100 shares D. L. and W. R. R. Co., price \$131$\frac{1}{2}$, dividends 4.6%.

150 shares Int. Nickel, common, price \$94, dividends 1$\frac{3}{4}$%.

220 shares National Biscuit Co., pref., price \$140.25, dividends 1$\frac{1}{4}$% quarterly.

300 shares Westinghouse Elect., price \$76.25, dividends 1% quarterly.

7. What can an investor afford to pay for stock yielding 2% quarterly, to realize 6% upon his investment?

8. Which is the better investment, and how much—United Fruit Co. stock at 137$\frac{1}{2}$, paying $\frac{3}{4}$% quarterly, or U. S. Smelting at 42$\frac{1}{4}$, paying 1$\frac{1}{2}$% quarterly, brokerage in each case $\frac{1}{8}$%?

9. Pacific Telephone and Telegraph stock, paying 1$\frac{1}{2}$% quarterly, was bought at a price that yielded the purchaser 4.8% on his investment. How much was paid for 128 shares of this stock?

10. Which is the better investment — stock at 127$\frac{3}{4}$ which pays semi-annual dividends of 3$\frac{1}{2}$%, or stock at 71$\frac{1}{8}$ which pays 1% quarterly, allowing the usual brokerage in each case?

11. John Kearny had a farm mortgage of \$3500 which paid 5$\frac{1}{2}$% interest. When the mortgage was due he received the money and invested it in National Biscuit Co. stock at 172$\frac{1}{2}$, brokerage $\frac{1}{8}$, which paid a quarterly dividend of 1$\frac{3}{4}$%. At this rate, what will be the change in his income in one year, if he puts the remainder, above an integral number of shares, in a savings bank at 4% interest compounded semi-annually?

Purchase of Stock on Margin

Frequently sales of stock are made on a margin, that is, a certain per cent of the par value of the stock is paid in cash and the remainder is paid at some future time. The cash payment is referred to as the *margin*.

In this form of purchase the broker either loans the remainder of the purchase price or borrows it at a bank and gives the stock as security for the loan.

In any event the purchaser does not get possession of the stock. If the stock declines in price the broker may demand a further deposit of cash and if the purchaser is unable to meet this demand, the stock is sold by the broker to prevent loss to himself.

EXAMPLE. On June 1, H. M. Owen purchased through Elliott and Company 200 shares Union Pacific at 152$\frac{5}{8}$, depositing 10%

of par value as margin at the time of purchase. On July 3, he sold out at $162\frac{3}{8}$. What was his profit, allowing $\frac{1}{8}\%$ for brokerage in buying and selling and figuring interest at 6%?

CHICAGO, ILL., June 1, 19__

M H M. OWEN

DECATUR, ILL.

In Account with

ELLIOTT AND COMPANY

DATE 19—			AMT.		DA.	INT.		DATE 19—			AMT.		DA.	INT.	
June	1	To 200 shares Union Pacific at 152 5/8	30525	00	32	162	80	June	1	By cash margin	2000	00	32	10	67
	1	To brokerage	25	00	32		13	July	3	By 200 shares Union Pacific at 162-3/8	32475	00			
July	3	To brokerage	25	00					3	To bal. int. (red)				152	26
	3	To int. bal.	152	26											
	3	To bal. due you	3747	74											
			34475	00		162	93				34475	00		162	93

Mr. Owen's profit is the difference between the balance due him and the margin originally advanced. \$3747.74 − \$2000.00 = \$1747.74.

WRITTEN. Prepare the following in statement form, as in the model. Find time by compound subtraction.

1. On May 15, I bought 250 shares of Norfolk and Western at $185\frac{1}{4}$, on a 10% margin. On July 24 the stock was sold at $182\frac{3}{8}$. Allowing $\frac{1}{8}\%$ brokerage in buying and selling, and figuring interest at 6%, did I gain or lose, and how much?

2. How much profit would be made by buying 150 shares Republic Iron and Steel at $69\frac{3}{4}$, on a 10% margin, and selling 39 days later at $77\frac{1}{2}$, if the usual brokerage is charged in buying and selling, and interest is at 7%?

3. James Bartlett bought 50 shares Delaware and Hudson at 147, brokerage $\frac{1}{8}\%$, in a 10% margin, on May 24. On June 30 a semi-annual dividend of $3\frac{1}{2}\%$ was paid. On July 12 the stock was sold at $151\frac{3}{8}$. Interest, 6%. What was Mr. Bartlett's profit?

CHAPTER XXXIII

BONDS

A **bond**, in a financial sense, is a written promise, under seal, to pay a specified sum of money at a fixed time, and to pay interest thereon at a stipulated rate at fixed intervals.

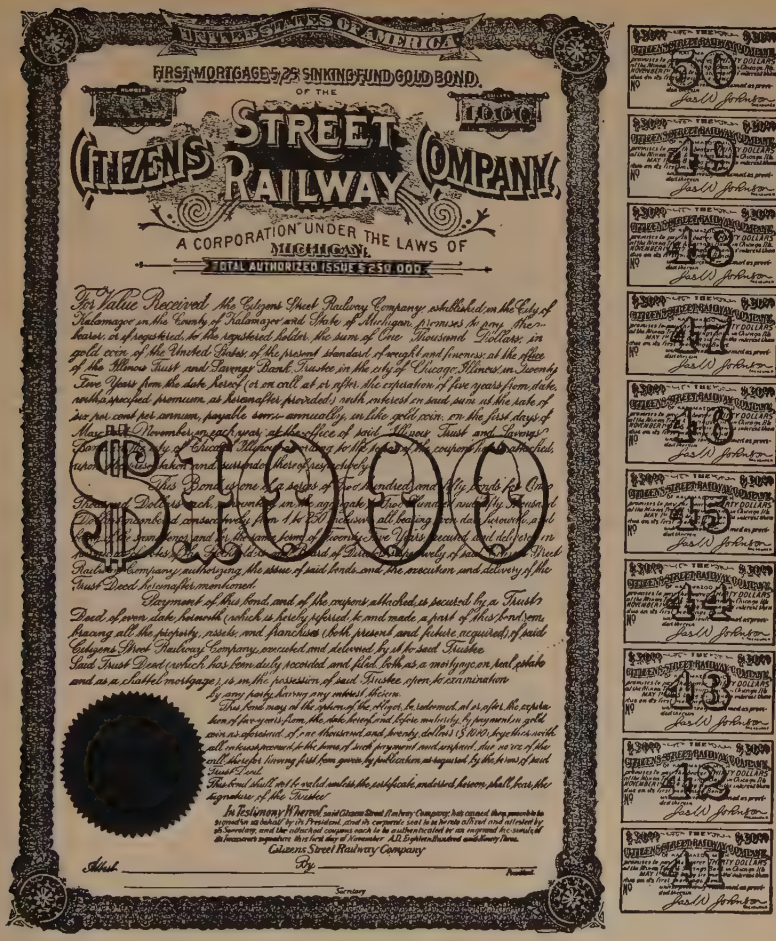
Broadly speaking, the issuance of bonds is a device for borrowing from many investors small amounts which in the aggregate make a large sum. Suppose that X, a corporation, wants to borrow $100,000. It is unable or unwilling to borrow the entire amount on a note payable to one person. It therefore issues 1000 bonds for $100 each, which are sold to the public. Security for the bonds, if any, is placed in the hands of a trustee, who holds it in trust for the bondholders.

A **coupon bond** is a bond to which are attached interest coupons, one coupon for each item of interest to be paid. Each coupon shows the date and amount of interest due, and as each falls due it is detached by the bondholder and presented for payment.

A **registered bond** is one which is payable only to the order of a bondholder specifically named (registered) on the books of the person, firm, or corporation issuing the bonds.

It may be registered as to principal only, in which case the interest would be represented by coupons payable to bearer; or it may be registered as to principal and interest, in which case the interest as well as the principal would be payable only to the order of the person in whose name it was registered.

It is customary to list a bond in such a way as to show by whom it was issued, when it is due, the rate of interest it bears, and some other important fact or facts. Thus *U. S. 4s, 1925 registered* are 4% registered bonds issued by the United States, payable in 1925. *Westinghouse convertible 5s, 1920 coupons*, are 5% coupon bonds issued by the Westinghouse Company, payable in 1920 but convertible into stock, if desired, under stipulated conditions. *First Mortgage* bonds are bonds secured by a first mortgage. *Refunding* bonds are bonds issued to raise money for the redemption of bonds previously issued. *Extension* bonds are bonds issued to secure money for extensions of properties. *Sinking Fund* bonds are bonds providing under the terms of their issue, for the creation of a sinking fund for their retirement. *Debentures* are unsecured bonds. *Income bonds* are bonds which pay interest only if justified by the income of the business for which the bonds were issued. *Consolidated* bonds (as U. S. Consol. 2s, 1930) are usually bonds issued in place of several previous issues, which have been consolidated in the one issue, and so on.


UNITED STATES OF AMERICA
FIRST MORTGAGE 5% SINKING FUND GOLD BOND
OF THE
$1000
CITIZENS STREET RAILWAY COMPANY
A CORPORATION UNDER THE LAWS OF MICHIGAN
TOTAL AUTHORIZED ISSUE $250 000
For Value Received, the Citizens Street Railway Company established in the City of Kalamazoo in the County of Kalamazoo and State of Michigan promises to pay the bearer, or if registered, to the registered holder, the sum of One Thousand Dollars, in gold coin of the United States of the present standard of weight and fineness at the office of the Illinois Trust and Savings Bank Trustee in the city of Chicago, Illinois, in twenty five Years from the date hereof or on call at or after the expiration of five years from date, with a specified premium as hereinafter provided, with interest on said sum at the rate of five per cent per annum, payable semi-annually, in like gold coin, on the first days of May and November, on each year, at the office of said Illinois Trust and Savings
$1000
Statement of this bond, and of the coupons attached, is secured by a Trust
Deed of even date herewith (which is hereby referred to and made a part of this bond) con
veying all the property, assets, and franchises (both present and future acquired) of said
Citizens Street Railway Company, executed and delivered by it to said Trustee.
Said Trust Deed (which has been duly recorded and filed both as a mortgage on real estate
and as a chattel mortgage) is in the possession of said Trustee open for examination
by any party having any interest therein.
This bond may at the option of the holder be redeemed at or after the expiration of five years from the date hereof and before maturity by payment in gold coin as aforesaid of one thousand and twenty dollars ($1020) buy the owner will all interest accrued to the time of such payment and unpaid due on the call the right having first been given by publication as required by the terms of said Trust Deed.
This bond shall not be valid unless the certificate endorsed hereon shall have the signature of the Trustee
In Testimony Whereof said Citizens Street Railway Company has caused these presents to be signed for it on behalf by its President, and its corporate seal to be hereto affixed and attested by its Secretary, and the interest coupons made to be authenticated by the engraved fac-simile of the Secretary's signature this first day of November A.D. Eighteen Hundred and Ninety Three.
Citizens Street Railway Company
Attest _____ By _____ President
Secretary
$2000 THE $2000
CITIZENS STREET RAILWAY COMPANY
50
$2000 THE $2000
CITIZENS STREET RAILWAY COMPANY
49
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$2000 THE $2000
CITIZENS STREET RAILWAY COMPANY
41

COUPON BOND

To find the cost of a bond.

Since bonds draw interest, it is understood that the purchaser must pay the quoted price plus interest accrued since the last payment of interest.

Thus, in case of a $100.00 bond, interest payable Jan. 1 and July 1, and sold on Mar. 1 at $97\frac{1}{2}$, the price would be found by adding to $97.50 the interest on the par value of the bond ($100.00) for two months.

EXAMPLE. On Sept. 10, I bought \$5000 Morris & Co. $4\frac{1}{2}$s at $88\frac{7}{8}$ and interest, brokerage $\frac{1}{8}$. If interest is payable Jan. 1 and July 1, what did they cost?

\$5000 × (.88$\frac{7}{8}$ + .00$\frac{1}{8}$) = \$4450.
Int. on \$5000 July 1 to Sept. 10 (2 mo. 9 da.) = \$43.13.
\$4450 + \$43.13 = \$4493.13, cost.

WRITTEN. Find the cost of each of the following bonds including brokerage at $\frac{1}{8}\%$, and interest.

	Amount	Name of Bond and Rate	Price	Interest Payable	When Purchased
1.	\$10,000	American Can 5s.	94$\frac{1}{4}$ and int.	Feb. 1 and Aug. 1	Mar. 22
2.	15,000	Chi. Ry. Purchase 4s	70$\frac{1}{4}$ and int.	Jan. 1 and July 1	Aug. 15
3.	28,800	Diamond Match Debenture 6s	105$\frac{1}{2}$ and int.	June 1 and Dec. 1	Oct. 5
4.	5,000	Schwarzchild & Sulzberger 6s	100$\frac{1}{4}$ and int.	June 1 and Dec. 1	Sept. 20
5.	83,000	Swift and Co. 5s	101$\frac{1}{2}$ and int.	Jan. 1 and July 1	Apr. 18
6.	24,000	Union Ele. R.R. 5s	88 and int.	Apr. 1 and Oct. 1	June 25
7.	3000	Cudahy & Co. 5s	100$\frac{5}{8}$ and int.	May 1 and Nov. 1	Aug. 20
8.	55,000	Chicago Ry. 5s	99$\frac{1}{2}$ and int.	Feb. 1 and Aug. 1	May 9
9.	92,000	South Side Elev. Ry. Co., gold 4$\frac{1}{2}$s	96$\frac{1}{2}$ and int.	Jan. 1 and July 1	June 18

To find the present value of a bond.

EXAMPLE. What can I afford to pay for a 6% bond, par value \$100.00, interest payable semi-annually, if it matures in $15\frac{1}{2}$ years, to earn 4%?

To find the present value of a bond, two elements must be considered: (1) The relation of the rate of interest on the bond to the rate it is desired to earn; (2) the time the bond has to run.

If the first element only were considered, I would be willing to pay \$150 for such a bond (\$3.00 ÷ .02 = \$150) because the interest on \$150 at 4% for a half year (\$3.00) is the same as the interest on the bond for the same period (3% of \$100 = \$3.00).

But suppose I should pay \$150. In $15\frac{1}{2}$ years the bond would be redeemed for \$100 and I should lose \$50.00. It is evident that I must pay less than \$150, by an amount which in $15\frac{1}{2}$ years will amount to \$50, at 4% interest compounded semi-annually (semi-annual interest is used for consistency, since the interest on the bonds is semi-annual). The compound interest tables show that \$1.00 will amount, compounded 31 times at 2%, to \$1.84758882. \$50 ÷ 1.84758882 = \$27.06. \$150 − \$27.06 = \$122.94, the present value of the bond.

NOTE. The present value of a bond is the total of the present worths of all future returns of interest and principal. Calculations may be made from present worth tables.

NOTE. The present value, plus compound interest to maturity at the rate of earning desired, must equal the total of all returns, each plus interest at the rate the bond bears, from the time it is made until maturity. Compound interest tables are needed to make this calculation.

Bond Tables. To facilitate calculations of bond values, tables are used, of which a specimen is shown herewith. Since interest on bonds is usually payable semi-annually, these tables are compiled on this basis.

Assuming that the student will wish some further explanation of the reasoning involved in figuring the value of a bond, and will wish to verify at least one of the figures given in the bond tables, the following short example and solution are presented.

15½ YEARS—Interest Payable Semi-annually.

Net per annum	BONDS BEARING INTEREST AT THE RATE OF						
	7%	6%	5%	4½%	4%	3½%	3%
4	134 41	122.94	111.47	105.73	100.00	94.27	88.53
4.10	133 03	121 64	110.25	104.56	98.86	93.17	87.47
4⅛	132.68	121.32	109.95	104.26	98.58	92.89	87.21
4.20	131.66	120.35	109.05	103.39	97.74	92.08	86.43
4¼	130.99	119.72	108.45	102.82	97.18	91.55	85.91
4.30	130.32	119.09	107 86	102.25	96.63	91 02	85.40
4⅜	129.32	118.15	106.98	101.40	95.81	90.23	84.64
4.40	128.99	117 84	106 69	101.12	95.54	89.96	84.39
4½	127.68	116.61	105.54	100.00	94.46	88.93	83.39
4.60	126 39	115.40	104.40	98.90	93.40	87.90	82.41
4⅝	126.07	115.09	104.12	98.63	93.14	87.65	82.16
4.70	125.12	114.20	103.28	97.82	92.36	86.89	81.43
4¾	124.49	113.60	102.72	97.28	91.84	86.40	80.95
4.80	123.86	113 01	102.17	96.75	91.32	85 90	80 48
4⅞	122.93	112.14	101.35	95.95	90.56	85.16	79.77
4.90	122.62	111 85	101 08	95 69	90 31	84.92	79 53
5	121.40	110.70	100.00	94.65	89.30	83.95	78.60
5.10	120.19	109.56	98 94	93 63	88.31	83 00	77 69
5⅛	119.89	109.28	98.67	93.37	88.07	82.76	77.46
5 20	118.99	108.44	97.89	92 61	87.34	82.06	76.78
5¼	118.40	107.89	97.37	92.11	86.85	81.60	76.34
5.30	117 82	107.34	96 86	91.62	86.37	81 13	75.89
5⅜	116.95	106.52	96.09	90.88	85.66	80.45	75.23
5.40	116.66	106.25	95.84	90 63	85.43	80.22	75 02
5½	115.51	105.17	94.83	89.66	84.49	79.32	74.15
5⅝	114.10	103.85	93.59	88.46	83.34	78.21	73.08
5¾	112.71	102.54	92.37	87.29	82.21	77.12	72.04
5⅞	111.34	101.26	91.18	86 13	81.09	76.05	71.01
6	110.00	100.00	90.00	85.00	80.00	75.00	70.00
6¼	107 38	97 54	87.70	82.79	77.87	72.95	68.03
6½	104.84	95.16	85.49	80.65	75.81	70.97	66.13
6¾	102.38	92.86	83.34	78 58	73.82	69.06	64.30
7	100.00	90.63	81.26	76.58	71.90	67.21	62.53

A BOND TABLE

EXAMPLE. What is the value on Jan. 1, 1928, of a 5% bond for $100 maturing July 1, 1929, interest payable semi-annually, in order to yield to the purchaser 3% on his investment?

(*a*) Amount for which bond will be redeemed July 1, 1929 ..... $100.00
Interest received by bondholder July 1, 1929 ..... 2.50
Value of bond before interest was paid $102.50

(*b*) Since bondholder was promised 3% interest on his investment (.015 each half-year) the present worth of the bond on Jan. 1, 1929, was $102.50 ÷ 1.015 = ... $100.98
Interest received by bondholder Jan. 1, 1919 ..... 2.50
Value of bond before interest was paid $103.48

(*c*) Since bondholder was promised .015 int. each half year, the present worth of the bond on July 1, 1928 was $103.48 ÷ 1.015 = ... $101.95
Interest received by bondholder July 1, 1928..... 2.50
Value of bond before interest was paid..... $104.45

(*d*) The present worth of the bond on Jan. 1, 1928 was $104.45 ÷ 1.015 = ... $102.91

WRITTEN. Use the tables for all problems after the 3d.

1. What can I pay for a $7\frac{1}{2}\%$ bond, par value $500, interest payable semi-annually, if it matures in 10 years, to earn 6%? (Follow the reasoning given at the bottom of page 396.)

2. What is the value on Jan. 1, 1928, of a 7% bond for $250, maturing July 1, 1929, interest payable semi-annually, to yield 4%? (Follow the plan of the solution given on page 397.)

3. Prove the above problem by working backwards from your result.

4. Using the table, find the value of a 7% bond for $1000, interest payable semi-annually, to mature in $15\frac{1}{2}$ years, to net 4% per annum.

5. What is the value on Jan. 1, 1928, of a 6% bond maturing on July 1, 1943, if interest on investment is calculated at $4\frac{1}{2}\%$?

6. A 5% bond, par $100, maturing in $15\frac{1}{2}$ years, interest payable semi-annually, cost me $94.83. What rate of interest did I receive on my investment?

7. Brown paid $400 for bonds yielding 4% per annum payable semi-annually. If the par value of his bonds was $500, and they matured in $15\frac{1}{2}$ years, what rate did they net him?

8. Smith owns the bonds listed below, all maturing in $15\frac{1}{2}$ years, interest payable semi-annually in each case. What is his total investment? His total returns?

$5000 Amer. Tel. & Tel. Conv. $4\frac{1}{2}$s, bought to net 5%.
$8000 Armour $4\frac{1}{2}$s, bought to net $5\frac{1}{2}\%$.
$12,500 Brooklyn R. T. Conv. 4s, bought to net 5%.
$6000 Gen'l Motor 6s, bought to net $5\frac{1}{2}\%$.

9. On Oct. 25, 1924, you gave H. E. Smith your note for $3000 at 6%, due in 4 years, interest payable annually. The several installments of interest were evidenced by coupons attached to the principal note, each bearing simple interest at 6%. What was the present worth of the note and coupons? If nothing was paid on

either principal or interest until Oct. 25, 1928, what amount was then due? Write the principal note and coupons.

10. What is the value, on July 1, 1926, of a 6% bond for $1500, maturing July 1, 1928, interest payable semi-annually, to yield 5%? Prove this.

11. What is the present worth of a 7% bond for $1000, maturing in 12 years, to net 5%, if the interest on the bond is payable semi-annually?

CHAPTER XXXIV

ANNUITIES

An **annuity** is an amount of money payable at stated intervals (usually yearly) through a given number of years.*

An insurance company, upon the death of B, who was insured for $10,000, offered to pay his widow $2310 a year for five years. This payment of $2310 a year was an annuity.

The word *annuity* is loosely used to mean any sort of an annual payment, as a pension, an allowance, or a contribution to a sinking fund. Its use here, however, is confined strictly to *equal periodical installments necessary to retire an interest-bearing debt.*

NOTE. Sinking funds, as hereafter more fully treated, usually are created for the retirement at some future date of a debt of a specified amount, which is either non-interest bearing or the interest on which is paid from year to year so that the principal does not increase.

EXAMPLE. An insurance company owes Mrs. B. $6000. Instead of paying this at once the company offers to make five equal annual payments, the first payable at once. What is the amount of this annuity, on a 5% basis?

The last payment will be made in 4 years.

Installments plus interest must equal debt plus interest.

$6000. at 5% compounded annually for 4 years will amount to (see compound interest table) $6000 \times 1.2155063 = \$7293.04$.

Installments of $1.00 a year paid annually for 5 years will at the end of 4 yr., at 5% compounded annually, amount to $4.525631 (see compound interest amount table). Adding $1.00 paid at beginning of 5th yr. gives $5.525631.

$\$7293.04 \div 5.525631 = \$1319.86$, amount of annuity.

EXAMPLE. I borrowed $6000 for 5 yr., at 5%. I desire to pay this back in five payments. What annual payment is required?

The first payment will be made in one year; the last, in five years.

$6000. at 5% compounded annually for 5 years will amount to (see compound interest table) $6000 \times \$1.2762816 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\$7657.69$

Installments of $1.00 a year paid annually for 4 years (one each at beginning of 2d, 3d, 4th, and 5th yr.) will amount to $4.525631 (see compound interest amount table). Adding $1.00 paid at end of 5th yr. gives $\ldots\ldots\ldots\$5.525631$

$\$7657.69 \div 5.525631 = \$1385.85$, amount of annuity.

* The number of years, in an insurance annuity, is often the number of years shown by the table of mortality to be the beneficiary's expectation of life.

WRITTEN

1. Mrs. A. has a claim of $5000 against the B. Insurance Company. The company offers to pay this claim, with interest at 4%, in four equal annual installments, the first payable at once. If Mrs. A. accepts this offer, how much money will she receive each year?

2. What would be your answer to problem 1 if the first installment had not been payable for a year?

3. Jordan has an endowment policy for $7500 which has just matured. Not needing the money at once, he asks the insurance company to pay it to him, with interest at 5%, in 10 equal annual installments, the first payable at once. What is the amount of the annuity?

4. Brown borrows $7500 at 5%. What annuity will discharge this debt in 10 years, if the first payment is to be made one year from the date of the loan?

5. You are employed by a life insurance company to prepare them a schedule of annuities per $1000 to be payable annually for 5, 10, or 20 years, on a 4% basis.

6. Jones borrowed $9000 on Jan. 1, 1927, for six years at 6%, giving a note for $9000 with six serial coupons attached, each for 1 year's interest. The first interest coupon was due Jan. 1, 1928. Each coupon bore simple interest at 6%. What is the amount of each of 6 annual investments, all but the last drawing 6% compounded annually, investments plus interest to equal the amount of the debt plus interest at maturity, no interest having been paid?

7. I issue a mortgage today for $5600 due in 3 years. It bears simple interest payable annually at 5%, and each item of interest draws simple interest until paid. What sum of money set aside each year with interest compounded annually at 5% per annum will enable me to pay the mortgage and all interest at maturity, no interest having been paid?

8. Smith issues a note today for $6200 due in 3 years. It bears interest compounded annually at 4%. What sum of money set aside each year at 4% interest compounded annually will enable him to pay the note at maturity?

CHAPTER XXXV

FINANCING

Most of the subjects in this chapter are treated from the standpoint of the corporation. This is in most instances for convenience (because corporation terminology is rich in words precisely adapted to the purpose) and not from necessity. Problems in capitalization, borrowing, repaying loans, profit distribution, etc., are quite as important in a privately owned business as in a corporation.

Capitalization

The **capital value** of an earning, at a rate of profit specified, is the amount of capital which at that rate would yield the amount of the earning.

The amount of the capital of a proposed corporation is often determined by "capitalizing" its prospective profits, at a given rate.

EXAMPLE. What is the capitalized value of an earning of $6000 per annum, on a 5% basis?

$6000 \div .05 = $120,000, the amount of capital which at 5% would yield $6000.

WRITTEN

1. Wilson has a business which earns $3600 annually. A corporation is formed and the business is taken over and capitalized on a 5% basis. What was the capital stock of the corporation?

2. Wilson's assets before incorporation (problem 1) were $15,825, and his liabilities $3225. Upon incorporation he was paid for his business in stock of the new company, receiving an amount of stock equal to the net assets of the business plus twice the annual profits. How much stock did he receive?

3. Wilson (see preceding problems) was made general manager of the business at a salary of $2500 a year. A dividend of 6% was declared the first year. How much more did Wilson make than he had made when he was sole owner of the business?

4. A young man whose salary had been $2500, accepted a position with a corporation under an arrangement that he should receive a salary of $1800 and that enough stock would be assigned

to him on a basis of 6% dividends, to make his income double what it had been. How much stock was assigned to him?

5. Logan and Warren, proprietors of a department store, incorporate on the basis of a 5% capitalization of average profits for the past five years. Logan owns a 60% interest in the business, and Warren 40%. If the profits for the past five years have been $15,980, $16,900, $17,590, $18,370, and $20,100, how many shares in the new corporation does each man get?

6. The following schedule of earnings and expenses was prepared for a partnership that it was desired to incorporate:

	Gross Earnings	Expenses		Gross Earnings	Expenses
1919....	$21,400	$18,600	1922....	$30,900	$24,200
1920....	24,750	20,540	1923....	35,850	28,200
1921....	28,300	23,760	1924....	38,700	31,460

Capitalize the average net earnings at 5%; at 6%; at 8%. Compute the capital to the nearest $1000 in each case.

7. A lease was taken on a building at annual rentals for 20 years as follows:

Sept. 1, 1914, to Aug. 31, 1915	$4000
Sept. 1, 1915, to Aug. 31, 1916	4400
Sept. 1, 1916, to Aug. 31, 1919	5000
Sept. 1, 1919, to Aug. 31, 1924	6000
Sept. 1, 1924, to Aug. 31, 1929	7500
Sept. 1, 1929, to Aug. 31, 1934	9000

The tenant was to pay all taxes on the property and keep it in good condition, except for ordinary waste and depreciation. On a basis of 4%, what was the property worth when leased, and what was it worth at the time of the expiration of the lease?

8. A teacher's salary was automatically increased from $1350 to $1800 per year upon the completion of a course of study specified by the board of education. Capitalized on a basis of 6%, what was his special education worth?

9. A cow produces annually 8000 pounds of milk at $1.20 per hundred. Her feed for one year costs $50.51, and annual expenses other than feed are $36.94. Capitalizing the profit on this cow at 9%, and not figuring on depreciation, what is she worth? If she has 7 years of usefulness, at the end of which time she can be sold for $17.50, what is her average depreciation?

10. If every year a cow gives 6000 pounds of milk at $1.16 per cwt., and her feed costs $34.31, and other expenses are $28.25, what is the cow worth if interest is worth 6%? How much is her average annual depreciation, not figuring on depreciation, if she has 7 years of usefulness and is then worth $20.00?

11. Assume that in a certain county 398 farmers had attended only the district school, and that their labor incomes averaged $618.00 per year. 25 had attended an agricultural college, and their labor incomes averaged $922.00. Capitalizing the difference in incomes at 5%, what was the extra schooling worth?

Bond Issues

When a corporation issues bonds it is a borrower and the bondholders are lenders. Before one lends money he wishes to know (1) whether the principal loaned is secure and (2) whether the interest promised can be paid.

The security of the principal is usually amply provided for in the contract under which the bonds are issued, by the pledge of collateral. Whether or not interest payments can be met out of profits is a question the answer to which must be determined by a study of the statements issued by the corporation.

These questions are of importance to those financing the corporation as well as to the lender, for an unsatisfactory answer to either may mean the failure of the corporation.

In every business there are expenses which continue whether the business operates or not, and expenses that must be met if operations are to continue. When a bond issue is proposed, the interest on the bonds, if issued, will be an additional expense.

The bonds may be issued to raise money for current expenses, or in order to extend the business.

Having analyzed the statements issued by a given corporation at the time a bond issue is under consideration, and with the above considerations in mind, ask yourself the following and other similar questions:

Is the business on a paying basis?

If so, does it pay enough to cover the interest on the bonds?

If not, are the receipts from bond sales to be used in such a way as to increase the profits?

EXAMPLE. A corporation proposes a $200,000 issue of 6% extension bonds, maturing in 10 years. The bonds are to be amply secured. Its statement of condition and its statement of

income for last year are given herewith. (*a*) Will its profits, based on last year's business, be sufficient to cover the interest on the bonds? (*b*) Can it pay the bonds when they mature?

Assets		Liabilities	
Cash	$ 75,000	Notes and accts. pay	$250,000
Notes and accts. rec	280,000	5% bonds	150,000
Real estate	300,000	Reserves	40,000
Machinery and tools	150,000	Capital stock	350,000
Materials	90,000	Surplus	105,000
	$895,000		$895,000

Income (Last Year)

Sales	$378,500
Manufacturing expenses	291,800
Manufacturing profit	$ 86,700
General expense, including reserves and int. on outstanding bonds	33,750
Net profits	$ 52,950
Dividends, 6% on capital stock	21,000
Balance carried to surplus account	$ 31,950

Solution

(*a*) 6% of $200,000 = $12,000, interest on proposed bonds. $52,950 ÷ $12,000 = 4.41+. Net income is 4.41 times interest on proposed bonds.

(*b*) $52,950 − $12,000 = $40,950, amount remaining each year after paying interest on proposed bond issue.

$40,950 × 10 = $409,500, amount of profits in excess of interest in 10 years, which is ample for retirement of bonds, without taking into consideration increased earnings from increased capital.

Written

1. A railroad proposes to issue $1,000,000 of $4\frac{1}{2}\%$ bonds, maturing in 30 years. The money is to be used in improving the roadbed and replacing worn-out equipment. Its statements follow:

Assets		Liabilities	
Cash	$245,000	Notes and accts. pay	$765,000
Notes and accts. rec	178,400	5% bonds	1,500,000
Roadbed and equipment	3,879,500	Reserves	218,000
Marketable securities	258,580	Capital stock and surplus	2,078,480
	$4,561,480		$4,561,480

Gross income	$647,520
Operating expenses and taxes	478,694
Net income from operations	$168,826

If $1,000,000 of the capital stock is preferred stock with a 6% dividend guaranteed on it, and this dividend and interest on outstanding bonds must be deducted from net income from operations, will the present earnings of the road justify the proposed bond issue? Will the road be able to redeem the bonds at maturity without touching its capital or surplus?

2. Assume that the rehabilitation of the road, in problem 1, will occasion an increase of 12% in gross income, expenses and taxes remaining the same. What will your answers to problem 1 be in this case?

3. What will the cost of the above bonds be, $14\frac{1}{2}$ years after their issue, to net $4\frac{1}{2}\%$ to the investor?

Book Value

The stockholders of a corporation are the owners of its net assets. The book value of all the stock they hold, therefore, as previously defined, is the amount of the net assets of the company.

The book value of one share is found by dividing the net assets of the company by the number of shares of its stock.

EXAMPLE. Find the book value, per share, of the stock of the Standard Oil Co. of Kansas when its balance sheet was as follows:

ASSETS		LIABILITIES	
Cash on hand	$ 61,345	Accts. and notes pay.	$ 500,133
Accts. and notes rec.	1,558,496	Capital stock	2,000,000
Properties	1,103,290	Undivided profits	1,371,204
Marketable securities	398,594		
Merchandise	749,612		
	$3,871,337		$3,871,337

The net assets can be found either by deducting the total amount of the notes and accounts payable from the amount of the total assets or by finding the total of capital stock and undivided profits. The number of shares of stock can be determined from the statement. Let the student perform the solution for himself.

WRITTEN

Find the book value of the stock in each of the following cases:

	Assets	Bonds and Mortgages	Other Liabilities	Capital	Surplus
1.	$15,066,290	$9,225,000	$3,137,754	$2,000,000	$ 703,536
2.	57,781,562	19,021,500	11,097,299	25,000,000	2,662,763
3.	56,068,494	11,700,000	15,185,311	20,000,000	9,183,183

4. Find the book value per share of the stock of a bank when its total assets are $4,398,203, if its capital is $200,000 and surplus $89,044, if it owes its depositors $4,052,446, and other liabilities aggregate $56,713.

5. The balance sheet of a corporation follows:

Assets		Liabilities	
Cash	$ 266,965	Notes payable	$ 1,500,000
Accounts receivable	4,500,250	Accounts payable	1,879,023
Mdse. and material	15,660,083	Reserve for insurance	384,076
Plant	10,325,088	Capital stock	5,000,000
Other property	7,625	Surplus	21,996,912
	$30,760,011		$30,760,011

A stock dividend of $2,000,000 was declared, this being legal in the state under which the corporation held its charter. (This means that $2,000,000 worth of new stock was issued and distributed proportionately among the stockholders without cost to them.) What was the book value of the stock before the stock dividend? After?

6. If the new stock in the above problem had been regularly issued and sold at par for cash, what would have been the book value per share after the sale?

The Arithmetic of Credits

Current assets (also called *quick* or *liquid*) are cash and assets which can be quickly converted into cash.

Current liabilities are all debts arising from the ordinary or current transactions of business.

Just how quickly assets must be available to be called "current," and just how soon debts must be payable to be called "current" are questions which cannot be settled in every case by the application of a definite general rule.

In the problems below, consider as current the assets and liabilities mentioned in the following list:

Current Assets	Current Liabilities
Cash	Accounts payable
Accounts receivable	Notes payable
Notes receivable	Accrued interest
Merchandise, $\frac{1}{3}$ off	Accrued taxes
Materials, $\frac{1}{5}$ off	Accrued salaries
Listed securities	Dividends payable
Dividends receivable on above	

It is important to know how the total of current assets and the total of current liabilities compare. The amount of credit to which a business House is entitled depends largely upon this.

Some accountants say that current assets should be at least $2\frac{1}{2}$ times current liabilities. What the ratio should be is a matter of opinion. At least it is clear that current assets should always exceed current liabilities.

Written

Report as to the ratio between current assets and current liabilities in each of the following cases:

1. Hart, Schaffner, & Marx Co.

Assets		Liabilities	
Cash	$ 1,365,612	Accounts payable	$ 373,453
Accts. and notes rec.	1,716,884	Accr. salaries and wages	133,066
Mdse. inventory	2,746,456	Accrued taxes	61,726
Prepaid expenses	77,271	Capital	19,600,000
Machinery	300,000	Surplus and res.	1,037,978
Goodwill	15,000,000		
	$21,206,223		$21,206,223

2. Standard Gas and Electric Co.

Assets		Liabilities	
Cash	$ 33,773	Dividends payable	$ 78,566
Accts. rec.	81,184	Accrued salaries	70,292
Notes rec.	1,035,351	Accts. pay.	126,610
Listed securities	23,755,595	Notes pay.	131,229
Dividends rec. on above	91,408	Preferred stock	11,784,950
Unlisted secur.	10,208,600	Common stock	9,343,150
Dividends rec. on above	52,568	Bonds, all kinds	13,136,000
Sinking fund	51,500	Preferred stock scrip	471,398
Office furniture	1	Surplus	679,245
Misc. assets	511,460		
	$35,821,440		$35,821,440

3. Sulzberger and Sons Co.

Assets		Liabilities	
Property acct.	$25,478,718	Capital stock	$20,000,000
Investment in subsidiary companies	6,272,652	Preferred stock	10,000,000
Mdse.	9,929,844	Debenture notes	9,300,000
Accts. and bills rec.	6,621,988	Accrued interest	110,833
Mort. and invest. sec.	268,214	Dividends payable	175,000
Cash	3,025,346	Bills payable	6,651,861
Ins., int., disc., etc., prepd.	391,142	Accounts payable	1,519,079
		Surplus	4,231,131
Total	$51,987,904	Total	$51,987,904

4. Western Grocer Co.

Assets		Liabilities	
Cash on hand	$ 260,747	Bills pay	$ 698,300
Accts. and notes rec.	821,247	Accts. pay	112,741
Inventories	1,107,076	Res. for taxes, etc.	36,217
Real estate	510,040	Preferred stock paid in	346,700
Corporation stocks, unlisted	275,300	Common stock paid in	1,900,000
Machinery, etc.	173,178	Undivided profits	400,999
Horses, wagons, etc.	14,039	Reserve for deprec.	46,670
Goodwill	380,000		
Total Assets	$ 3,541,627	Total Liabilities	$ 3,541,627

Profit Distribution

The gross earnings of a corporation are first used to pay fixed expenses and operating expenses. What is then left may be used to meet the requirements of the bond interest, corporate reserves, and dividends. If there is a balance remaining, it may be transferred to the surplus account.

Written

1. A corporation has outstanding $1,500,000 of 6% preferred stock, $600,000 of common stock; $1,000,000 of First Mortgage $4\frac{1}{2}\%$ bonds, to redeem which 3% of their par value must be appropriated annually to a sinking fund; $500,000 collateral Trust

6% bonds, to redeem which $2\frac{1}{2}\%$ of their par value must be appropriated annually to a sinking fund. The net income of the corporation this year is $236,950. After all interest payments and appropriations called for in the foregoing have been made, how much is available for the surplus account?

2. A corporation has the following stock and bonds outstanding:

Common stock	$1,000,000.
First preferred (6%)	2,000,000.
Second preferred (7%)	1,000,000.
First Mortgage bonds (5%)	2,000,000, sinking fund 2% annually.
Second Mortgage bonds (6%)	1,500,000, sinking fund 1% annually.

The corporation has been paying a dividend of 2% quarterly on the common stock and the directors wish to continue these payments and also to increase the surplus account annually by a sum equal to 1% of all outstanding stock, which is the custom of the corporation. What must the net earnings of the company be, to meet all requirements?

3. A street railway company has the following outstanding stock and bonds:

Preferred (6%)	$2,000,000.
Common	3,000,000.
First Mortgage bonds (5%)	2,500,000, sinking fund $2\frac{1}{2}\%$ annually.
Sinking fund, gold (6%)	2,000,000, sinking fund 1% annually.

It is desired to pay a dividend of $4\frac{1}{2}\%$ on the common stock, to make improvements costing $450,000, and to increase the surplus account by an amount equal to 1% of all outstanding stock. What must the net earnings be, to meet all requirements?

4. A business has been making a return of 10% annually on a capital of $100,000. The directors issue $30,000 additional stock, 7% preferred. What per cent will the business then return on the common stock, the earnings remaining the same?

5. A factory earns 9% on a capital stock of $250,000, common. Without reducing the amount of common stock, the directors issue bonds for $80,000 at 5% and issue $60,000 of 7% preferred non-participating stock (stock that does not participate in profits,

beyond the rate guaranteed). What rate of dividend can they declare on the common stock?

6. A corporation has been earning $100,000 gross per annum and its ratio of operating expenses to income has been 75%. A year of depression comes on and its gross earnings decline 35% while the ratio of expenses to income increases to 80%. If its annual interest charge is $12,500, what is its surplus for each year?

7. The Reading Co. has $70,000,000 common stock paying dividends of 6%, $28,000,000 of 4% 1st preferred stock, and $42,000,000 of 4% 2d preferred stock. What must its net earnings be to pay dividends on these?

8. The Colorado and Southern Railway has $31,000,000 common stock paying 2%; $8,500,000 of 4% 1st preferred stock; $8,500,000 of 4% 2d preferred stock; and $49,300,000 of bonds averaging 4.31%. What must be its net earnings to meet the interest and dividend charges on these?

9. A corporation was obliged to borrow $1,000,000 at a time when 30-year bonds could not be sold at par unless bearing $5\frac{1}{2}\%$. It raised the money by giving 5-year notes bearing $6\frac{1}{2}\%$, and at maturity it refunded the loan by issuing 25-year bonds, bearing $4\frac{1}{2}\%$ (that is, it issued the bonds and with the proceeds paid the notes). What did it save if no allowance is made for expenses in putting out these issues or for loss of use of money?

10. The provisions of the First Mortgage Bonds issued by the Mayville Light and Power Co., these bonds being now outstanding, prevent the issuing of more bonds unless the net earnings of the company for the year previous shall be at least one and one-half times one year's interest on outstanding bonds and those proposed to be issued. There are outstanding $50,000.00 in 5% bonds and $75,000.00 in 4% bonds. Earnings, net, for last year, were $13,500.00. How large a bond issue, at 5%, is the company permitted to bring out?

11. The Middle West Utilities Co. (a holding company) had a total income of $911,969.20. It had 6% bonds outstanding to the par value of $3,500,000. If its total expenses and taxes were $212, 948.05, its income after paying expenses and taxes was how many times its total interest charge?

Sinking Funds

A *sinking fund* is the amount accumulated by setting aside a sum of money each year for a fixed period of years, so that, with compound interest, the sums set aside will amount to a certain debt *at the end* of the fixed period of time.

Naturally those who lend money wish assurance of its repayment. The creation of a sinking fund to pay the debt at maturity gives them this assurance. This is of importance to the borrower also, as the sinking fund is a guaranty against the eventual impairment of capital.

The amount of debt is paid to the creditor at maturity. The debt itself is usually no larger at maturity than at its origin, as interest has probably been paid periodically throughout its life.

The amount of the annual contribution is frequently spoken of as a sinking fund, but this is not strictly correct, as the sinking fund is the *result* of these contributions and their accumulations of interest.

EXAMPLE. The Sprague Water Power Co. has issued \$1,000,000 of $5\frac{1}{2}\%$ bonds due in 5 years. What amount should the company annually set aside in a sinking fund at 5% to equal the principal debt at maturity?

Naturally no appropriation to the sinking fund is made at the beginning of the first year. The presumption must be that the money borrowed was *all* needed. The first appropriation was made at the beginning of the second year.

Two methods of solution are given below. Study both carefully and use the method which seems the easier to you.

Using Compound Interest Amount Table

\$1.00 invested at the beginning of each year for 4 years (the 2d, 3d, 4th, and 5th year of the loan) at 5% amounts to \$4.525631.

Adding \$1.00 appropriated at the end of the 5th year of the loan gives \$5.525631. \$1,000,000 ÷ 5.525631 = \$180,974.82, amount of annual appropriation.

Using Compound Interest Tables

Each \$1.00 of 1st payment compounded for 4 yr.	\$1.215506
Each 1.00 of 2d payment compounded for 3 yr.	1.157625
Each 1.00 of 3d payment compounded for 2 yr.	1.1025
Each 1.00 of 4th payment compounded for 1 yr.	1.05
Each 1.00 of 5th payment compounded for 0 yr.	1.00
For each \$1.00 paid annually for 5 years, the amount of the debt cancelled will be	\$5.525631

\$1,000,000 ÷ \$5.525631 = \$180,974.82, amount of the payment.

It is the custom for corporations, states, and cities to buy back their own bonds with their sinking fund annuities when the interest rate on the bonds they wish to retire is higher than can be realized on the sinking fund securities and the bonds can be purchased for an amount near par.

Written

1. The Eastern Construction Co. issued \$100,000 in bonds payable in 20 years, and created a sinking fund to pay the bonds at maturity. On a 4% basis, how large an annual appropriation to the sinking fund was made?

2. A railroad company borrows \$1,000,000 by issuing sinking fund bonds payable in 15 years. If it can net $4\frac{1}{2}\%$ on its sinking fund investments, how large an annual appropriation should be made?

3. A city issued \$350,000 of 20-year bonds to raise money for constructing a municipal electric lighting plant. If the city can realize 4% on its sinking fund securities, how much must be the annual payment to the fund?

4. A corporation, trying to borrow money on its sinking fund bonds, advertises that it will set aside each year \$6,000.00 for this fund. The bonds it is trying to sell amount to \$250,000, and are payable in 20 years. How much more or less than sufficient to pay this indebtedness in 20 years will be the fund so established if the corporation can realize 5% upon its investments?

5. Harding leased a lot for 20 years at an annual rental of \$2500. He erected on it a building at a cost of \$50,000. What must be the annual income from the building in order to cover: (*a*) The annual rent of the land; (*b*) interest of 8% per annum on the cash investment; and (*c*) an annual appropriation to a sinking fund which at 4% would equal the cost of the building at the expiration of the land lease?

6. A city issued \$250,000 of 20 yr., 5% bonds to erect a city hospital. It was agreed that the city should provide a sinking fund at 3% sufficient to retire the entire issue at maturity. How much did the city appropriate annually? If the taxable value of all property within the city at the time this issue of bonds was made amounted to \$325,560,000, how many mills per dollar of valuation would be necessary to raise the first contribution to the sinking fund?

Federal Plan for Farm Loans

According to the government's plan for farm loans, a farmer may borrow on his land, or borrow to buy land, and may pay back his loan by a plan which is called *amortization*.

Amortization is the payment of a debt by a series of equal payments, at regular intervals, the payments covering both principal and interest. (The principle is also applied to other computations.)

These payments are made to the creditor, and are not mere appropriations. Each payment covers the interest accrued at the time it was made, the balance being applied on principal. Each successive payment covers more principal than the preceding payment, and less interest.

EXAMPLE. On Apr. 1, 1920, a farmer borrowed $1000 from the farm loan association of his district, for 4 years at 4%, under an agreement that principal and interest were to be paid in 4 equal payments. How much should the farmer pay back each year?

The question to be decided is, what periodical payment will in 4 years at 4% equal a debt of $1000 bearing interest at 4%?

Consulting the *compound interest amount table* we find that $1.00 invested at the beginning of the 2d, 3d, and 4th year (3 payments) will amount to $3.246464. This, plus a final payment of $1.00 on which no interest is figured, gives $4.246464 as the amount realized in 4 years from a series of annual investments of $1.00 at 4% compound interest.

Consulting the *compound interest table*, we find that $1 compounded for 4 periods at 4% amounts to $1.16985856. Multiplying by 1000 gives $1169.85856 as the amount of $1000 in 4 years at 4% compound interest.

$1169.85856 ÷ $4.246464 = $275.49, the annual payment.

The foregoing is proved by the following schedule, which also shows how much of each payment is applied to principal and how much to interest. Accept $275.49 as the correct amount, and work out each step for yourself.

Schedule of Amortization

DATE	PAYMENT	INTEREST ON BALANCE	AMORTIZATION OF PRINCIPAL	BALANCE AFTER EACH PAYMENT
Apr 1, 1924	$	$	$	$1000.00
Apr. 1, 1925	275.49	40.00	235.49	764.51
Apr. 1, 1926	275.49	30.58	244.91	519.60
Apr. 1, 1927	275.49	20.78	254.71	264.89
Apr. 1, 1928	275.49	10.60	264.89	00.00

WRITTEN

1. A farmer bought 160 acres of land at \$9 per acre. He had enough cash to pay one third of the purchase price, and borrowed the remaining two thirds from the government farm loan association of his district. He agreed to return the loan with interest at 5% in 5 equal annual payments. What should be the amount of each annual payment?

Prepare a schedule of amortization.

2. The Vinton Mfg. Co. installed machinery at a cost of \$70,000. They paid \$20,000 cash and agreed to pay the balance and interest at 6% in 5 equal annual payments.

Prepare a schedule of amortization.

3. Samuel Wilkins borrowed \$3000 from a farm loan association, agreeing to pay back the loan and interest at 4% in 10 equal annual payments. He made the first five payments and then sold his property, the purchaser agreeing to assume the unpaid portion of the debt. How much did Mr. Wilkins pay on the principal and how much did the buyer assume?

Prepare a schedule of amortization.

4. A farmer borrowed \$3000 and agreed to repay the principal and interest at 6% in 10 years by making 20 equal semi-annual payments. Find the amount of each payment and prepare a schedule of amortization.

How Bond Premiums and Discounts Affect the Income Tax

Bond premiums and discounts represent losses and gains which in figuring annual income should be spread equitably over the years which they cover. Failure to do this will result in too heavy taxation in some one year.

Thus, if a profit which should cover ten years be entered in the books of account as of one year, the income tax that year would be much too heavy. Furthermore the taxes for the ten years would be less if profit were equitably distributed than if all the profit were to appear in one of the ten years.

A premium on a bond is the amount by which its market value is in excess of par. As the bond will eventually be redeemed at par, it is clear that the premium which is included in the value of the bond will gradually disappear. This gradual disappear-

ance is computed by a plan of amortization which is developed in the two examples following:

Bonds sell at a premium when the rate of interest they bear is greater than the rate of interest which is to be realized from the investment. The longer the life of the bond the greater the premium.

EXAMPLE. What can a bank afford to pay in cash on Jan. 1, 1920, for a $100,000 issue of 5% bonds, interest payable semi-annually, in order to realize upon its investment the equivalent of 4% interest compounded semi-annually, if the bonds are payable in 2 years?

The returns to the bank will be the four interest payments, due in $\frac{1}{2}$ yr., 1 yr., $1\frac{1}{2}$ yr., and 2 yr., respectively, and the principal, due in 2 yr. It can afford to pay the present worth of these future returns calculated on a basis of 4% compounded semi-annually. Consulting the compound interest table for our rates, we find:

P. W.	1st int. payt. = $2500 ÷ 1.02 =	$ 2450.98
P. W.	2d int. payt. = 2500 ÷ 1.0404 =	2402.92
P. W.	3d int. payt. = 2500 ÷ 1.061208 =	2355.81
P. W.	4th int. payt. = 2500 ÷ 1.08243216 =	2309.61
P. W.	of principal = 100,000 ÷ 1.08243216 =	92,384.54
	Total bank can afford to pay	$101,903.86

EXAMPLE. As each payment of $2500 bond interest is received, how much of it is for interest on investment and how much is applied to extinguishment of premium?

As each payment of interest on bonds is received, it is applied first to interest on the bank's investment and the balance is applied to amortization of premium.

Schedule of Amortization

DATE	AMOUNT RECEIVED	2% INTEREST ON INVESTMENT	AMORTIZATION OF PREMIUM	BANK'S INVESTMENT AFTER EACH PAYMENT
Jan. 1, 1926	$	$	$	$101,903.86
July 1, 1926	2500	2038.08	461.92	101,441.94
Jan. 1, 1927	2500	2028.84	471.16	100,970.78
July 1, 1927	2500	2019.42	480.58	100,490.20
Jan. 1, 1928	102500	2009.80	490.20	00.00

WRITTEN

1. The Central Light and Power Co. is issuing $50,000 of 5 yr., 6% bonds, interest payable semi-annually on Jan. 1 and July 1 of each year. How much can be paid for these bonds to realize 5%? At what price should these bonds be quoted, taking the nearest eighth of one dollar greater than the cost?

2. Make a schedule showing how much of the premium paid should be considered as loss each year, in the preparation of the income report.

3. One month and 24 days after the end of the first year these bonds are sold at 103 and accrued interest. What profit (including interest) does the purchaser of the bond issue make by the sale?

4. At what price should these bonds be quoted at the end of the second year? Give the result to the nearest eighth of a dollar in excess of their worth.

5. What was paid for $5000 of 6% bonds, interest payable semi-annually, if the bonds had 3 years to run and were sold at a price to net the purchaser 4%?

A discount on a bond is the amount by which its market value is less than its par value.

Bonds sell at a discount when the rate of discount they bear is less than the rate of interest which is to be realized from the investment. The longer the life of the bond the greater the discount.

Suppose a bond to be sold at a discount. As it will eventually be redeemed at par, its worth gradually approaches par. It is clear that the discount should not be considered a lump sum discount (or a profit made in one day) but should be distributed uniformly throughout the period covered by the bonds. How this distribution is accomplished through a sinking fund, is developed in the two examples following:

EXAMPLE. On Jan. 1, 1927, the Simplex Manufacturing Co. issued $100,000 of 2 yr. 4% bonds, interest payable on Jan. 1 and July 1 of each year. What could the purchaser pay for the issue, to net 6% compounded semi-annually on his investment?

The returns to the purchaser will be the four interest payments and the payment of principal when due. To net 6% on his investment, he can pay the present worth of these items as follows:

P. W. 1st int. payt.	$= \$ 2000 \div 1.03 =$	\$ 1941.7476
P. W. 2d int. payt.	$= 2000 \div 1.0609 =$	1885.1918
P. W. 3d int. payt.	$= 2000 \div 1.092727 =$	1830.2833
P. W. 4th int. payt.	$= 2000 \div 1.12550881 =$	1776.9741
P. W. principal payt.	$= 100{,}000 \div 1.12550881 =$	88,848.7048
		\$96,282.9016

EXAMPLE. As each payment of \$2000 interest on the foregoing bonds is received, how much of it is for interest on investment, and how much placed in the sinking fund to diminish the discount? How much profit should show on each year's income report?

As each payment of interest on the bonds is received, it is applied to pay the interest on the purchaser's investment. The balance of interest on investment is charged to sinking fund.

Sinking Fund Schedule

DATE	2% INTEREST RECEIVED	3% INTEREST ON INVESTMENT	SINKING FUND	DISCOUNT	PURCHASERS' INVESTMENT
Jan. 1, 1926	\$	\$	\$	\$3717.10	\$ 96282.90
July 1, 1926	2000	2888.49	888.49	2828.61	97171.39
Jan. 1, 1927	2000	2915.14	915.14	1913.47	98086.53
July 1, 1927	2000	2942.60	942.60	970.87	99029.13
Jan 1, 1928	2000	2970.87	970.87	00.00	100000.00

Note that the interest on the bonds has never been sufficient to cover the interest the purchaser was to receive on his investment. To make up for this deficiency, the amount of the shortage (\$888.49 on July 1, 1919) is taken out of the discount and placed in the sinking fund, which is really a part of the purchaser's investment. The purchaser's investment is shown above as increased by the amount of each of these sinking fund appropriations until it reaches \$100,000, while the discount is shown as decreased by each of these amounts until it reaches \$00.00.

WRITTEN

1. The Hudson Gas Co. on Apr. 1, 1926, issued \$100,000 in bonds payable in 2 years, bearing interest at 5% payable semi-annually on Apr. 1 and Oct. 1 of each year. At what price should these bonds be purchased in order that the purchaser might realize 6% compounded semi-annually upon the investment?

2. Make a schedule showing the amount of the contribution to the sinking fund each half year (problem 1), necessary in order to realize 6%. How much of the discount should be considered as profit each year in preparing the income schedule?

3. At what price should the above bonds be sold at the end of the first year in order that the purchaser may not lose by the sale? Give the result in the next higher eighth of a dollar.

4. On June 2, 1920, $10,000 of these bonds were sold at the value obtained by the 3d problem, and accrued interest. Give the total cost of the bonds to the purchaser on June 2, 1920.

5. On Jan. 1, 1925, a bank bought an entire issue of $200,000 of 4% bonds, due in 3 years, interest payable Jan. 1 and July 1. The bonds were bought to net 6% on investment. On Jan. 1, 1926, the bank was required to make its statement of earnings, to be used as the basis for its income tax. How much of the discount on the bonds should show as a part of 1925 profits?

6. A corporation paid $90,000 for a coal mine. Engineers calculated that the mine would yield coal for 20 years at the rate at which the coal could be removed by the machinery available. The machinery cost $40,000, and its junk value at the end of 20 years was estimated at $5,000. How much money should be set aside from profits each year to make a sinking fund at 4% which would at the end of the 20 years be sufficient to repay the investors their entire investment?

Real Estate

1. Edward Renwick leased the S. E. corner (100x101) of Van Buren and La Salle Sts. for ninety-nine years, paying therefor $25,000.00 a year, Mr. Renwick to pay the taxes. He erected on it a building costing $1,180,000.00. He leased the building to the Hotel Sherman Company for ninety-nine years at $97,000.00 a year, the lessees to pay taxes on both ground and building. He mortgaged the building for $650,000.00 at 6%. Capitalizing the first ground lease at 4% what was the ground worth? What was Mr. Renwick's annual net return upon his investment?

2. Alexander has a lease on the Union Restaurant property, 80.5 ft. frontage by 81.25 ft., for 100 years at an annual rental of $22,500. If this be capitalized on a 4% basis, what is the land

worth per front foot? What per square foot? If capitalized on a 5% basis, what is the property worth per square foot?

3. The Northwestern University leased a plot of land, 50′ x 132′, to Mr. Witkower at a graded rental beginning with $900 a year from July 1, 1924, to Apr. 1, 1925; $1050 a year for the succeeding ten years; $1200 a year for the next ten years; $1350 for the following ten years; and $1500 a year for the last ten years. Mr. Witkower then leased this same property on Oct. 1, 1924, for $3050 for the first year; $4200 for the succeeding ten years; $4850 for the next ten years; $5500 for the following 10 years; and $6000 for the remainder of the term. Making no allowance for interest, what is Mr. Witkower's profit on this lease?

4. At the beginning of Mr. Witkower's lease in 1924, what was the value of the property if capitalized on a basis of 5%? What was the value, at the expiration of the lease, if capitalized at 5%? What was the average increase per year in the value of the property?

5. An apartment house is purchased for $123,500, the buyer assuming a mortgage of $73,500 at 5%. The annual gross rental is estimated at $14,500. Taxes, $21.32 per M on a valuation of $75,000; insurance, 25¢ per $100 on a valuation of $80,000; general expenses and repairs estimated at $1560. The income is what per cent upon the total investment? What per cent upon his own capital does the owner receive?

6. An apartment house and land (100′ x 150′) sold for $100,000. There were 18 apartments in the building, one-half of them containing 5 rooms and the remainder 4 rooms. The buyer assumed a mortgage of $56,250 bearing interest at 5% annually. What was the average interest charge per month on each room?

7. If taxes in the above problem (6th) average $19.78 per $1000 of valuation, insurance 25¢ per $100 on a valuation of $60,000, and general expenses and repairs amount to $850, at what average amount per month must each apartment be rented to realize 7% net upon the owner's investment in the property?

8. A building that cost $350,000 is erected upon land valued at $100,000. The building is estimated to contain 33,600 sq. ft. of rental space. What must be the rental average per square foot per annum to produce 12% gross income upon total cost? What should be the monthly rent for an office 15′ x 18′ in this building?

Promoting and Financing

1. A railway company is organized Jan. 1 with a capital stock of \$30,000,000.00, of which $\frac{1}{2}$ is 5% non-cumulative preferred, and $\frac{1}{2}$ common. Bonds are issued for \$22,500,000.00 at 4%, running fifty years. If the stock is sold to a syndicate, the preferred at 87 and the common at 81, what does it bring?

2. If the bonds are dated March 1, interest payable September and March, and are sold April 15 to a syndicate at 92 what amount does the company receive?

3. A construction company is organized with a cash capital of \$2,500,000.00. Out of this capital the construction company purchases a going mine for \$500,000.00. The railway company contracts with this construction company to build and equip its road, agreeing to pay therefor at the rate of \$8250.00 each of its common and preferred stock and \$16,500.00 of its bonds for each mile equipped. The cost of the road to the construction company averages \$18,000 per mile. What amount of cash will the construction company have on hand on the completion of 110 miles of road?

4. What is the total par value of each kind of stock received by the construction company at this time, and what amount of bonds will it now be entitled to receive from the railway company?

5. If it pledges these bonds April 1 with bankers for a loan at 60% of their face, what amount can it borrow?

6. How many additional miles of road can it now build and equip, using all but \$9,000 of its cash?

7. On the completion of these additional miles June 1, how much of each security would the construction company be entitled to receive from the railway company?

8. For what loan could it pledge the bonds, using the same facts as in problem 5? After building 30 miles of road, how much cash will the construction company have?

9. What is the total number of miles of road constructed? What amount does the construction company owe bankers?

10. The bonds are sold Dec. 15 to a syndicate at 89, and interest. What amount do they bring? After paying its loans with interest at 6% what amount of cash does the company now have on hand?

Conversion of Bonds

Convertible bonds are bonds which may be exchanged, under specified conditions, for stock or for other bonds. The prices at which such conversion will be made are specified, as to both the bonds traded and the stock or bonds traded for.

Thus, bonds convertible at 95 into stock at 105 means that the bonds will be accepted at 95 and the stock will be sold for 105, when the exchange is made.

WRITTEN

1. The Santa Fe Railroad has issued bonds convertible at par into common stock at \$110. How much common stock will be received in exchange for \$23,100 worth of bonds?

2. If 5% bonds can be converted at \$105 into 6% stock at \$110, will the holder of such a bond increase or decrease his income by making the exchange, and how much?

3. I own bonds of the Commonwealth Power Company which are now due and the par value of which is \$5000. One condition of the bond is that it is convertible into a like amount of the Company's 6% preferred stock at par, and in addition common stock of the Company of the par value of 30% of the bonds. The preferred stock is selling at 93 and the common at 70. What will be my gain if I convert the bonds and sell the stock, brokerage on the stock $\frac{1}{8}$%?

4. If the purchaser of the stock held it as an investment, and the preferred paid 6% and the common 4%, what would be his annual income from his investment, including brokerage, and his per cent of income?

5. Mr. Griffith has \$10,000 worth of 5% bonds convertible at market price, $102\frac{1}{2}$, into common stock of the same company at par. The common stock is expected to yield 7%. If Mr. Griffith makes the exchange will he increase or decrease his income and how much?

General Review Problems

1. Which is better and how much, \$5500 in cash or \$1500 at the end of each year for 4 years, if money is worth 4%?

2. A woman's husband dies leaving her a life interest in a part of a farm, which pays her annually \$900. The heirs owning the

remaining interests in the property insist on a sale and the property cannot be divided. If her expectation of life is 6 years, and money is worth 6%, what sum should she be paid for her interest?

3. Ames has an endowment insurance policy for $1000 payable at once. He also has the option of taking $211.99 in cash and 4 additional payments of a like sum made in 1, 2, 3, and 4 yr. respectively. If in either case he gets 4% interest compounded semi-annually in a savings bank, which plan is better and how much, at the end of the fourth year?

4. A man owing $500 due in 6 mo., $700 in 1 yr., $900 in 2 yrs. and $1000 in 3 yrs., all without interest, wishes to settle these debts by making 3 equal annual payments, in 1, 2, and 3 yr. respectively. What must each payment be if money is worth 5%?

5. A man spends 25 cents a day for tobacco and drink. If he should deposit the same gross amount at the end of each year in a savings bank bearing 4% payable semi-annually for 5 yr., what would it amount to?

6. A man pays for a house $1000 down and $1000 a year for 4 years with interest at 5% on deferred payments. How much money does he pay, including interest payments?

7. A railroad pays $720 a year to each of two watchmen to guard a crossing. How much can it afford to spend for elevating the tracks over that crossing if money is worth $4\frac{1}{2}$%?

8. A man owes a mortgage, due in 5 yr., on an apartment house. When he renews the mortgage he wishes to reduce the loan $6000. If money is worth 5% what amount must he set aside at the end of each year to equal $6000?

9. What is a coal mine worth which will yield $10,000 annually for 10 yr. before it is exhausted, if money is worth 5%?

10. A man buys a piano for $600. He pays $100 in cash and agrees to pay the balance in 5 bi-monthly payments of $100.00 each, each payment to include 6% on the unpaid balance. What is the amount of each payment?

11. An engine when new cost $10,000. Its life is 10 yr. Its junk value at the end of 10 yr. is $500. What is it worth at the end of 5 yr. if depreciation is the same each year?

12. By an expenditure of $50,000 for a dam, horse power worth $9700 per annum can be produced at an annual expense of $1300. If money is worth 5%, what is the site worth?

CHAPTER XXXVI

EQUATION OF ACCOUNTS

Equation of accounts is the process of finding the date upon which the balance on an account may be paid without loss of interest to either debtor or creditor.

Suppose that on July 1 Brown buys from Jones two bills of goods, each for $100. One is payable July 15, the other Sept. 15. If both bills were paid on July 15, Brown would lose. If both were paid Sept. 15, Jones would lose. But both may be paid on Aug. 15 without loss to either party.

Equation, or averaging, of payments is of especial importance in the settlement of bills that are given a long dating. The averaging is done, and payment of the total is made, in advance of the dates when some of the bills fall due — often in advance of the receipt of some of the goods.

When the Items Are All Debits or All Credits

EXAMPLE. We have the following charges against Burwell and Potter: Jan. 20, 1927, $750, terms 60 days; Feb. 29, terms 30 days, $500; Mar. 6, terms 30 days, $400. Find the equated date of maturity.

BURWELL AND POTTER

DATE 1920	TIME	DUE	AMOUNT	
Jan. 20	60 da.	Mar. 20	$750 \times 16 =$	12,000
Feb. 29	30 da.	Mar. 30	$500 \times 6 =$	3000
Mar. 6	30 da.	Apr. 5	$400 \times 0 =$	000
			1650	$15{,}000 \div 1650 = 9$

Focal date is Apr. 5. Counting back 9 days gives Mar. 27, due date.

EXPLANATION: Assume the due date of last bill (Apr. 5) as the day when all are being paid. The day so assumed is called the *focal date*.

If the balance (in this case, the total of the 3 bills) were paid on this date, Burwell and Potter would owe us interest for 16 days on the 1st item, for 6 days on the 2d item, and for 0 days on the 3d item.

Interest on $750 for 16 days is equivalent to interest on $1 for 12,000 days. Interest on $500 for 6 days is equivalent to interest on $1 for 3000 days. No interest would be due on the 3d item. The total is equivalent to interest on $1

for 15,000 days, which in turn is equivalent to interest on the total, $1650, for 15,000 ÷ 1650 or 9.1— days. 9.1 being less than $9\frac{1}{2}$, we call this 9 days.

The conclusion reached is that if Burwell and Potter should wait until Apr. 5 to pay, they would owe us interest for 9 days. They must therefore pay us 9 days before Apr. 5, which gives Mar. 27 as the equated date of payment.

PROOF. To prove this solution, test it to see whether the interest on items due before Mar. 27 is equal to the discount on items due after Mar. 27. Make this proof, using any rate of interest.

There will be a slight discrepancy, owing to the dropped fraction of a day in the solution, but if the discrepancy is less than it would be on Mar. 26 or Mar. 28, your proof is established.

The foregoing solution is by the so-called "product" method. Solve the following by the same method.

WRITTEN. Find the average due date in each of the following cases:

1.	Jan. 20	$375.00	**5.**	Oct. 15	$164.00
	27	264.00		25	471.00
	31	115.00		31	238.00
	Feb. 4	260.00		Dec. 10	178.00
2.	Apr. 14	$268.00	**6.**	June 1	$425.71*
	26	147.00		18	380.57
	June 5	230.00		July 5	535.83
	21	292.00		22	362.20
				Aug. 10	250.14
3.	June 10	$180.00	**7.**	May 2	$540.36
	25	340.00		22	865.30
	Aug. 7	250.00		June 12	200.74
	24	300.00		July 14	175.96
				Aug. 10	320.18
4.	Apr. 5	30 da. $212.50	**8.**	May 5	30 da. $195.18
	17	30 da. 183.15		27	60 da. 214.37
	25	60 da. 247.12		June 10	90 da. 563.40
	May 14	60 da. 260.82		24	60 da. 215.80

* Omit cents in multiplications, but not in finding balances.

9.	Sept.	1	30 da.	$114.75	**12.**	Jan. 3	30 da. $218.40
		12	60 da.	207.46		20	60 da. 198.75
		25	2 mo.	435.00		Feb. 12	60 da. 246.37
		30	2 mo.	227.45		20	90 da. 312.10
10.	Mar.	2	60 da.	$198.74	**13.**	Oct. 5	30 da. $137.50
		18	60 da.	247.50		18	30 da. 265.27
	Apr.	12	60 da.	320.85		Nov. 12	30 da. 320.70
		29	60 da.	390.40		29	30 da. 475.00
11.	Jan.	15	90 da.	$263.80	**14.**	May 10	2 mo. $218.70
		27	90 da.	150.60		29	2 mo. 745.83
	Feb.	15	90 da.	186.78		June 18	2 mo. 864.95
		28	90 da.	280.15		30	2 mo. 900.00

When the Account Contains Both Debits and Credits

EXAMPLE. Find the equated date for the payment of the following account.

Wolverine Motor Co.

19—	Dr.			19—	Cr.		
June	2	Mdse.	$356 27	June	10	Cash	$250 00
	21	Mdse.	267 40		25	Cash	250 00
July	15	Mdse.	473 68				
	24	Mdse.	130 70			Bal.	728 05
			$1228 05				$1228 05

Take July 24 as the focal date.

Dr. Side		Cr. Side	
Date	Amt.	Date	Amt.
June 2	356×52 = 18,512	June 10	250×44 = 11,000
21	267×33 = 8,811	June 25	250×29 = 7,250
July 15	474× 9 = 4,266		18,250
	31,589		
	18,250		
	13,339		

EXPLANATION. Had the balance of the account been paid on July 24, the creditor would have been entitled to interest equivalent to the interest on $1 for 31,589 days, and the debtor would have been entitled to interest on $1 for 18,250 days. Deducting 18,250 from 31,359 leaves 13,339.

13,339 ÷ 728 = 18, the number of days that must be counted back from July 24. July 24 − 18 da. = July 6, the equated date.

NOTE. Had the balance of interest been in favor of the debtor (a larger total of products on the credit side than on the debit), we would have counted forward from the focal date to get the equated date.

Prove the foregoing solution.

When to Count Forward and When Backward

When the balance of products and the balance of the account are on the same side, it indicates that interest is due the same party to whom the account is due, hence the account should have been paid earlier. Count backward from the focal date.

When the balance of products is on one side and the balance of the account is on the other, it indicates that the interest is due the party who owes the account, and he is therefore entitled to hold the balance longer. Count forward from the focal date.

Had we selected as the focal date the earliest date on the account instead of the latest date, conditions would have been reversed, as the products would not then have represented interest items, but discounts. The above rules would in such a case be reversed.

WRITTEN. Find the equated date of payment for each of the following accounts:

ROGERS AND BLAIR

1. Sept.	3	Mdse.	$ 265.40	Sept.	10	Cash	$ 200.00
	21	Mdse.	194.60		25	Cash	300.00
	29	Mdse.	342.20	Oct.	1	Cash	250.00
Oct.	8	Mdse.	197.63				

ALLEN AND JONES

2. Apr.	5	Mdse.	$ 420.74	Apr.	5	Cash	$ 300.00
	18	Mdse.	258.29		19	Cash	300.00
	29	Mdse.	370.36		25	Cash	70.00
May	8	Mdse.	1240.92	May	1	Cash	750.00
	15	Mdse.	1163.70		10	Cash	350.00

HENRY ADAMS AND COMPANY

3. June	1	Mdse.	$1175.20	June	2	Cash	$ 800.00
	15	Mdse.	1180.18		20	Note	1000.00
	29	Mdse.	1370.00	May	1	Cash	2000.00
May	8	Mdse.	1457.92				
	10	Mdse.	1065.76				

A. F. HARRIS

4. Nov.	10	Mdse. 30 da.	$284.60	Dec.	14	Cash	$ 250.00
	15	Mdse. 30 da.	174.40		17	Cash	175.00
	20	Mdse. 30 da.	167.30		23	Cash	100.00

F. E. BRIGGS

5. Apr.	1	Mdse. 60 da.	$974.46	May	24	Cash	$1200.00
	16	Mdse. 30 da.	700.00	June	29	Cash	600.00
	24	Mdse. 30 da.	675.37				
May	5	Mdse. 60 da.	850.30				

Cash Balance

The cash balance of an account on a given date is the amount of cash that will pay it at that time. If no interest is allowed or charged on any items, it is the balance of the account. If interest is to be charged on overdue bills, the cash balance must include such interest.

The simplest method of solution is one by which interest is charged on all items overdue on the date when the cash balance is figured and credited on all payments made before that date.

EXAMPLE. Find the cash balance of the following on Aug. 1, 1927, if interest at 6% is to be figured on all items.

HENRY B. SIMON & COMPANY

1927						1927			
Apr.	6	Mdse.	30 da.	$478	25	Apr.	16	Cash	$300
	20	"	30 da.	392	40	May	1	"	500
May	1	"	30 da.	278	16	June	1	"	300
	17	"	30 da.	570	58				

INTEREST ON BILLS

$ 478.25	— May 6 to Aug. 1 — 87 da.	$ 6.93
392.40	— May 20 to Aug. 1 — 73 da.	4.77
278.16	— May 31 to Aug. 1 — 62 da.	2.87
570.58	— June 16 to Aug. 1 — 46 da.	4.37
$1719.39		$18.94

INTEREST ON PAYMENTS

$ 300	— Apr. 16 to Aug. 1 — 107 da.	$ 5.35
500	— May 1 to Aug. 1 — 92 da.	7.67
300	— June 1 to Aug. 1 — 61 da.	3.05
$1100		$16.07

$1719.39 − $1100 = $619.39, bal. due on acct.
$18.94 − $16.07 = $2.87, bal. due on int.
$619.39 + $2.87 = $622.26.

Equation of Accounts, Interest Method

The cash balance can be made the basis of a method for finding the equated date of payment, thus:

Take Aug. 1 as the focal date and figure interest on all items up to that date. Find the balance of the account and the balance of interest. Thus far the problem is the same as a problem in cash balance.

Int. on $619.39 for 1 day at 6% = $.10323$\frac{1}{6}$.

$2.87 ÷ $.10323$\frac{1}{6}$ = 28 −, the number of days that must be counted back from the focal date.

Aug. 1 − 28 days = July 3, the equated date of payment.

Merchants' Rule in Partial Payments

The so-called "Merchants' Rule" for finding the balance due on a note or account on which partial payments have been made, is identical, in its method of application, to the plan just described for finding the cash balance of an interest-bearing account.

The Merchants' Rule is: Find the amount of the note or other obligation, including interest to the date of final payment. Then find the amounts due on the various partial payments, including interest, from the time each was made until the date of final payment, and find the total of these. The difference between the two totals is the amount due.

At this time turn back to the illustrative example under Partial Payments, United States Rule, and solve this example by the merchants' rule stated above. What is the difference in result, and why?

WRITTEN.

Find the cash balance of each of the following accounts, as of the date separately specified in connection with each:

1.

B. JOHNSON & COMPANY

19						19			
May	4	Mdse	30 da.	$318	20	June	4	Cash	$250
	12	"	60 da.	139	64	July	10	"	100
	18	"	60 da.	284	32	July	20	"	200

Find the cash balance on Aug. 1, figuring interest at 6%.

2.

G. C. MEYERS & BRO.

1927						1927			
Sept.	1	Mdse.	30 da.	$286	40	Oct.	1	Cash	$175
	15	"	30 da.	138	62		12	"	200
	27	"	30 da.	216	17		28	"	150
Oct.	8	"	30 da.	378	40	Nov.	10	"	200
	18	"	30 da.	870	29			"	

Find cash balance on Dec. 1, 1927, int. at 6%.

3.

J. L. HUDSON & CO.

1926						1927			
Dec.	3	Mdse.	60 da.	$736	82	Jan.	10	Cash	$500
	14	"	30 da.	354	60		19	"	360
	24	"	30 da.	247	25		25	"	750
1921									
Jan.	1	"	10 da.	736	14				
	17	"	30 da.	354	80				

What is the cash balance on Mar. 1, 1927, int. at 6%?

4.

R. L. BROWN

1927						1927			
Nov.	2	Mdse.	Cash	$240	25	Nov.	2	Cash	$100
	10	"	10 da.	217	30		10	"	150
	18	"	30 da.	492	76		20	"	207 55
	26	"	30 da.	547	80	Dec.	1	"	250

What is the cash balance on Jan. 1, 1928, int. at 5%?

5. Find the equated date of payment of each of the preceding problems by the interest method, using your solutions already prepared for the first part of your work on each problem.

6. A note for $1000 was dated Jan. 12, 1928, with interest at 6%. The following payments were made: Mar. 8, 1928, $250; May 22, 1928, $370; July 18, 1928, $196.40. What was the balance due on Sept. 1, 1928, Merchants' Rule?

7. What was due on Oct. 1, 1927, by the Merchants' Rule, on a note for $1140.82, with interest at 7% dated Mar. 5, 1927, if the following payments have been made: Apr. 24, 1927, $360; June 1, 1927, $225; Aug. 22, 1927, $100?

CHAPTER XXXVII

PARTNERSHIP

A **partnership** is an association of persons who combine their capital, labor, and skill in agreed proportions in the conduct of some business, and who divide the profits and bear the losses of the business in agreed proportions.

An *active* partner is one who takes an active part in the management of the business, as contrasted with a *silent* partner, who furnishes capital but is not active. A *general* partner is one who is responsible for the debts of the partnership to the extent of his entire property, as contrasted with a *special* or *limited* partner, who is responsible only to the extent of his investment. Firms in which there are special partners are in some states required to include the word "Limited (Ltd.)" in the firm name. A *dormant* partner is one who is secretly a partner; he is not personally liable for any obligations contracted with any person who did not know of his connection with the firm at the time credit was extended.

Distribution of Profits. Profits may be divided (or losses borne) in any one of many ways, among which are the following:

(*a*) Equal division.

(*b*) Division into shares fixed in advance.

(*c*) Division in proportion to original investment.

(*d*) Division in proportion to average investment.

(*e*) One or more partners to draw a salary, remaining profits to be divided in some agreed proportion.

(*f*) Interest to be allowed each partner on his investment, remaining profits to be divided in some agreed proportion.

Withdrawals and Additions. A withdrawal decreases the investment of the partner who makes it (unless it is a withdrawal of profit only). Additional investment increases the investment balance of the partner who puts the new funds into the business. Profits, if allowed to remain in the business, increase investment. Losses, unless covered by additional funds, decrease investment.

Dissolution. Partners may make any agreement they wish in regard to the distribution of net assets upon dissolution. In the absence of other agreement, however, the net assets remaining after all creditors were satisfied would be disposed of as follows:

1st. All advances to the firm by partners, as distinguished from capital, would be returned.

2d. Each partner would be paid the amount of his present investment, or if funds were insufficient for this then a share of the assets would be paid to each proportionate to his investment.

3d. Any remaining assets would be distributed among the partners as profits, in the proportions agreed upon for profits.

All of the cases enumerated under *Distribution of Profits* are illustrated in the problems. None need explanation except case (d).

Division in Proportion to Average Investments

The average investment for a given period is the amount which if invested for the entire period would be equivalent in earning power to the various amounts which were actually invested, each for the period for which it was invested.

EXAMPLE. A and B were partners. Their original investments, withdrawals, and additions to investment, were as shown below. Their net profit for the year ($3400.00) was divided in proportion to their average investments. How much profit did each receive?

(A)			(B)		
Jan. 1	Invested	$6000.00	Jan. 1	Invested	$6000.00
Apr. 1	Withdrew	720.00	Mar. 1	Invested	600.00
Sept. 1	Withdrew	480.00	Aug. 1	Invested	840.00

A'S INVESTMENT

$6000 invested for 3 mo. is equivalent to $18,000 for 1 mo.
5280 invested for 5 mo. is equivalent to 26,400 for 1 mo.
4800 invested for 4 mo. is equivalent to 19,200 for 1 mo.
Total investment is equivalent to $63,600 for 1 mo.
$63,600 ÷ 12 (months) = $5300, A's average investment.

B'S INVESTMENT

$6000 invested for 2 mo. is equivalent to $12,000 for 1 mo.
6600 invested for 5 mo. is equivalent to 33,000 for 1 mo.
7440 invested for 5 mo. is equivalent to 37,200 for 1 mo.
Total investment is equivalent to $82,200 for 1 mo.
$82,200 ÷ 12 (months) = $6850, B's average investment.
$5300 + $6850 = $12,150, total average investment.

$\frac{5300}{12150} \times \frac{\$3400}{1} = \$483.13$, A's share of profits.

$\frac{6850}{12150} \times \frac{\$3400}{1} = \$1916.87$, B's share of profits.

WRITTEN

1. Miller and Scribner began business with a capital of $4500 furnished by Miller. Scribner was to manage the business and receive one-half the profits. At the end of the year the net assets of the firm amounted to $7800. If this was allowed to remain in the business, what was the investment of each partner at the beginning of the second year?

2. Sankey and Brown engaged in business. Sankey invested $2500 and Brown invested $1500. The profits and losses were to be divided equally. At the end of the first year their losses amounted to $1650. What was the investment of each at the end of the year?

3. Banker and Corwin are partners. Their original investments were $2000 and $1500 respectively. Their profit for the year is $3697.85 and is to be divided in proportion to original investment. To how much profit is each entitled?

4. Sherman and Thomas were partners under an agreement that Sherman was to get 60% of the profits and Thomas 40%. Their investments on Jan. 1 were Sherman $1600, Thomas $4000. On July 1, their total investment was $9850. On Dec. 31 it was $8700. What was the interest of each on July 1? What was the interest of each on Dec. 31?

5. Dennis, Doyle, and Harmon were partners with original investments of $5000, $6000, and $3000 respectively. Profits were to be divided in proportion to original investments. If their gain for the first year was $3450 and no part of it was withdrawn, what was the interest of each at the end of the year?

6. Summer and Brown invested in a business $8000 and $4000 respectively. At the close of the first year they had cash, $2620.83; accts. rec., $5857.25; notes rec., $3729.87; stock of goods, $15,392.78; fixtures, $1478.50; delivery equipment, $1741.35; rent prepaid, $570; and they owed on account, $6473.60, and on notes $3576.90. Brown was entitled to a salary of $200.00

a month. Profits were to be divided in proportion to original investment. No salary or profit having been withdrawn, what was the investment of each partner at the end of the year?

7. Cross Bros. and Dodge are partners; their investments follow: W. H. Cross, $11,350; F. G. Cross, $11,300; R. L. Dodge, $24,000. At the close of the year their books show the following accounts: Cash, $6387.54; accounts receivable, $12,581.97; notes receivable, $4392.86; real estate, $5400; inventory of goods, $23,892.76; fixtures, $1173.89; delivery equipment, $3187.94; Merchants' National Bank stock, $2475; insurance prepaid, $391.50; due employes, $270.34; notes payable, $2386.45; accounts payable, $9228.30. Withdrawals: W. H. Cross, $1250; F. G. Cross, $1420; R. L. Dodge, $2100. On accounts receivable an allowance of 5% for bad debts is to be made and 10% depreciation is to be provided for on delivery equipment. Withdrawals are not considered losses but are deducted from investments. The profits are to be divided as follows: Cross Bros., $\frac{1}{4}$ each; Dodge, $\frac{1}{2}$. What was the investment of each at the close of the year?

8. On Jan. 1, John M. Lodge and Robert E. Owen invested $10,000 and $15,000 respectively, under an agreement that each partner was to be allowed, out of profits, 7% interest on his investment, the balance of profit to be divided equally. On May 1, Owen increased his investment $3000; and on Sept. 1, Dodge withdrew $1000 of his investment. The profits for the year were $6250. To how much profit was each partner entitled?

9. A and B were partners. Their original investments, withdrawals, and additions to investment were as shown below. Their net profit for the year ($5000) was divided in proportion to their average investments. How much profit did each of the partners receive?

(A)			(B)		
Jan. 1	Invested	$6000.	Jan. 1	Invested	$6000.
May 1	Invested	720.	Apr. 1	Withdrew	360.
Oct. 1	Invested	480.	Aug. 1	Withdrew	600.

10. D, E, and F were partners under the same agreement as that stipulated in the previous problem. Their profit for the year was $8436.00. What was the investment of each on Dec. 31?

(D)	(E)	(F)
Jan. 1 Invested $3000.	Jan. 1 Invested $4000.	Jan. 1 Invested $5000.
Mar. 1 Invested 540.	June 1 Withdrew 780.	Apr. 1 Invested 1200.
Sept. 1 Invested 720.	Aug. 1 Withdrew 300.	Nov. 1 Withdrew 900.

11. C and D were partners. C invested $8000 and D invested $3000 at the beginning of business. They were to share profits equally. They were in business one year and decided to dissolve partnership. All property was sold and all debts paid, leaving $5198.70 cash. During the year C had drawn out $1600 and D had drawn out $2100. State how the cash was divided between the partners.

12. M and N engaged in business. Each invested $2500. M was to get $\frac{2}{3}$ of the profits. During the year M loaned the firm $1500 without interest. At the end of the year they dissolved partnership, having $11,500 in cash, and owing $2000 to outside creditors. Show what was done with the $11,500 cash. How much did N receive?

13. G and H are partners with original investments of $8000 and $6000 respectively. Profits are to be divided in proportion to original investments. At the end of the year the firm has $3000 in cash as its only asset, and no debts. During the year, G has withdrawn $9000 and H has withdrawn $7000. How much of the $3000 cash should each receive?

CHAPTER XXXVIII

HOW TO READ THE STOCK MARKET

Table I shows the prices of certain stocks during the day, the change over the preceding day, the last quotations at the close, the net change, and the number of shares sold on the Stock Exchange at New York City on Aug. 5, 1918.

Table II shows the sales and prices on Dec. 24, 1919, of the stocks listed in Table I. It is to be interpreted in the same manner as Table I.

TABLE I

Sales	Name of Stock	Prev. Close	Open	High	Low	Close	Net Change
400	Balt. & O.	54	$54\frac{1}{8}$	$54\frac{3}{8}$	54	54	0
200	do preferred	57	57	57	57	57	0
200	Colo. Fuel & Iron	45	$45\frac{1}{8}$	$45\frac{1}{8}$	45	45	0
500	Mo. Pacific	$23\frac{3}{8}$	$23\frac{1}{4}$	$23\frac{1}{8}$	$23\frac{1}{8}$	$23\frac{1}{8}$	$-\frac{1}{4}$
300	Norfolk & Western	103	103	$103\frac{1}{4}$	103	$103\frac{1}{4}$	$+\frac{1}{4}$
600	Wabash	$9\frac{1}{4}$	$9\frac{1}{2}$	$9\frac{1}{2}$	$9\frac{1}{4}$	$9\frac{1}{4}$	0
100	Woolworth	$112\frac{1}{2}$	111	111	111	111	$-1\frac{1}{2}$

TABLE II

Sales	Name of Stock	Prev. Close	Open	High	Low	Close	Net Change
6400	Balt. & Ohio	32	32	$32\frac{5}{8}$	$31\frac{3}{4}$	$32\frac{1}{4}$	?
1200	do preferred	$46\frac{1}{2}$	46	46	$45\frac{1}{4}$	$45\frac{1}{4}$	?
200	Colo. Fuel & Iron	40	$39\frac{5}{8}$	40	$39\frac{5}{8}$	40	?
1000	Mo. Pacific	$25\frac{3}{4}$	$25\frac{5}{8}$	$25\frac{7}{8}$	$25\frac{1}{2}$	$25\frac{3}{4}$	?
1400	Wabash	$8\frac{1}{8}$	$8\frac{1}{8}$	$8\frac{3}{8}$	8	8	?
100	Woolworth	$121\frac{3}{8}$	122	122	122	122	?

The first line of this table is to be read as follows: On August 5, 1918, there were sold on the New York Stock Exchange 400 shares of common stock of Baltimore and Ohio Railroad. The first sale (called "Open" in the table) was made at $\$54\frac{1}{8}$ per share; the highest price was $\$54\frac{3}{8}$; the lowest price was \$54, which was also the price of the last sale of the day (called "Close" in the table). Net change means the difference in price between

the last sale of the day and the last sale of the previous day. In Baltimore and Ohio there was no *net* change, although the price *fluctuated during the day.*

The second line in Table I means this: There were sold 200 shares of Baltimore and Ohio preferred stock at $57 per share, there being neither net change nor fluctuation during the day.

1. Tell the meaning of the rest of Table I.

2. Tell the meaning of Table II for each stock.

3. Look on the stock market page of your newspaper for quotations on the stocks listed in this table.

Stock Market and Cost Accounting Problems

1. Compute the loss of a man who bought 300 shares of Wabash Railroad stock at $9\frac{1}{2}$ and sold it at $8\frac{3}{8}$, brokerage $\frac{1}{8}\%$ per share in each transaction.

2. Compute the gain of the owner of 100 shares of Woolworth stock if it was bought at its lowest price in Table I and sold at its highest price in Table II, brokerage $\frac{1}{8}\%$ in each transaction.

The following formulas are sometimes used in the solution of stock market problems.

The dividend = par value × rate of dividend.
Par value × rate of dividend = market value × rate of income.

3. When United States Steel 7% stock was selling at 105, was the rate of income more or less than 7%? Why?

4. When 4% railroad stock was sold at 85, was the rate of income as much as 5%? At what price must this stock be bought to earn exactly 5% income?

5. When a certain bank stock was selling at 115, its dividend rate was 10%. Find the rate of income.

6. If a buyer is satisfied with a 5% income on a 10% bank stock, what can he afford to pay for the stock?

7. What rate of dividend on a capitalization increased 50%, is equivalent to a 6% dividend on the former capitalization?

8. I have 50% more stock in a certain company this year than I had last year, but the rate of dividend is only 5% this year as

against 6% last year. What per cent more or less money did I receive in dividends this year than last?

9. A corporation capitalized at $700,000 and a corporation capitalized at $300,000 were bought by Kuhn, Loeb & Co. and merged into one corporation capitalized as follows: Preferred Stock $600,000; Common Stock $900,000. The shareholders of the old companies received for each share of their old stock $60.00 in preferred stock and $90.00 in common stock in the new company. The preferred stock in the new company yields 6% which is the rate of the average dividends of the old companies. What rate of dividends on the common stock of the new company will make the stockholders' receipts from dividends the same under the new corporate firm as under the old?

10. The Acme Co., a corporation, has a capitalization of $500,000 in 5% preferred stock and $1,000,000 in common stock. Its earnings available for dividends in 1920 were $50,000. What rate of dividends could it have declared upon its common stock? (The dividends on the preferred stock must be paid before the dividends can be paid on the common stock.)

11. I own bonds of the Commonwealth Power Co. which are now due and the par value of which is $5,000. One condition of the bond is that it is convertible into (exchangeable form) a like amount of the company's 6% preferred stock at par; and in addition, common stock of the company of the par value of 30% of the bonds. The preferred stock is selling at 93 and the common at 70. What would be my gain if I convert the bonds and sell the stock, brokerage on the stock being $\frac{1}{8}$%? If the bonds were converted into stock and the stock held as an investment and the preferred paid 6% and the common 4%, what would be my annual income from the investment and the per cent of income?

12. Wilson bought 100 shares of Illinois Central stock at 135, yielding a dividend of 7%. Later he bought 60 shares at 115, yielding the same dividend. What is the average cost of his investment per share? What rate of interest is he receiving on his total investment? If the market value of this stock is 120, which would bring him the better terms and how much: to keep the stock or sell it and invest in 105% guaranteed stock at 101, brokerage in each case $\frac{1}{8}$%?

13. A corporation issued $100,000 of 6% 20-year bonds, subject to call at 105 after three years. At the end of three years it called the bonds and issued 5% 17-year bonds at 95. If the corporation had to borrow 4% additional cash necessary to do this, how much did it borrow? How much less interest was paid out annually under the new plan?

14. A factory about to be built on the Pacific Coast requires a lot costing $100,000. The plant (an improvement) will cost $300,000, and the manufactured product (personal property) will average $200,000 assessed valuation each year for 20 years. If Seattle, Washington, taxes lots, improvements, and personal property, the rate being $2.10 and Vancouver, B. C., taxes land only, no tax being placed upon improvements or upon personal property, with the rate of $2.80, how much will the company that owns the factory save in taxes in 20 years by selecting Vancouver?

15. A factory can buy for $38.50 one of the metal parts needed to make its finished product. It can manufacture this part for $36.75 but to do so must purchase machinery at a cost of $150,000. The machinery will last eight years during which time it will make 100,000 of these parts, making 12,500 each year. Figuring interest on the investment at 5%, will the factory save or lose money by installing the machinery? How much, the first year?

16. The capital stock of the Rock Island Railroad Company in 1911 was $75,000,000. Its bonded debt was $98,816,150. If it paid an average interest of 5% on its bonds, what per cent dividend could it declare and still keep on hand undivided profits amounting to $4,567,495.19, if its earnings for the year were $17,008,302.69?

17. The First National Bank is capitalized at three hundred thousand dollars. Its depositors' accounts, non-interest-bearing, amount to more than two million dollars, of which an average of one million five hundred thousand dollars is constantly loaned out at an average rate of interest of 6%. If the other income of the bank is just enough to meet its current expenses, what dividend can it pay on its capital stock?

18. B, a banker, received a deposit of $10,000 from A, upon which he agreed to pay him 6% interest. The same day he

loaned the $10,000 for 30 days at 5% to C, who paid it at once to D, a depositor in B's bank. On the same day D deposited the money and began to draw checks against it. At the end of 30 days D had drawn out all the $10,000, but an analysis of his account showed that his balance for the month averaged $5,000. If B kept this $5,000 out at 5% interest, did B lose or gain in all the transactions with this money and how much?

19. The Central Canning Company's sales for March, 1914, in their six departments, and the costs of goods sold in those departments were as follows:

	Dept. A	B	C	D	E	F
Sales	$1260.32	$688.50	$210.58	$3595.30	$1168.82	$1109.06
Cost of goods sold	876.46	530.50	178.32	2642.34	760.33	933.70

Find the total sales, and the total cost of goods sold, for the six departments. Find the profit in each department and the total profit. Find the per cent of profit in each department and the average per cent of profit. How much would the average per cent of profit be increased by dropping the two departments which paid the smallest per cent of profit, assuming that this would have no effect on the volume of sales in the four departments remaining? Had the sales in Department A been 10% more and the cost of goods sold 10% less, what would have been the per cent of profit in this department?

20. The operating expenses of Smith & Brown's business for January, 1914, were $863.56. Advertising, traveling, and miscellaneous expenses for the month amounted to $753.10. During the month $168.22 was received for interest on loaned capital. Smith's investment for the month was $7200.00. Brown's investment was $3600.00 for the entire month and $2000.00 for 12 days. Deducting the interest gains from the expenses, and adding interest on the capital invested at 6% and also adding $140.52 for bad debts, what was the cost of doing business for the month? If the cost of the goods sold was $15,651.94, what per cent was the cost of doing business of the cost of the goods sold?

21. A man was constantly 6 months behind on the rent for his store, which was $225.00 a month, and constantly 4 months behind on the rent for his home, which was $50.00 a month. His

payroll averaged $1000.00 a month and he was constantly 2 months behind in paying his employees. He paid all business bills 3 months after they fell due, and his bills amounted to $20,000 annually. It cost this man $2250.00 a year for living expenses. If money was worth 6%, what per cent of his living expenses was borne by his creditors?

22. A man divided his estate among four sons and two daughters, the latter sharing equally. The oldest son received $\frac{1}{5}$ of the whole estate, the second son $\frac{1}{6}$, the third $\frac{1}{7}$, and the fourth $\frac{1}{8}$. The oldest son received $2400 more than the youngest son. How much did each son and daughter receive and what was the value of the whole estate?

23. A man who has $6000 invested so that he makes 6% annually on his investment, buys with this money a city property for $6000 which pays him $65 per month rent. If repairs, taxes, and insurance cost him $350 per year, does he gain or lose by the transaction and how much?

24. The transportation revenue of a certain railroad for the year ending June 30, 1911, was freight, $43,368,395.66; passenger, $20,240,528.03; mail, $1,577,219.40; express, $2,053,549.44; miscellaneous, $800,107.57. What per cent was each of the whole?

25. The average number of miles of track operated by the Rock Island lines for the year ending June 30, 1911, was 8,026.09. For the preceding year the average mileage was 8,043.59. What was the increase or decrease? the per cent of increase or decrease?

How Business Houses Figure Profits

Most business houses use as a basis for figuring profits the manufacturing cost of the article sold. Of course, overhead expenses, freight, and selling expenses are treated as separate items and finally enter into the cost of the article sold. A large number of business houses compute the rate of profit on the selling price of the article rather than the manufacturing cost. Whether the basis of figuring be the selling price or the cost of the goods, the same results are finally obtained since overhead, freight, and the cost of selling must ultimately be included as part of the cost of the article sold.

It will be seen from the following problems that the cost price (manufacturing cost) is the true basis for figuring all profits whether the business house figures profits on the selling price or the cost price of the article. It is simply a different way of reaching the same result. For example, the following rule for solving problems where it is desired to estimate the cost of doing business and at the same time realize the desired net profit clearly shows that the manufacturing cost of the article is the true basis of solution of all such problems.

To find the price at which an article will have to be sold in order to cover the cost of doing business and at the same time realize the desired net profit: *Add together the per cent of cost of doing business (including overhead and selling cost) and the desired per cent of net profit. Deduct this amount from 100% and divide the cost price of the article by this result.*

1. At what price must a publisher sell a school book that cost him $66\frac{2}{3}¢$ (manufacturing cost) in order to realize a net profit of 25% if he estimates that the selling expense is 20% and overhead 5%?

2. At what price must a merchant sell hats that cost him $3.00 if the overhead expense is 15% and the selling expense 20%?, to realize a profit of 25%?

3. A certain article costs $1.00 wholesale including freight. What will it have to be sold for to bring a net profit of 10% after allowing 18% for the cost of doing business?

4. What per cent profit must be added to the cost of an article so that 25% may be deducted from the asking price and a profit of 20% still realized on the cost?

5. What must be asked for goods costing $20.00 in order that a discount of 20% may be allowed and a profit of 20% be maintained?

6. Find the selling price of a hat costing $3.50 if store expense is 20% of cost and the merchant wishes to make 20% above cost and expenses.

7. The manufacturing cost of an article is $1.50. At what price must this article be sold to yield the merchant a profit of 20% if the salesman is allowed 25% of the selling price for making the sale?

Miscellaneous Problems

1. A merchant after reducing his marked price $33\frac{1}{3}\%$ sold a hat for \$3.00 and still gained $33\frac{1}{3}\%$. What was his marked price? his cost?

2. A merchant sells an article for \$150, gaining 20%. If he had bought it for $\frac{1}{5}$ less and sold it for \$30 more, what would have been the gain per cent?

3. A suit of clothes was marked at \$57.00 which was 50% more than the cost. It was sold at a reduction of 30% from the marked price. What per cent of profit was made on the first cost?

4. A merchant lost 20% of his goods by fire. He then sold what was left at a gain of 30%, receiving \$2500 more than he paid for the whole. What did the goods cost him?

5. An article costing \$3.50 is so marked that after deducting a discount of 20% from the list price, it is sold at a profit of $14\frac{2}{7}\%$. What was the list price?

6. At what price must a merchant mark a hat that cost \$2.00 so as to make 20% after discounting his marked price 40%?

7. Type metal is composed of $\frac{17}{20}$ lead, $\frac{2}{25}$ antimony, and $\frac{7}{106}$ tin. Lead is worth $4\frac{1}{2}¢$ a pound, antimony, $10\frac{1}{2}¢$ a pound, and tin, 52¢ a pound. If the manufacturer estimates that it costs him $1\frac{1}{2}¢$ a pound to make it and $\frac{1}{10}$ of selling price to sell it, what profit does he make on 300 pounds if he sells at 11¢ a pound?

8. Green and Jaeger publish a catalog of their hardware every year on Jan. 1. All hardware is listed at retail price. Retail dealers can buy at $33\frac{1}{3}\%$ off of catalog price. Out-of-town jobbers can buy at $12\frac{1}{2}\%$ less than the price to retail dealers. Jobbers in the same city can buy at 10% less than out-of-town jobbers. On June 1 all stoves were reduced 10% from catalog price. How much did Hobart & Co., city jobbers, pay on June 15 for 10 stoves listed at \$65.00 each, if they were allowed a discount of 2% for cash?

9. E. R. Bowen & Co. allow 5% discount for cash in 10 days. Jones & Graff, customers buying \$2,500.00 worth of goods, are unable to pay the entire bill within the 10 days. E. R. Bowen & Co. agree to accept \$1,500.00 on account, and allow discount

on the part of the bill covered by the payment. For what amount should E. R. Bowen & Co. credit Jones & Graff upon receipt of $1,500.00 from the latter?

10. Green & Co.'s letterheads cost $2.65 per M. Their printed envelopes cost them $1.35 per M. A typewriter ribbon costing them 75¢ will last one stenographer four weeks. They hire correspondents for $30.00 a week who dictate 100 letters a day and stenographers to whom they pay $22.00 a week who take and transcribe 50 one-page letters a day each, and use 30¢ worth of notebooks and pencils a week each. Estimating waste of paper and envelopes at 10% and adding $\frac{1}{2}$¢ a letter for the proportion of general office expenses chargeable to stenographic expense, what does it cost the firm to send out one letter under 2¢ postage?

11. From the following statistics, find what it costs to manufacture a box of 1 doz. pairs of kid gloves: materials, $300.00 a thousand pairs; labor making the gloves, $35.00 a hundred pairs; boxes and tissue wrappers, $45.00 a thousand pairs; packing in boxes of 1 doz. pairs each, $100.00 a thousand boxes. Figure waste on materials and boxes at 2% and overhead charges at $.0527 a box.

12. A merchant buys gloves at $9.00 a dozen f. o. b. Gloversville, N. Y. He pays 65¢ for express charges on 25 dozen pairs. A clerk who is paid $10.00 for 50 hours' time per week consumes 2 hours unpacking the gloves, marking them, and placing them in stock and on display. What do the gloves cost per pair ready to sell? Adding 15% for overhead and selling charges, what is the gross cost per pair sold? What per cent profit does the merchant make if he sells the gloves for $1.50 per pair?

13. (a) On April 15, Turner bought through his broker 20 shares of Continental Tobacco Company's common stock at 96, the broker advancing the purchase price. On May 15, upon instructions from Turner, the broker sold the stock at 97$\frac{1}{2}$. Did Turner gain or lose, if the interest charged by the broker on the money advanced for the purchase was 6%, and brokerage was $\frac{1}{8}$% for buying and $\frac{1}{8}$% for selling? (b) If the sale had been made at 96 what would have been Turner's loss? (c) If the sale had been made at 99, and while Turner had the stock a

dividend of 4% had been paid, what per cent would he have made on the money invested?

14. If the output of a bakery is 20,000 loaves of bread a day, what is the cost of production per loaf, the daily outlay being $474.50 for flour, $125.00 for other ingredients, and $64.50 for labor? What is the gross profit on each loaf, if the entire output is sold at 4¢ a loaf? Allowing $62.50 a day for running expenses, and deducting 5% from sales for allowances made to customers and bad debts, what is the baker's daily profit or loss?

15. Brown and Co., grocers, make it a rule to mark goods for sale at 50% above cost. If 10% of their stock spoils and has to be thrown away, 10% has to be sold at cost, and the rest is sold at marked price, what per cent profit do they make on a given lot of groceries?

16. Shubert & Co. handle Perrin's men's kid gloves. They plan to keep sizes in stock in the ratio shown by the following scale of sizes:

Sizes	7	$7\frac{1}{4}$	$7\frac{1}{2}$	$7\frac{3}{4}$	8	$8\frac{1}{4}$	$8\frac{1}{2}$
Quantities (dozens)	$\frac{1}{4}$	$\frac{1}{2}$	1	1	1	$\frac{1}{2}$	$\frac{1}{4}$

If they have on hand $\frac{1}{2}$ doz. of each size except size $7\frac{1}{2}$, how many of each size shall they specify in an order for 12 doz.?

17. Shubert & Co. plan to keep in stock Monarch O. F. & B. shirts in the ratio of quantities shown by the following scale:

Sizes	14	$14\frac{1}{2}$	15	$15\frac{1}{2}$	16	$16\frac{1}{2}$	17
Quantities (dozens)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

There are on hand $\frac{1}{2}$ doz. No. 14, $\frac{1}{12}$ doz. No. $15\frac{1}{2}$, and $\frac{3}{4}$ doz. No. 17. How many of each size shall they specify in an order for 50 doz.?

18. How many of each size of Crown derby hats shall Shubert & Co. include in an order for 18 doz., to maintain the proportions shown by the following scale, if sizes on hand are as shown:

Sizes	$6\frac{3}{4}$	$6\frac{7}{8}$	7	$7\frac{1}{8}$	$7\frac{1}{4}$	$7\frac{3}{8}$	$7\frac{1}{2}$
Scale (dozens)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{8}{12}$	$\frac{8}{12}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{12}$
On hand (dozens)	$\frac{1}{6}$	$\frac{1}{12}$	0	$\frac{5}{8}$	$\frac{1}{4}$	1	$\frac{1}{12}$

Watered Stock

Some corporations are prohibited by law from declaring dividends in excess of a certain per cent. If a corporation with a

capital stock of two million dollars gains two hundred thousand dollars and is not allowed to declare a dividend greater than eight per cent, what will the directors of the corporation do?

They may give to the stockholders five hundred thousand dollars worth of new stock. Then they can declare a dividend of eight per cent on two million five hundred thousand and divide the two hundred thousand among the stockholders, without violating the law. The five hundred thousand dollars worth of stock certificates issued add no value to the corporation but it enables the corporation to declare dividends on a two million five hundred thousand basis instead of a two million basis.

Stock issued simply to swell the capital stock is called *watered stock*. Sometimes the managers of a corporation issue watered stock when they wish to keep the public in ignorance of the true rate of dividends.

New York Stock Exchange

The principal stock exchange or stock market in America is in New York City. Most of the stock bought and sold at the exchange is dealt in not to raise money or to invest money, but to make money on the rise and fall in the price of stocks. Dealers sell stock with the expectation that the price will fall. They buy stock with the expectation that the price will rise. Dealers who try to drive the price of stock up by buying are called "bulls" and dealers who try to drive the price down by selling are called "bears."

Besides stock of the various corporations, corn, wheat, cotton, etc. are sold. When wheat is sold the seller agrees to deliver the wheat to the purchaser within a certain period on demand but the purchaser seldom demands the wheat. He either sells the wheat or settles for it at the market price. A dealer may sell a million bushels of wheat if he has not a kernel of the grain, but if the purchasers demand the wheat, he must deliver it, and will have to buy it in the market. If those who have bought the wheat in the exchange have also bought up the real wheat in the market, they may compel the sellers of wheat in the exchange to buy it at a high price. There are various schemes for sending the price of stocks up and down.

Buying on a Margin

Stock brokers often buy stocks on a margin. If a broker offers to buy cotton for his speculator, on a margin of 10%, he means that he will put in 90% of his money and 10% of the speculator's money, buy stock, and hold it to the gain or loss of the speculator. The broker changes a commission for buying or selling stock and interest on the money he puts in for the speculator. The speculator may give instructions to his broker as to when to buy and sell stock, or he may let the broker use his own judgment. If the stock that the broker buys advances in price, he may sell it and remit to the speculator the gain from the sale less the commission and interest charges. If the stock that the broker buys falls in price he may hold it for a time, expecting it will advance again. If the stock is sold at a loss, the loss and the broker's charges are taken out of the margin.

When the price of stock falls so low that there is danger that the margin may not cover the loss, the broker sells the stock to save his money invested. Buying on a margin is a hazardous risk and is calculated to stimulate a speculative spirit which frequently results in great losses and insolvency on the part of the speculator.

Review Stock Market Problems

1. A speculator sent $510 to a broker to be invested in stock on a 10% margin. The broker bought at 102 and 12 days later sold it at 105, brokerage $\frac{1}{8}$% in each case. What was the speculator's net gain if the broker charged 6% interest?

2. A broker received $1500 to invest in stocks on a margin of 10%. He bought 120 shares of stock at 105, held it 15 days and sold it at 108. If the broker charged $\frac{1}{8}$% for both buying and selling and 8% interest on his money invested, what was the speculator's gain?

3. A broker received a remittance of $1200 as a 10% margin and purchased 200 shares of stock at 60. 19 days later he sold the stock at 82. If the broker charged $\frac{1}{8}$% for buying and selling and 6% interest, what was the speculator's gain?

4. A broker received $2000 for an 8% marginal investment. He bought 300 shares of stock at 83, held it 3 days, and

sold it at 94. Find the broker's profit, commission $\frac{1}{8}\%$, interest 6%.

5. A speculator sent his broker \$5000 as a 10% margin for investment in bank stock. The broker invested \$5000 in stock at 125. The same day the stock fell to 113 and the broker sold out to save his investment. What did the speculator lose, brokerage $\frac{1}{8}\%$?

CHAPTER XXXIX

COOPERATIVE MARKETING AND FARM PROBLEMS

Cooperative marketing is the group marketing of a large farm commodity through one central selling channel. An individual farmer can not materially affect the market for his produce. By joining with neighbor farmers in the shipment of grain, fruit, stock, or other farm products, lower marketing costs, better prices, and better distribution are obtained. This also results in the standardization and grading of the product, better care in packing (especially fruit), and the extension of the market.

It aids in reducing price fluctuation. As prices increase, the market is flooded which usually results in a rapid decrease in price. In the case of grain the producers flood the market at reaping time. This is partly because they need the money and partly because they do not have storage facilities. The demand for grain is distributed more or less throughout the year. If the farmer can deliver his grain to the cooperative association to be distributed as demanded by the consumer, it is certain that the farmer would realize a better price and the consumer would have a more regular supply. The law of supply and demand as applied to the consumer does not regulate the price since it is often determined by the producer flooding the market.

Better business conditions result also, for the farmer is able to realize a part of his cash at the time of reaping the grain and to receive the balance in periodical installments. In this way he can better finance his farm and living operations. The bank will lend him money on his grain receipts. His credit will be good, for the grain receipts will show that he will receive cash at a certain time. This plan may be applied to other products as well as grain.

Cooperative marketing associations are organized for the marketing of farm products. They are made up of farmer members who unite in the marketing of their product. They are not organized for profit. The income received from the sale of the product is distributed among the farmer members on the basis of the amount of the product they had to sell, after all necessary operating expenses have been paid.

Cooperative Marketing Applied to Dairying

A cooperative dairying association consists of a number of farmers who have united for the marketing of their milk. They each subscribe to stock in the association. This is needed to provide capital to begin operations. The association endeavors to secure the best price possible for the product and to do this must dispose of every part of the milk. A part of the milk is sold on the market, a part is used for ice cream and cheese, and the balance

is converted into a by-product or surplus which must be sold. The amount to be paid the farmer is determined by the price received for these products. For every one hundred pounds of milk there is a certain portion or per cent, of butter fat. This depends upon the quality of the milk. It varies among different cows, and also with the same cow if she is not getting the proper feed. The association selects some standard of butter fat upon which the price of milk is based. Four (4) per cent is used by many as the standard, although in some instances it is thought to be too high and 3.5 per cent is used.

To Determine the Price the Cooperative Association May Pay For Milk

EXAMPLE. Find the average price of 4 per cent milk when 70 per cent of the milk is market milk for which the association can pay 66 cents per pound of butter fat; 20 per cent of the milk is for cheese and ice cream for which they can pay 60½ cents per pound of butter fat; and 10 per cent of the milk is surplus for which they can pay 52½ cents per pound of butter fat.

SOLUTION:

4×66¢	=$2.64	prices of 4 per cent milk for 100 pounds
4×60½¢	= 2.42	
4×52½¢	= 2.10	
70% of $2.64	=$1.848	
20% of 2.42	= .484	
10% of 2.10	= .210	
	$2.542	or

$2.54 is the average price of 4 per cent milk

Problems

1. What price can a cooperative dairying association pay for 3.5 per cent milk if 65 per cent is market milk worth 68 cents, 25 per cent is milk used for by-products worth 62 cents, and the balance is surplus milk worth 50 cents per pound of butter fat?

2. If market milk is worth 67 cents, milk for cheese and ice cream 63 cents, and surplus milk 51½ cents, how much can a cooperative association pay for 4 per cent milk if 68 per cent is market milk, 21 per cent is milk for cheese and ice cream, and the balance surplus?

3. The Hoosier Dairying Association has established a price of $2.30 for 3.5 per cent milk. What may it pay for market milk if 20 per cent of the milk purchased is for by-products worth 61 cents and 10 per cent is surplus worth 50 cents per pound of butter fat?

4. A cooperative milk association pays $2.52 per hundred weight for 4 per cent milk. If 70 per cent of the milk is for the market and worth 67 cents per pound; 20 per cent is for cheese and ice cream worth 58 cents per pound; what can it pay per pound of butter fat for the balance which is surplus milk?

Purchase of Milk From the Farmer by Cooperative Associations

The farmer sells his milk to the association to which he belongs. They haul this milk to the plant in trucks and prepare it for resale. If they find the proper market they are able to pay the farmer a good price for his milk. They are at least able to compete with trade marketing associations better than the farmer alone could compete with them. The price received by the farmer depends upon the test and price used by the association as their standard. We have already learned how this is determined.

EXAMPLE. When 4 per cent milk is worth $2.54 per hundred weight, what should a farmer receive for 9650 pounds of 3.4 per cent milk?

SOLUTION. For every .1 per cent above or below the 4 per cent milk test which is used as a standard, 4 cents is added or subtracted respectively.

3.4 per cent is .6 per cent below the standard (4 per cent)

$6 \times 4 = 24$ the number of cents to be subtracted from the standard price for 4 per cent milk.

$\$2.54 - .24 = \$2.30$ the price to be received per cwt.

$9650 \times \frac{2.30}{100} = \$221.95$ the total price received.

Problems

1. When 4 per cent milk is worth $2.50 per hundred weight, what should a farmer receive for 12,342 pounds of 3.2 per cent milk?

2. How much will a farmer receive for 2,752 pounds of 4.5 per cent milk if the standard price for 4 per cent milk is $2.68 per hundred weight?

3. Find the value of 8,763 pounds of 3.5 per cent milk if the standard price for 3.5 per cent milk is $2.64 per hundred weight.

4. What is 3.2 per cent milk worth if 4 per cent milk is worth $2.54 per hundred weight?

5. If the standard milk test is 3.5 per cent and the price is $2.70, what does the milk test for which the farmer receives $3.10 per hundred weight?

TABLE SHOWING the price per hundred weight of milk using 3.5 per cent as a basis for milk which is worth $2.50 per hundred weight:

TEST	PRICE PER CWT.	
3.	$2.30	
3.1	2.34	
3.2	2.38	For every .1 per cent the milk tests above or
3.3	2.42	below the standard (3.5) the price increases or
3.4	2.46	decreases, respectively, 4 cents per hundred
3.5 standard	2.50	weight.
3.6	2.54	
3.7	2.58	
3.8	2.62	
3.9	2.66	
4.	2.70	This table may be used as a basis for oral or
4.1	2.74	written problems.
4.2	2.78	
4.3	2.82	
4.4	2.86	
4.5	2.90	
4.6	2.94	
4.7	2.98	
4.8	3.02	
4.9	3.06	
5.	3.10	

6. The price of 4 per cent milk is $2.54. Which is better, to sell 3.4 per cent milk at 4 cents per .1 per cent below the standard or to sell at straight 63½ cents per pound of butter fat?

7. Henry Johnson has kept an accurate record of the milk received from his four cows for one month. Cow number one gave 964 pounds of milk which tested 4.4 per cent; cow number two 573 pounds of 3.1 per cent milk; cow number three 976 pounds of 4.5 per cent milk; cow number four 896 pounds of 4.7 per cent milk. How much income has each cow produced if 4 per cent milk is worth $2.58 per hundred weight? Assuming that his cows are all the same age and have had the same feed and pasture, what would you suggest that he do with cow number two? If cow number one is worth $100, what should cow number two be worth for dairy purposes?

8. Which is better to sell 4.6 per cent milk for 63½ cents per pound of butter fat or, if the standard price of 4 per cent milk is $2.54 per hundred weight, add 4 cents per .1 per cent above the standard?

9. One hundred farmers have 5000 pounds of milk to sell daily. If each farmer should market his own milk it would take him on an average 30 minutes daily. His time is worth 25 cents per hour. In a cooperative association one man averages 5000 pounds every two hours. His time costs the farmer 75 cents per hour. Should the farmer sell his own milk or sell through the association provided that the same price is obtained each way? What would be his gain in six months time?

Orderly Marketing

Cooperative marketing tends to encourage orderly marketing. By this, farmers market their product in conformity with the demand of the consumer rather than with the supply of the producer. The unloading of products at irregular periods does not conform to the economic needs of the consumer. The price of live stock is largely determined by the number delivered each day. If more hogs are delivered than the packer can use he will lower the price to reduce the quantity delivered, and increase the price to secure the number needed.

Problems

1. The National Stockyards report the following receipt of hogs and the price paid per hundred for the period of five days beginning December 1:

December 1—	8000 hogs average 200 pounds @	$10.00
2—	10000 hogs average 200 pounds @	9.60
3—	4000 hogs average 200 pounds @	10.10
4—	12000 hogs average 200 pounds @	9.50
5—	6000 hogs average 200 pounds @	10.05

If the law of supply and demand justified the delivery of 8000 hogs each day at $10.00, what would the farmers as a whole have gained or lost by receiving an amount based on this price?

2. The market for sheep in Buffalo during the week of October 5 justified the price of $13.00 per hundred-weight. The receipts of Monday and Tuesday on the Buffalo market were 10,000 sheep each day and the price was $13.10. Wednesday the receipts

jumped to 12,000 and the price dropped 30 cents. Thursday the receipts were 13,000 and there was an additional drop of 10 cents. Friday the receipts were 10,000 and the price advanced 30 cents. Saturday the receipts were 8,000 and the price advanced another 10 cents. The average weight of the sheep was 110 pounds. The daily consumption called for 10,500 sheep. Would orderly marketing have gained for the farmers in general? If so, how much?

3. The demand of the consumer calls for a daily receipt of 10,000 cattle in a certain stock yard. The unorderly marketing of the producer caused the following fluctuation on receipts and prices for the week of January 15:

Monday	10,000 average weight 800 pounds @	$7.50
Tuesday	8,000 average weight 800 pounds @	8.00
Wednesday	12,000 average weight 800 pounds @	7.45
Thursday	9,000 average weight 800 pounds @	7.85
Friday	11,000 average weight 800 pounds @	7.50
Saturday	10,000 average weight 800 pounds @	7.60

The law of supply and demand for that week justified a price of $7.50. Would the farmers have gained or lost by having orderly marketing and how much?

Cooperative Marketing Associations Purchase Supplies For Members

Cooperative Associations assist their members to obtain the best seeds, fertilizers, fuel, and other necessary supplies by quantity purchases. They are able thereby to get better quality and lower prices through the medium of their local association. Orders are placed through the local association but deliveries are made direct to customer. The local association pays the manufacturer or mine and charges the customer a small fee for handling or the goods may be billed at current market prices. At the end of the period any surplus remaining is returned as a patronage refund.

Problems

1. The local organization purchased 15 ton of 0-16-0 fertilizer at $24.25 per ton, 25 ton of 4-24-4 at $59.30 per ton, and 12 ton of 3-8-6 at $35.25. The patronage refund made by the state organization was 9 per cent of the list price. What would the local organization receive in patronage refund?

2. The list price given in the above problem is $3 below regular elevator price for 0-16-0, $3.50 for 3-8-6, and $6 below for 4-24-4. Find the total saving by cooperative purchasing.

3. A purchases 50 ton of hominy from the local elevator at $42 per ton. B purchases 50 ton of hominy from a Federal Farm Bureau at $42 per ton. The patronage refund on Farm Bureau handled feeds is 5 per cent. How much does B save by using the Farm Bureau purchasing department.

4. A purchased chicken mash for his chickens at $4.10 per hundred weight at a local dealer. B purchased chicken mash at $4.15 at the Farm Bureau feed storage. During the year each bought 1000 pounds of mash. At the end of the period B received a patronage refund of $5 from the Farm Federation. Should B buy from the local dealer or Farm Federation. How much was his gain or loss if he remained with the federation?

5. Of the 1927 crop, a cooperative sold 75,000,000 pounds of Grade A tobacco @ 35¢ per pound and 25,000,000 pounds of Grade B tobacco @ 25¢ per pound.

The salary of officers and employees and other operating expenses amounted to $125,000. Deductions for the purchase of warehouses amounted to 50¢ per hundred pounds of tobacco and 1 percent of the gross sale price was placed in a reserve fund to be used in case of emergencies.

In addition to the above, some scrap tobacco was sold for $10,000 and an income of $2,500 was obtained by renting out two of the association's warehouses. What was the average net price per pound, returned to the growers?

Miscellaneous Farm Problems

1. A farmer pays $10 per year for membership in a cooperative cotton association. By belonging to this association he is able to save $2.25 an acre on fertilizer and $1.15 an acre on improved seed. He is also able to realize 3½ cents more per pound on his yield of 420 pounds per acre. If he grows 40 acres of cotton, what are his net savings by being a member of the association?

2. Farm "A" and farm "B" are identical except that the cost of marketing the products of farm "B" is $60 per year less than farm "A". If this condition is permanent and money is worth 6 per cent, which farm is worth the more and how much?

3. A boy has nine bee hives in his colony that produce 90 pounds of comb honey per colony. What is his income if he sells the honey at 20 cents per pound?

4. How many plants will be required for a strawberry patch containing $\frac{1}{2}$ acre if the plants are set 18 inches apart in a row and the rows are 3 feet apart, if the patch is 10 rods long?

5. What rate of interest can a farmer pay if he can buy a farm for $10,000 cash rather than pay $11,000 for the farm on which a five year loan at 6 per cent interest can be secured?

6. It takes a farmer one hour each day to drive his cows 30 rods to pasture. If his time is worth 25 cents per hour and a fence could be constructed for a lane at 70 cents per rod, how much would he save in six months time by constructing the lane?

7. If a tractor can plow at the rate of 3 miles an hour, what area will a 3-furrow twelve inch plow turn in 10 days of 8 hours each?

8. Two hours each day are required to pump water for the stock of a farmer. His time for this is worth 15 cents per hour. How long would it take to pay for a gasoline engine costing $90? The cost of gasoline and oil for each day the engine is used will amount to 15 cents.

9. A farmer has 40 acres of land divided into 10 acre fields. He also owns 15 dairy cows, to each of which he feeds silage 26 pounds, hay 7 pounds, corn 2 pounds, oats 2 pounds, and oil meal 1 pound per day. He has a four year rotation of corn, oats, and clover. The corn yields 60 bushels per acre in one field, 10 ton silage in another. The oats yield is 50 bushels per acre, and hay yield 3 tons per acre. If the herd is sold at the end of 200 days, how much feed will be left for other stock?

10. How much corn must be sold at 55 cents per bushel to pay for the oil meal used in the above problem if oil meal is worth $35 per ton?

11. A man loses $1.04 on each $100 worth of sheep sold and gains $15 on each $100 of hogs sold. What will be his gain or loss on sale of 100 sheep at $7 per head and 50 hogs weighing 200 pounds each at 9 cents per pound?

12. If a bushel of corn will produce 12 pounds of fat when fed to hogs, what must the farmer get per pound for his hogs, allowing 2 cents per bushel for fertilizer purposes for feeding on the place, in order to get an equivalent to 66 cents per bushel for his corn?

13. Corn is worth 63 cents per bushel. Pork is worth 9 cents per pound. If a bushel of corn causes the hog to gain 9 pounds, how much per bushel will the farmer gain or lose by feeding his corn to hogs?

14. A man has a mower worth $60, a binder worth $150, and other machinery worth $600. These tools depreciate 10 per cent per year from being left in the weather. He can build a shed for them for $200 that will depreciate 4 per cent per year. What will be saved in 10 years by building the shed?

15. About 4.5 pounds of corn are required to produce one pound of hog fat. Jones has 500 bushels of corn. He buys fifty 60 pound feeder hogs at 12 cents per pound. Market hogs are worth $10.50 per hundred weight. Corn is worth 95 cents per bushel. Will Jones gain or lose by buying hogs and feeding them if the feeding of the corn on the place is worth 2 cents per bushel for fertilizer purposes?

16. Simmons is offered 13 cents for 200 feeders averaging 125 pounds. He has no corn and can buy none for less than 80 cents per hundred weight. He thinks if he makes his hogs weigh 200 pounds he can get $11.50 per hundred weight for them. Had he better sell the feeders or buy corn and feed them to weigh 200 pounds? (3.5 pound of corn is required to produce one pound of hog fat.)

17. Smith has farming machinery and tools valued at $1500. His neighbor just across the road offers to rent him shelter for his tools for $20 per year. The machinery and tools depreciate 50 per cent of their value in 5 years from being left out of shelter. Had Smith better rent the shelter or leave his tools in the weather?

18. A certain farm of 100 acres is worth $100 per acre. The state exercises the right of eminent domain and builds a state road along one edge of the farm which takes 3 acres of the farming land. He receives $300 for this land. His taxes are raised

$25 per year. At the end of 5 years he sells the farm for $115 per acre. Did the state road make or lose him money? How much?

19. John desired to start farming but had no money. His banker offered him $2000 at 6 per cent for 5 years. John considered this an opportunity and borrowed the money. By paying cash for his implements he saved $250. During the next 5 years his rent cost him $2500, living expenses $1500, other expenses such as horse shoeing, machinery repair, etc. cost him $200. He fed most of his crops to stock purchased for $2000 and stock produced on his farm. His gross income was $10,000 and by taking inventory he found his machinery and other assets valued at $1200. Is John worth more or less at the end of 5 years and how much?

20. A owns a farm worth $110 per acre. A state road is built along one side of his farm which adds $10 per acre to the value of his land. Before this road was built, he was forced to haul to market with a team and wagon. Market was 10 miles from his farm. Eight hours were required to go and return. The team was worth $2.00 to work at home. He now purchased a truck for $1000 to haul to market. Gas, oil, and wear and tear is considered 4 cents per mile of driving. His time used in going to market is now one hour.

(a) How much does the road add to the land value of A's 100 acre farm.

(b) If he goes to market on an average of 20 times a year, how much does he gain or lose by the new road per year in going to market?

(c) How much does he gain or lose if his taxes are raised 20 cents per acre?

21. A farmer has $1\frac{1}{2}$ acre in the middle of a field on which all crops drown out. An outlet for the swamp is 25 rods away. The remainder of the field yields him a net income of $10 per acre during an average year. It costs 90 cents a rod for tile and 96 cents a rod for digging the ditch, laying the tile, and filling in. If the tile lasts 15 years, would it pay to ditch the swamp? Find the gain or loss per year.

22. A herd of 15 cows produced an average of 4800 pounds each of milk yearly with an average test of 3.8 per cent butter fat. Whole milk of this grade may be contracted for at $1.50 per hundred weight or 33 cents per pound of butter fat. The skim milk remaining when cream is sold is worth $5.25 per cow for hog feed. The extra cost of separating the milk is $15 per year. In which case will the farmer receive the greater net income?

23. Three cows costing an average of $70 per head will give 170 pounds of butter each per year which sells at 39 cents per pound. Feed, labor, etc. to care for each cow costs $60.50 per year. Can one afford to borrow the cost price of the cows at 6 per cent interest? What is the net profit (in per cent) on the investment?

24. Mary Jones, the wife of John Jones, decides to make some pin money by raising chickens. She buys a brooder for $25 and 500 young chicks at 12½ cents each. She pays for feed for year $100. By special records she finds that she has been able to raise 300 of these chickens and sells 150 of them for fries at $1.00 each. She picks out 20 of her best cockerels and sells them for stock at $2.00 each. They have killed 10 young fries for their table. She is offered $1.50 each for the remaining pullets but decides to keep them for stock. What is her profit for her summer's work if the brooder may now be sold for $20.00?

25. H. A. Arthur sold 40 hogs to the Consolidated Union Stock yards. Their total weight was 12,000 pounds. What should he receive for these hogs at $8.80 per hundred if the stock-yards charge 10 cents each for yardage and 20 cents each for commission.

26. If the Consolidated Union Stockyards should ship the hogs purchased in the above problem to Johnson Brothers, Packers, Cleveland, Ohio at $9.00 what would be their gross profit for handling the shipment?

27. L. A. Barnes sold to the National Stockyards 30 hogs, 10 cattle, 40 sheep and 4 calves. The market for the day was $9 per hundred on hogs, $12 per hundred on cattle, $15 per hundred on sheep and $14 per hundred on calves. Total weights were as follows: hogs, 6,000 pounds; cattle, 10,000 pounds; sheep, 4400

pounds; and calves, 800 pounds. The charge for yardage on hogs was 10 cents each; sheep 10 cents each; cattle 25 cents each; and calves 15 cents each. The commission charges on hogs were 20 cents each; sheep, 15 cents each; cattle, 75 cents each; and calves, 25 cents each. What would be the amount of check issued in payment?

28. A farmer received $501 for 30 hogs weighing 6000 pounds at $8.50. How much yardage and commission has been charged?

29. An acre of land for potatoes was fertilized as follows: Two tons of barnyard manure costing $2.25 per ton; 100 pounds of acid phosphate costing $21.00 per ton; 50 pounds of muriate of potash costing $52.50 per ton. What was the cost of fertilizing.

30. If the yield without fertilizer is 70 bushels per acre, and fertilizer referred to in the above problem doubles the yield, what would be the gross increase from fertilizing a ten acre field if the potatoes were sold at $2 per bushel.

31. A farmer can purchase a tractor for $800 on 12 months time or $750 cash. If money is worth 6 per cent which would be the better proposition and how much?

32. Harold Dale is very anxious to know which of his stock of three cows he should keep, so decides to keep a record of each cow for one month.

Total of record as follows:

	Total Pounds Milk Per Day	Test	Price for 3.5 Milk
Daisy	40	4.2	$2.50
Bossy	48	4.	2.50
Jersey	33	5.	2.50

Since the expense for each cow is about the same, which should he keep and why?

33. Jack Smith has delivered to the Hoosier Dairy Marketing Association 2752 pounds of milk during the month of January. This milk has a test of 4.8 per cent butter fat. If the association is able to pay at the rate of $2.70 for 3.5 per cent butter fat, what should Jack receive for his milk?

GLOSSARY

Acceptance. Agreement of drawee in writing on the face of a draft to pay same; a draft which has been accepted.

Accident insurance. Agreement of an insurance company to pay an indemnity in case of damage to the person by accident.

Ad valorem duty. A duty based upon the value of the goods imported.

Agent. One empowered to act for another.

Amortization. The payment of a debt by a series of equal payments at regular intervals, the payments covering both principal and interest.

Amount. The face of a note plus the interest due.

Area. The surface included within any given lines.

Assessed valuation. The valuation placed upon property for purposes of taxation.

Bank. An institution which receives money for safekeeping, loans money, and transmits money and credits.

Bank discount. Interest on a loan paid to a bank in advance.

Bank draft. A draft drawn by one bank against another.

Banker's method of calculating interest. Count the time by the exact days and figure the interest on the basis of 360 days to the year. See page 181.

Bank pass book. A depositor's memorandum book in which bank deposits are entered.

Beneficiary. The person named in an insurance policy to receive the benefits thereof, if any.

Bill of exchange. A bill of exchange is a draft drawn in one country and payable in another.

Bond. An instrument under seal by which a party binds himself to the performance of certain stipulations; usually a certificate issued under seal by a corporation attesting its indebtedness to the holder of the certificate for a specified part of a capital loan.

Book value. Book value is that part of the net assets of the corporation, as shown by its books, which corresponds to the part of the capital stock which one share represents.

Broker. An agent who makes a contract between a buyer and a seller.

Brokerage. The percentage charged by a broker for his services.

Cable transfer. Cable transfers are accomplished in much the same way as telegraphic transfers of domestic exchange.

Capital. The net amount of money and property invested in business.

Capitalization. The capital value of an earning, at a rate of profit specified, is the amount of capital which at that rate would yield the amount of the earning.

Capital stock. The amount of shares at par which a corporation is authorized to issue.

Cash discount. Cash discount is a deduction from invoice price made in consideration of payment within a specified time.

Cashier's check. A bank's check against its own money signed by the cashier as an officer of the bank.

Certified check. A check upon the face of which is the certification of the drawee bank that the drawer has funds on deposit to cover it, which certification is at the same time the bank's guaranty that it will be paid.

Check. A depositor's written order upon a bank for the payment of money.

Check system in bookkeeping. Many bookkeepers check the accuracy of their footings by casting out 9's, 11's, and 13's. See page 40.

Circumference. Circumference is the distance around a circle.

Collector. A government official whose duty it is to collect customs and taxes.

Commercial discount. See trade discount.

Commission. A percentage charged by an agent for his services.

Commission merchant. One who sells goods for another, usually upon a percentage.

Common stock. Capital stock the holders of which have no guaranty or preference of any kind as in the distribution of profits or of assets.

Complement method in subtraction. A complement of a number is an amount which, if added to it, will make one unit of the next higher order. Subtraction is accomplished on the adding machine by adding the complement of the number to be subtracted, and deducting 1 from the next higher order.

Compound interest. Interest upon interest.

Consignee. Person to whom goods are consigned.

Consignment. Goods shipped; especially goods shipped to be sold on commission.

Consignor. The shipper.

Corporation. An organization authorized by law to carry on business along lines designated in its charter. The corporation is the organization and not the persons who compose it.

Cost (of goods). The amount expended to buy, make, or produce goods.

Cost accounting problems. Pages 138 and 139.

Coupon bond. A bond to which are attached separate certificates for each interest item.

Current assets. Current assets are cash assets which can be quickly converted into cash.

Current liabilities. Current liabilities are all debts arising from the ordinary or current transactions of business.

Customhouse. The office or building which is headquarters for the collector of customs.

Customs. Government taxes upon deposits and exports.

Date of maturity. The date on which an obligation falls due.

Demand note. A promissory note in which the maker obligates himself to make payment whenever demanded by the payee.

Departmental costs. The gross cost of each unit of manufacture is ascertained by dividing the general expenses among the departments, then dividing each departmental overhead, including its share of general expense, among the units of manufacture in the department, and adding this total overhead to the prime cost per unit.

Deposit. A sum of money placed in a bank for safe-keeping.

Depositor. One who makes a deposit in a bank.

Direct labor-hour method. Under this method the overhead expense is distributed among different jobs in proportion to the time spent on each job in direct labor.

Direct labor method. Under this method the burden is distributed over labor costs. The rate of burden is the ratio of the overhead charge to the total of direct labor.

Directors. Stockholders elected by the other stockholders of a corporation to manage the corporation.

Discount. A discount on a bond is the amount by which its market value is less than its par value.

Dividend. The amount paid to a stockholder as his share of the earnings of a corporation.

Draft. An order drawn by one party on another ordering the payment of a certain sum of money to a payee named therein.

Drawee. The person against whom a draft is drawn.

Drawer. The person who draws a check or a draft.

Duty. A tax levied by a government upon imports, exports, and goods manufactured in the country.

Earnings. Profits of a business or an investment.

Exact interest. Interest figured on the basis of 365 days to the year.

Exchange. Fee charged by a bank for remitting money.

Evolution. Evolution is the process of finding what number was used as a constant factor to produce a certain power.

Exponent. The power to which a number is to be raised is often indicated by writing a small figure to the right and slightly above the number to be raised. This small number is called the exponent.

Equation of accounts. Equation of accounts is the process of finding the date upon which the balance on an account may be paid without loss of interest to either debtor or creditor.

Face. The amount named in a note or other commercial paper as the principal amount to be paid.

Factory cost. The factory cost or production cost is the prime cost plus factory overhead.

Federal plan for farm loans. According to the government's plan for farm loans, a farmer may borrow on his land, or borrow to buy land, and may pay back his loan by a plan which is called amortization.

Fire insurance. Agreement of an insurance company to pay an indemnity in case of loss by fire.

Gain. The difference between the cost of goods and the amount received for them if the latter is greater.

Gross cost. The sum of the original cost of goods and expenses necessary to get them ready for sale or use.

Guaranty. A guaranty is a charge made by the agent to cover his personal risk in guaranteeing the goods to be of a certain quality.

Gross sales. Gross sales is the total of sales at invoice price.

Hypotenuse. The side of a right-angled triangle that is opposite the right angle.

Income on stock. Rate of per cent on his investment received in dividends by a stockholder.

Income tax. A federal tax levied upon all persons having an income in excess of a certain amount.

Indorsement. A writing on the back of a negotiable paper for the purpose of transferring it, or furnishing evidence of a payment.

Indorsement in blank. Indorsement which consists of the name of the payee only.

Indorsement in full. An indorsement which states the name of the person to whom the paper transferred becomes payable.

Indorsement without recourse. An indorsement which is accompanied by the words "without recourse," which signify that the indorser cannot be held liable.

Insurance. An agreement by which a company agrees to pay an indemnity in case of a loss of a specified nature.

Insured. The person for whom an insurance policy is issued.

Insurer. The company issuing insurance.

Interest. As commonly defined, interest is a compensation paid for the use of money.

Internal revenue. A tax upon certain goods manufactured within the United States.

International Postal money order. This is an order similar in nature to the domestic postal money orders, but payable in certain countries where, by treaty with our country, it will be honored.

Involution. Involution is the process of raising a number to one of its powers.

Life insurance. An agreement by an insurance company to pay an indemnity to a person named therein upon the death of the person named in the policy.

Legal rate of interest. The rate of interest allowed by law on interest-bearing obligations when no other rate has been agreed upon between the parties.

Listed stocks. The stock exchange publishes a list of stocks which may be bought and sold on the floor of the exchange. These are called listed stocks.

Letters of credit. A letter of credit is a circular letter issued by a bank in this country enabling its holder to secure currency at various times and places in foreign countries up to the total amount specified in it.

List price. The price of an article as printed in a catalog or other list.

Loss. The difference between the cost of goods and the amount received from them if the latter is less.

Machine-rate method of costs. Under this method a rate per hour of use is fixed upon for each machine, this rate being large enough to include all of the overhead charges.

Maker. The person who is the signer of a note.

Manufacturing costs. Manufacturing costs include materials, labor and factory expense. By the manufacturing cost is ordinarily meant the cost of manufacturing.

Market value. The value at which stock is quoted upon the open market.

Maximum legal rate. The highest rate of interest which may be legally collected even though by agreement.

Merchant's rule for figuring partial payments. The method that is common among business men for figuring interest on a note upon which partial payments have been made.

Multiplication table in aliquot parts. A standardized multiplication table by the use of aliquot parts of 100.

National bank. A bank operating under charter issued by the government.

Negotiable note. A note which when transferred becomes in the hands of the new holder collectible for the amount named on its face.

Net price. Price after all discounts have been taken.

Net proceeds. Proceeds after all allowances have been deducted.

Note. See promissory note.

Ordinary method of calculating interest. Count the time by compound subtraction and figure the interest on the basis of 360 days to the year. Consider a year as 12 months of 30 days each. This is the legal method for ordinary business transactions.

Overhead. By overhead we ordinarily mean the cost of conducting a business, including office rent, salaries of officials and helpers, etc.

Partnership. The relation existing between two or more persons who have combined their property, labor, or skill in the transaction of business for their common profit.

Partial payments. Payments of a part only.

Par value. Face value of stock.

Payee. Person to whom paid.

Percentage. Percentage is an application of decimal fractions in which one one-hundredth is the unit used in computations.

Perch. Stone work is measured either by the cubic yard or by the perch. A perch is equivalent to a mass $1\frac{1}{2}$ ft. thick, 1 ft. high, and a rod long, containing $24\frac{3}{4}$ cu. ft.

Perimeter. The perimeter of a figure is the distance around it. The perimeter of a surface is the combined length of its sides.

Personal property. All property that is not real property.

Policy of insurance. The paper issued by a company agreeing to insure.

Poll tax. A fixed sum assessed as a tax upon individuals without regard to property or income.

Port of entry. Any point at which goods are imported into one country from another.

Preferred stock. Stock issued under an agreement that dividends will be paid on it before dividends are paid on other stock or that the holders of such stock have a first claim upon the assets of the company in case of liquidation, or both.

Premium. The amount of money paid periodically to an insurance company by the insured to maintain a policy of insurance.

Prime cost. The prime cost or net cost of goods purchased is the original purchase price as shown by the invoice.

Principal. The amount of money on which interest is figured, or the amount of money loaned if no interest is figured. The party for whom an agent acts.

Prism. A prism is a solid the bases of which are equal, similar, parallel figures, and the sides of which are parallelograms. See pages 235 and 236 for other definitions on Solids.

Private bank. A bank that is operated without either a government or state charter.

Proceeds. Amount realized; in case of a discounted note, the net amount realized.

Profit or loss on trading. The profit or loss on trading is the difference between the gross cost of goods sold and the net returns from sales. The per cent of profit or loss is the amount of profit or loss expressed as a per cent; it may be expressed as a per cent of the prime cost, or of the gross cost, or of the net returns, or of the selling price.

Promissory note. An agreement in writing to pay a certain sum of money to a person named in the writing.

Purchase of stock on margin. Frequently sales of stock are made on a margin, that is, a certain per cent of the par value of the stock is paid in cash and the remainder is paid at some future time. The cash payment is referred to as the margin.

Rate of interest. The per cent of the principal which is paid annually for the use of the principal.

Real estate. Immovable property such as lands and the buildings thereon.

Registered bond. A bond which is registered in the books of the company or government issuing it, the registration showing the person to whom it is payable.

Restrictive indorsement. An indorsement which specifies the person to whom a commercial paper is transferred and requires the payee to pay to him personally, thus preventing the further transfer of the paper.

Root. The root of a number is that number which was used two or more times as a constant factor to give the number.

Savings banks. Banks in which deposits of savings are made, these deposits not being subject to check.

Selling price. The amount received for something sold.

Shipper. The person shipping goods.

Sight draft. A draft which is payable at sight or as soon as presented to the drawee.

Sinking fund. A sinking fund is the amount accumulated by setting aside a sum of money each year for a fixed period of years, so that, with compound interest, the sums set aside will amount to a certain debt at the end of the fixed period of time.

Specific duty. Duty on imports that is based upon quantity without reference to value.

Specific gravity. Specific gravity, or relative weight, is the ratio of the weight of a substance to that of an equal volume of another substance taken as a standard.

State bank. A bank that is operating under a state charter.

Stock quotations. Prices at which stocks have last been bought and sold, or offered, as usually quoted in newspapers.

Successive discounts. Two or more discounts successively applied in reducing the listed price of an article.

Tax. A sum of money levied by a government on the personal property, income, business, or person of an individual, firm, or corporation.

Tax rate. The rate per cent of tax levied on the property, income, or business of a person, firm, or corporation.

Tare. An allowance made for boxes, bags, wrapping paper, twine, and other materials used in packing.

Tariff. A schedule.

Terms of sale. The conditions of sale usually printed upon an invoice, including the length of time for which credit is given and the discounts allowed if paid within a specified time.

Time draft. A draft which is to be paid at some future time as specified or definitely indicated in its wording.

Time note. A note payable at some definite future time.

Trade discount. A deduction from list price, made to the trade.

Travelers' checks. Travelers' checks are checks issued by express companies in fixed denominations of $10, 20, $50, $100, and $200.

Trust companies. Corporations the primary purpose of which is to accept and execute trusts such as the administration of an estate.

United States Postal Savings Banks. Savings banks under the control of the United States government postal department.

United States Rule for figuring partial payments. The method prescribed by the government for figuring interest on an obligation when partial payments have been made.

Usury. A rate of interest or the charging of a rate of interest higher than the maximum legal rate.

Voids. In mixing concrete the small particles fill in the voids or spaces between the larger pieces. Voids really mean spaces.

APPENDIX

TABLES OF MEASURES

MEASURES OF EXTENT

Linear Measure		Surveyor's Linear Measure	
12 inches (in.)	= 1 foot (ft.)	7.92 inches	= 1 link
3 feet	= 1 yard (yd.)	25 links	= 1 rod
$5\frac{1}{2}$ yards	= 1 rod	4 rods	= 1 chain
320 rods	= 1 mile	80 chains	= 1 mile

Miscellaneous: 4 in. = 1 hand. 6 ft. = 1 fathom. 6068 ft. ($1\frac{1}{6}$ statute miles) = 1 knot, or geographical mile. 3 knots = 1 league.

Square Measure		Surveyor's Square Measure	
144 square inches	= 1 square foot	625 square links	= 1 square rod
9 square feet	= 1 square yard	16 square rods	= 1 square chain
$30\frac{1}{4}$ square yards	= 1 square rod	10 square chains	= 1 acre
160 square rods	= 1 acre	640 acres	= 1 square mile
640 acres	= 1 square mile	36 square miles	= 1 township

A "square" in practical measurements is 10 feet x 10 feet, or 100 square feet. In government surveys a square mile is called a section.

Cubic Measure		Circular Measure	
1728 cubic inches	= 1 cubic foot	60 seconds (″)	= 1 minute (′)
27 cubic feet	= 1 cubic yard	60 minutes	= 1 degree (°)

A cord is 128 cu. ft.
A perch is $24\frac{3}{4}$ cu. ft.
A load is 1 cu. yd.

A right angle is 90°.
A circumference is 360°.
A geographical mile is 1′.
A quadrant is $\frac{1}{4}$ of a circle.
A sextant is $\frac{1}{6}$ of a circle.

MEASURES OF CAPACITY

Liquid Measure		Dry Measure	
4 gills	= 1 pint (pt.)	2 pints	= 1 quart
2 pints	= 1 quart (qt.)	8 quarts	= 1 peck (pk.)
4 quarts	= 1 gallon (gal.)	4 pecks	= 1 bushel (bu.)

A standard barrel is $31\frac{1}{2}$ gal.
A standard hogshead is 63 gal.
A gallon is 231 cu. in.
One cubic foot is $7\frac{1}{2}$ gal., approx.

A stricken bushel is 2150.42 cu. in.
A heaped bushel is 2747.71 cu. in.
A dry quart is 67.2 cu. in.

MEASURES OF WEIGHT

Avoirdupois

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundred weight (cwt.)
2000 pounds	= 1 short or net ton
2240 pounds	= 1 long or gross ton

Troy

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweights	= 1 ounce
12 ounces	= 1 pound

A carat is 32 grains (troy). A pound troy weighs 5760 grains. A pound avoirdupois weighs 7000 grains. The term "carat" is also used to express fineness, and means $\frac{1}{24}$. "18 carats fine" means $\frac{18}{24}$ fine. A cubic foot of water weighs $62\frac{1}{2}$ lb.

MEASURES OF TIME

60 seconds (sec.)	= 1 minute (min.)	12 months	= 1 year (yr.)
60 minutes	= 1 hour (hr.)	365 days	= 1 year
24 hours	= 1 day (da.)	366 days	= 1 leap year
7 days	= 1 week	100 years	= 1 century
30 days	= 1 month		

Every fourth year is a leap year, except for centennial years not divisible by 400. The calendar months are: Jan., 31 da.; Feb., 28 or 29 da.; Mar., 31 da.; Apr., 30 da.; May, 31 da.; June, 30 da.; July, 31 da.; Aug., 31da.; Sept., 30 da.; Oct., 31 da.; Nov., 30 da.; Dec., 31 da.

MEASURES OF VALUE

U. S. Money

10 mills	= 1 cent (¢)
10 cents	= 1 dime
10 dimes	= 1 dollar ($)
10 dollars	= 1 eagle

English Money

4 farthings (far.)	= 1 penny (d.)
12 pence	= 1 shilling (s.)
20 shillings	= 1 pound sterling, or sovereign (£)
£1	= $4.8665

French Money

10 millimes = 1 centime
10 centimes = 1 decime
10 decimes = 1 franc ($.193)

German Money

100 pfennig (pf.) = 1 mark
1 mark = $.238

MISCELLANEOUS MEASURES

Quantity

12 units = 1 dozen (doz.)
12 dozen = 1 gross (gro.)
12 gross = 1 great gross

Paper

24 sheets = 1 quire
20 quires = 1 ream

METRIC TABLES

Linear Measures

10 millimeters (mm.)	= 1 centimeter (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 meter (m.), 39.37 in.
10 meters	= 1 dekameter (Dm.)
10 dekameters	= 1 hektometer (Hm.)
10 hektometers	= 1 kilometer (Km.), .62 mi.

Square Measures

100 square millimeters (sq. mm.)	= 1 square centimeter (sq. cm.)
100 square centimeters	= 1 square decimeter (sq. dm.)
100 square decimeters	= 1 square meter (sq. m.)
100 square meters	= 1 square dekameter (sq. Dm.)
100 square dekameters	= 1 square hektometer (sq. Hm.)
100 square hektometers	= 1 square kilometer (sq Km.)

Measures of Volume

1000 cubic millimeters (cu. mm.)	= 1 cubic centimeter (cu. cm.)
1000 cubic centimeters	= 1 cubic decimeter (cu. dm.)
1000 cubic decimeters	= 1 cubic meter (cu. m.)
1000 cubic meters	= 1 cubic dekameter (cu. Dm.)
1000 cubic dekameters	= 1 cubic hektometer (cu. Hm.)
1000 cubic hektometers	= 1 cubic kilometer (cu Km.)

Measures of Capacity

100 milliliters (ml.)	= 1 centiliter (cl.)
10 centiliters	= 1 deciliter (dl.)
10 deciliters	= 1 liter (l.)
10 liters	= 1 dekaliter (Dl.)
10 dekaliters	= 1 hektoliter (Hl.)
10 hektoliters	= 1 kiloliter (Kl.)

Measures of Weight

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 gram (g.)
10 grams	= 1 dekagram (Dg.)
10 dekagrams	= 1 hektogram (Hg.)
10 hektograms	= 1 kilogram (Kg.)

Comparisons

1 meter	= 39.37 in.
1 liter	= { 61.022 cu. in. .908 dry quart 1.0567 liquid quarts
1 kilogram	= 2.2046 lb. avoir.
1000 kilograms	= 1 metric ton (approx. 1 long ton)

VALUES OF FOREIGN COINS IN UNITED STATES MONEY

(Proclaimed by the Secretary of the Treasury, October 1, 1903)

Country	Standard	Monetary Unit	Value in Terms of U. S. Gold Dollar
Argentine Rep.	Gold	Peso	$0.965
Austria-Hungary	Gold	Crown	.203
Belgium	Gold	Franc	.193
Bolivia	Silver	Boliviano	.408
Brazil	Gold	Milreis	.546
British Possessions, N.A. (except Newfoundland)	Gold	Dollar	1.000
Central American States			
Costa Rica	Gold	Colon	.465
British Honduras	Gold	Dollar	1.000
Guatemala, Honduras, Nicaragua, Salvador	Silver	Peso	.408
Chile	Gold	Peso	.365
China	Silver	Tael	.602-.671
Columbia	Silver	Peso	.408
Cuba	Gold	Peso	.926
Denmark	Gold	Crown	.268
Ecuador	Gold	Sucre	.487
Egypt	Gold	Pound (100 piasters)	4.943
Finland	Gold	Mark	.193
France	Gold	Franc	.193
German Empire	Gold	Mark	.238
Great Britain	Gold	Pound Sterling	4.866½
Greece	Gold	Drachma	.193
Haiti	Gold	Gourde	.965
India	Gold	Pound sterling†	4.866½
Italy	Gold	Lira	.193
Japan	Gold	Yen	.498
Liberia	Gold	Dollar	1.000
Mexico	Silver	Dollar	.443
Netherlands	Gold	Florin	.402
Newfoundland	Gold	Dollar	1.014
Norway	Gold	Crown	.268
Persia	Silver	Kran	.075
Peru	Gold	Sol	.487
Portugal	Gold	Milreis	1.080
Russia	Gold	Ruble	.515
Spain	Gold	Peseta	.193
Sweden	Gold	Crown	.268
Switzerland	Gold	Franc	.193
Turkey	Gold	Piaster	.044
Uruguay	Gold	Peso	1.034
Venezuela	Gold	Bolivar	.193

†The sovereign is the standard coin of India, but the rupee ($0.3244⅓) is the money of account, current at 15 to the sovereign

STATUTORY WEIGHTS OF THE BUSHEL

States and Territories	Wheat	Rye	Oats	Barley	Buckwheat	Shelled corn	Corn on Cob	Potatoes, Irish	Onions	Turnips	English Beans	Apples	Flaxseed	Millet seed	Timothy seed	Clover seed
United States	60	56	32	48	48	56	...	60	..	..	60	...	56	...	..	..
Alabama	60	56	32	47	..	56	70	60	..	55	60	...	..	...	..	..
Arizona	60	56	32	45	..	54	...	...	...	..	60	...	..	...	..	..
Arkansas	60	56	32	48	52	56	70	60	57	57	60	50	56	50	60	60
California	60	54	32	50	40	52	...	...	...	..	...	...	..	...	..	..
Colorado	60	56	32	48	52	56	70	60	57	..	60	...	..	...	45	60
Connecticut	60	56	32	48	48	56	...	60	52	50	60	48	55	...	45	60
Delaware	60	...	..	..	..	56	...	...	..	..	...	...	..	...	..	..
Dist. of Columbia	60	56	32	..	..	56	...	60	..	..	...	...	..	...	..	..
Florida	60	56	32	48	..	56	70	60	56	54	c60	48	..	50	..	..
Georgia	60	56	32	47	52	56	70	60	57	55	60	...	56	...	45	60
Hawaii	60	56	32	48	..	56	...	...	..	..	...	...	..	...	..	..
Idaho	60	56	36	48	42	56	...	60	..	..	...	45	56	...	..	60
Illinois	60	56	32	48	52	56	70	60	57	55	60	...	56	...	45	60
Indiana	60	56	32	48	50	56	d68	60	48	55	60	...	..	50	45	60
Iowa	60	56	32	48	52	56	70	60	57	..	60	48	56	50	45	60
Kansas	60	56	32	48	50	56	70	60	57	55	60	48	56	50	45	60
Kentucky	60	56	32	47	56	56	e70	60	57	60	60	...	56	50	45	60
Louisiana	60	f32	32	f32	..	56	...	..	..	..	...	..	..	...	..	..
Maine	60	...	30	48	48	56	...	60	52	50	60	44	..	...	45	..
Maryland	..	...	26	..	..	..	...	56	..	..	...	...	..	...	..	..
Massachusetts	60	56	32	48	48	56	...	60	52	..	60	48	55	g35	45	60
Michigan	60	56	32	48	48	56	70	60	54	48	60	48	56	50	45	60
Minnesota	60	56	32	48	50	56	70	60	52	52	60	50	..	48	45	60
Mississippi	60	56	32	48	48	56	72	60	57	55	60	...	56	50	45	60
Missouri	60	56	32	48	52	56	70	60	57	42	60	v48	56	50	45	60
Montana	60	56	32	48	52	56	70	60	57	..	60	h45	56	...	45	60
Nebraska	60	56	32	48	52	56	70	60	57	55	60	...	56	50	45	60
New Hampshire	60	56	32	..	..	56	...	60	..	..	62	...	..	...	..	..
New Jersey	60	56	30	48	50	56	...	60	57	..	60	50	55	...	..	64
New York	60	56	32	48	48	56	...	60	57	..	60	48	55	...	45	60
North Carolina	60	56	32	48	50	56	...	...	..	..	...	...	55	...	..	60
North Dakota	60	56	32	48	42	56	70	60	52	60	60	...	56	50	42	60
Ohio	60	56	32	48	50	56	68	60	55	60	60	50	56	50	45	60
Oklahoma	60	56	32	48	42	56	70	60	52	60	60	...	56	...	42	60
Oregon	60	56	32	46	42	56	...	60	..	..	...	45	..	...	..	60
Pennsylvania	60	56	32	47	48	56	...	56	50	..	...	...	..	...	..	60
Rhode Island	60	56	32	48	48	56	70	60	50	50	60	48	56	50	45	60
South Dakota	60	56	32	48	42	56	70	60	52	60	60	...	56	...	42	60
Tennessee	60	56	32	48	50	56	70	60	56	50	60	50	56	50	45	60
Texas	60	56	32	48	42	56	70	60	57	55	60	45	56	50	45	60
Vermont	60	56	32	48	48	56	...	60	52	60	62	46	..	...	45	60
Virginia	60	56	30	48	52	56	70	56	57	55	60	45	56	50	45	60
Washington	60	56	32	48	42	56	...	60	..	..	...	45	56	...	..	60
West Virginia	60	56	32	48	52	56	...	60	..	..	60	...	56	...	45	60
Wisconsin	60	56	32	48	50	56	70	60	57	42	60	50	56	50	45	60

c Velvet beans in hull, 78.
d Before Dec. 1, 70.
e May 1 to Nov. 1, 68.
f So in statutes, but an evident error.
g Japanese barnyard millet.
h Peeled. 40.

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Trade discount.....214
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Page 6. **1.** (*a*) 238. (*b*) 235. (*c*) 197. (*d*) 176. (*e*) 199. (*f*) 186. (*g*) 160. (*h*) 214. (*i*) 220. (*j*) 220. (*k*) 229. (*l*) 173. Total, 2447.

2. (*a*) 200. (*b*) 200. (*c*) 260. (*d*) 170. (*e*) 230. (*f*) 200. (*g*) 240. (*h*) 227. (*i*) 282. (*j*) 217. (*k*) 252. (*l*) 214. Total, 2692.

3. (*a*) 315. (*b*) 265. (*c*) 300. (*d*) 350. (*e*) 284. (*f*) 405. (*g*) 440. (*h*) 410. (*i*) 403. (*j*) 349. (*k*) 368. (*l*) 410. Total, 4299.

4. (*a*) 404. (*b*) 500. (*c*) 470. (*d*) 540. (*e*) 405. (*f*) 580. (*g*) 497. (*h*) 405. (*i*) 533. (*j*) 479. (*k*) 353. (*l*) 394. Total, 5560.

5. (*a*) 631. (*b*) 593. (*c*) 743. (*d*) 604. (*e*) 711. (*f*) 630. (*g*) 689. (*h*) 588. (*i*) 821. (*j*) 708. (*k*) 570. (*l*) 724. Total, 8012.

Page 8. **1.** $1673.54. **2.** $2113.83. **3.** $1518.78. **4.** $1754.33. **5.** $1931.64. **6.** $2763.44. **7.** $3178.97.

Page 9. **1.** 92,083. **2.** 81,134. **3.** 76,614. **4.** 86,421. **5.** 110,587. **6.** 81,332. **7.** 97,889. **8.** 239,870. **9.** 129,788. **10.** 109,880. **11.** 137,596. **12.** 268,770. **13.** 291,353. **14.** 183,614. **15.** $938. **16.** $628.51. **17.** $1,895.40. **18.** $424.53. **19.** $1,120.90. **20.** $527.16. **21.** $1,293.36.

Page 11. **1.** $1051.02. **2.** $1236.46. **3.** $1440.47. **4.** $2056.11. **5.** $1442.18. **6.** $4175.15.

Page 12. **1.** 1322, 1673, 1514, 1340, 1228. **2.** 916, 1887, 1309, 1500, 1370. **3.** 2220, 1571, 1666, 1572, 1128. **4.** 1283, 1334, 1521, 1587, 2378. **5.** 1323, 997, 1528, 1665, 1960. **6.** 1236, 867, 1287, 2184, 1485. **7.** M., $3114.92; Tues., $3082.88; Wed., $2967.35; Thr., $3185.92; Fr., $2673.83; S., $3889.77; total, $18,914.67. d.g., $4109.26; gro. $6031.31; clo., $2886.69; furn., $4172.; b. & s., $1715.41. **8.** $25,050.40, $153,500.51, $80,598.19, $79,908.29 $46,462.72, $21,747.98, $61,473.15, $28,648.74, $42,334.77, $45,496.11, $29,297.58, $5,208.49, $14,565.34, $345.09, $11,631.26, $89,254.33, $24,477.59, total, $760,000.54. City Line, $121,187.70; Rapid Ry. $461,435.31; D. M. & T., $99,994.86; D. J. & C., $77,382.67. **9.** 349.76 mi., 74.97 mi., 167.53 mi., 93.04 mi., 334.14 mi., 533.85 mi., 98.32 mi., 393.78 mi., 91.33 mi., 431.79 mi., 168.71 mi., 332.12 mi., total 3069.34 mi. Main track, 1680.95 mi.; second track, 338.63 mi., yards & s., 1049.76 mi. **10.** $8.13, $8.20, $7.65, $10.34, $9.30, $3.55, $6.05, $5.30, $3.25, $5.80, $5.17, $5.80, $3.85, $4.10, $7.30, $4.50, $4.15, $5.30, $2.70, $3.96, $3.30, $3.30, $3.95, $7.72, $9.05, $5.08, $6.85, total $153.65. Carf., $41.75; hotel, $64.00; bag., $19.90; incid., $28.00.

Page 17. **1.** $175.22. **2.** $2148.; Carpenter Bros. **3.** $4026.86. **4.** $363.25. **5.** $9313.43. **6.** M., $1342.48; Tu., $1594.49; W., $1739.42;

Th. $1468.69; F., $1398.77; S., $1945.23, total, $9489.08. Miss H., $2971.69; Miss W., $2026.92; Miss S., $1396.68; Miss C., $1677.40; Miss B., $1416.39. **7.** Mar., $42.30, $45.86, $45.09, $43.22; Apr., $48.97, $34.74, $38.10, $52.82; May, $32.20, $41.93, $47.12, $33.14, $41.42; June, $51.15, $47.59, $33.78, $43.16; July, $58.29, $40.66, $31.70, $37.81; Aug., $39.95; total, $931.00. No. 248, $193.64; No. 679, $155.60; No. 387, $140.33; No. 475, $151.46; No. 638, $135.05; No. 846, $154.92.

Page 20. **1.** 298,003. **2.** 378,909. **3.** 366,243. **4.** 342,478. **5.** 322,895. **6.** 236,951. **7.** 292,219. **8.** 334,772. **9.** J., $14,478.56; T., $13,601.44; H., $12,411.52; R., $9,769.90; W., $10,064.82; total, $60,326.24. Jan., $13,197.28; Feb., $14,075.78; Mar., $17,019.47; Apr., $16,033.71. **10.** Bow., $254,389; Cal., $1,450,093; Coml., $213,065,031.; Exch, $68,012,824.; Dro., $11,127,537; 1st N., $140,971,682; total, $434,881,556. Cap. St., $35,400,000.; surp., $23,810,000.; und prof., $5,879,022; notes, $12,647,597; dep., $357,144,937.

Page 23. **1.** 48,634. **2.** 64,965. **3.** 53,433. **4.** 12,906. **5.** 30,465. **6.** 18,496. **7.** 7653. **8.** 19,926. **9.** 2572. **10.** 45,753. **11.** 58,977. **12.** 11,778. **13.** 37,118. **14.** 2927. **15.** 14,261. **16.** 2888. **17.** 2409. **18.** 83,195.

Page 25. **1.** 755. **2.** 707. **3.** 723. **4.** 470. **5.** 262. **6.** 1562. **7.** 1633. **8.** 1532. **9.** 798. **10.** 805. **11.** 813. **12.** 570. **13.** 527. **14.** 1765. **15.** 1721. **16.** 1306. **17.** 1910. **18.** 1848. **19.** 1680. **20.** 1489. **21.** 1533. **22.** 2695. **23.** 2726. **24.** 2516. **25.** B & Co., $1869.55; S., $2635.70; C., $8248.33; P., $3836.09. **26.** Total morning bals., $14,589.16; deps., $3324.10; cks., $1323.59; evening bals., $16,589.67. **27.** Bar., $2960.91; Br., $3887.69; Bos., $11,488.04; Berk., $1530.49; Bur., $3863.43; Burl., $28,193.05; Brook., $37,068.75; Bark, $68.81 O. D.; Burlage, $1305.03; Bas., $310.43, Proof: Total morning bals., $89,035.24; deps., $10,518.18; cks., $9014.41; evening bals., $90,539.01. **28.** Bar., $3264.07; Br., $3948.48; Bost., $11,942.00; Berk., $1790.55; Bur., $3968.45; $30,739.89; Brook., $39,876.46; Bark., $196.84; Burlage, $1262.67; Bask., $297.68. Proof, total morning bals., $90,539.01; deps., $7831.19; cks., $1083.11; evening bals., $97,287.09.

Page 27. **1.** $22.87. **2.** $233.05. **3.** By registers, $100.21, $134.75, $64.05, $64.60, $75.85, $64.25, $87.60, $85.00, $102.45, $146.18. Total morning bals., $277.52; receipts, $1112.26; payts., $464.84; evening bals., $924.94. **4.** $1549.24. **5.** $1645.19. **6.** $13.882.27. **7.** Bal., $1281.65. **8.** $110.09 **9.** $333.63. **10.** $183.69. **11.** $243.10. **12.** $18,385.13. **13.** $1186.27. **14.** $8737.75; $346.54. **15.** $604.52. **16.** $156.23. **17.** $403.85. **18.** $274.36. **19.** $73.47, $195.82. **20.** $291.72. **21.** $196.15. **22.** $898.27. **23.** $260.40. **24.** $6121.59. **25.** $5011.83. **26.** $15,089.94, $14,210.68. **27.** $21,029.99, $19,140.23. **28.** $35,728.41. **29.** $28,031.86. **30.** $23,317.62. **31.** $75,263.91. **32.** $687.12. **33.** $642.50, $195.12. **34.** $3795.50, $939.06. **35.** $2080.55, $1682.38. **36.** $1560.07, $919.43. **37.** $1173.93, $1487.62,

$1058.12. **38.** $14,915.84. **39.** $2177. **40.** $2326.75. **41.** $9062.60. **42.** Mdse., $9781.75 profit; int. & dis., $188.89 loss; rent, $1265.75 loss; exp., $2589.70; net profit, $5737.41. **43.** $3993.73, $757.60, $2452.99, $388.74, $118.82, $1529.59, $1364.83, $567.37, $2145.06, $138.27, total $13,457. **44.** Total assets, $19,590.85, $21,403.25, incr. $1841.15, decr. $28.75. Liab., $19,590.85, $21,403.25, incr., $2886.90, decr., $1074.50. **45.** $9745.84, $10,759.61.

Page 41. **1.** Bal., $1334.25.

Page 42. **1.** $186.55. **2.** $288.89. **3.** $92.04. **4.** $174.54. **5.** $212.31, $432.37, $790.94, $400.11, $649.83, $394.03, total $2879.59. Total morning bals., $3106.21; cks., $613.22; deps., $386.60; evening bals., $2879.59. **6.** Gross prof., $1842.85, $742.78, $1335.37, $966.14, $1070.32, $2567.84, total $8525.30. Net prof., $653.47, ($192.29 loss), $571.13, $427.60, $390.44, $1553.50, total $3403.85. Sales, $27,519.67. Cost of g. s. $18,994.37. Exp., $5121.45.

Page 45. 67,882,360; 4,865,835; 29,055,432; 16,486,652; 50,609,440; 33,273,464.

Page 45. **1.** 1,413,992. **2.** 4,149,808. **3.** 2,762,230. **4.** 3,528,798. **5.** 1,029,584. **6.** 2,093,450. **7.** 3,828,906. **8.** 2,786,396. **9.** 2,698,724. **10.** 4,447,106. Total 28,738,994.

Page 46. **1.** 25,872. **2.** 47,531. **3.** 57,673. **4.** 67,485. **5.** 391,842. **6.** 513,843. **7.** 576,774. **8.** 677,292. **9.** 1,685,706. **10.** 8,022,927. **11.** 9,036,082. **12.** 4,658,302.

Page 47. **1.** 85,650; 129,300; 120,850; 64,900; 135,925; 53,700. **2.** 26,450; 17,300; 6,900; 14,700; 21,850; 31,450. **3.** 39,675; 18,375; 23,775; 51,075; 69,300; 22,875.

1. 224. **2.** 247. **3.** 272. **4.** 270. **5.** 342. **6.** 238. **7.** 432. **8.** 576. **9.** 364. **10.** 559. **11.** 952. **12.** 1275. **13.** 1326. **14.** 1104. **15.** 624. **16.** 918. **17.** 522. **18.** 1168. **19.** 833. **20.** 938. **21.** 1209. **22.** 1178. **23.** 611. **24.** 784.

Page 48. **1.** 648. **2.** 792. **3.** 1288. **4.** 1458. **5.** 1638. **6.** 1702. **7.** 2523. **8.** 928. **9.** 1295. **10.** 1656. **11.** 1914. **12.** 2278. **13.** 2886. **14.** 3268. **15.** 3588. **16.** 1134. **17.** 1610. **18.** 2024. **19.** 2793. **20.** 2752. **21.** 1445. **22.** 4674. **23.** 2670. **24.** 3239. **25.** 735. **26.** 4608. **27.** 1080. **28.** 3864. **29.** 1239. **30.** 5152.

1. 2275. **2.** 1296. **3.** 551. **4.** 1984. **5.** 2679. **6.** 1449. **7.** 1271. **8.** 2604. **9.** 5376.

Page 49. **1.** 650. **2.** 2754. **3.** 8832. **4.** 7395. **5.** 1054. **6.** 4485. **7.** 1974. **8.** 234. **9.** 5544.

1. 16,366. **2.** 73,032. **3.** 33,630. **4.** 54,808. **5.** 26,602. **6.** 83,655. **7.** 89,046. **8.** 46,973. **9.** 66,432.

Page 50. **1.** 69,812. **2.** 153,153. **3.** 198,744. **4.** 29,610. **5.** 83,385. **6.** 126,672. **7.** 80,337, **8.** 80,631. **9.** 201,996.

Page 51. **1.** 24,381. **2.** 47,208. **3.** 30,168. **4.** 26,325. **5.** 132,678 **6.** 325,686. **7.** 293,344. **8.** 333,303. **9.** 4,418,248. **10.** 1,703,343. **11.** 2,492,468. **12.** 2,205,294. **13.** 70,812. **14.** 126,801. **15.** 1,165,224.

1. 36,666. **2.** 3066. **3.** 10,395. **4.** 30,919. **5.** 18,192. **6.** 12,128. **7.** 53,256. **8.** 29,904. **9.** 11,400. **10.** 8892. **11.** 61,056. **12.** 3753.

Page 52. **1.** 21,328. **2.** 17,325. **3.** 15,725. **4.** 34,566. **5.** 5736. **6.** 11,823. **7.** 26,418. **8.** 23,328. **9.** 42,636. **10.** 17,496. **11.** 13,824. **12.** 25,662.

Page 53. **1.** $25.82. **2.** $57.80. **3.** $364.50. **4.** $1164.61. **5.** $232.95. **6.** $176.18. **7.** $175.81. **8.** $162.19. **9.** $1050.93. **10.** $407.72. **11.** $470.48.

Page 57. **5.** $1.80. **6.** $314.25. **8.** $5.74. **9.** $182.74; $182.02.

Page 59. **1.** 147,742. **2.** 139,342. **3.** 160,128. **4.** 158,670. **5.** 175,086. **6.** 73,584. **7.** 217,092. **8.** 274,320. **9.** 53,985, **10.** 194,712. **11.** 170,775. **12.** 97,917. **13.** 94,128. **14.** 177,452. **15.** 188,892. **16.** 15,624. **17.** 17,376. **18.** 33,088. **19.** 9,360. **20.** 37,191. **21.** 4270. **22.** 15,850. **23.** 26,650. **24.** 18,600. **25.** 3869. **26.** 38,236. **27.** 463,000. **28.** 52,900. **29.** 323. **30.** 768. **31.** 5,432,765.

Page 60. **1.** 5; rem., 11. **2.** 6; rem., 11. **3.** 8; rem., 11. **4.** 8; rem., 16. **5.** 7; rem., 430., **6.** 4; rem., 139.

Page 61. **1.** 11,994; rem., 4. **2.** 18,210; rem., 1. **3.** 9177. **4.** 27,381. **5.** 4784; rem., 7. **6.** 2181; rem., 11. **7.** 3511; rem., 1. **8.** 11,463; rem., 4. **9.** 2244; rem., 10. **10.** 8128; rem., 4. **11.** 165; rem., 102. **12.** 60; rem., 105. **13.** 290; rem., 237. **14.** 37; rem., 374. **15.** 78; rem., 269. **16.** 78; rem., 80. **17.** 230; rem., 194. **18.** 114; rem., 88. **19.** 67; rem., 108.

Page 62. **1.** 95. **2.** 129. **3.** 99. **4.** 26. **5.** 127. **6.** 106. **7.** 49. **8.** 73. **9.** 127. **10.** 127. **11.** 55. **12.** 249.

Page 63. **1.** 29. **2.** 28. **3.** 55. **4.** 36. **5.** 48. **6.** 27. **7.** 54. **8.** 30; rem., 5. **9.** 57. **10.** 29. **11.** 26. **12.** 31. **13.** 23. **14.** 34. **15.** 26. **16.** 44; rem., 20. **17.** 37. **18.** 32. **19.** 35. **20.** 49. **21.** 65; rem., 15. **22.** 60; rem., 27. **23.** 61; rem, 19. **24.** 47; rem., 4. **25.** 38; rem., 13. **26.** 47; rem., 11. **27.** 49; rem., 25. **28.** 45; rem., 7. **29.** 38; rem., 28. **30.** 70; rem., 1. **31.** 28; rem., 10. **32.** 81; rem., 15. **33.** 56; rem., 15. **34.** 48; rem., 51. **35.** 57; rem., 2. **36.** 18; rem., 33.

Page 64. **1.** 6321. **2.** 7324. **3.** 7553. **4.** 1234. **5.** 17,206; rem., 4. **6.** 52. **7.** 89. **8.** 58. **9.** 106. **10.** 81. **11.** 162. **12.** 256. **13.** 174.

1. Pri., 27,082; 1st R., 30,262; 2d R., 31,750; 3d R., 14,313; 4th R., 18,356; 5th R., 13,748; 6th R., 13,337; H.R., 11,003. **2.** $476.67. **3.** $2165.52, $2205.48, $551.70 loss, $2646.50, total $6465.80. **4.** $286.03, $613.59, $42.99,

$597.64, total $1540.25. **5.** 10,385,227; 6,485,280 **6.** 14,086. **7.** (*a*) $773.40, $63.73, $1124.45; (*b*) $22.10, $1102.35; (*c*) $366.25, $78.93, $269.52, $190.59; (*d*) $88.24, $102.35, $1102.35.

Page 67. **1.** (*a*) $4512.82; (*b*) $2877.75; (*c*) $3247.82; (*d*) $2975.00; (*e*) $4743.24; total, $18,356.63. **2.** (*a*) $292.19; (*b*) $266.74; (*c*) $486.36; (*d*) $634.55; (*e*) $137.07; total, $1816.91. **3.** 112,014. **4.** 197,883. **5.** 184,968. **6.** 70, rem. 112. **7.** 110, rem. 123. **8.** 174, rem. 396.

Page 72. **1.** 12. **2.** 9. **3.** 2. **4.** 1. **5.** 6. **6.** 7. **7.** 6. **8.** 8. **9.** 8. **10.** 17. **11.** 19. **12.** 23.

Page 73. **1.** 72. **2.** 120. **3.** 144. **4.** 1440. **5.** 720. **6.** 9360. **7.** 720. **8.** 54. **9.** 360. **10.** 150. **11.** 858,390. **12.** 7875. **13.** 27,360. **14.** 252. **15.** 2944.

Page 74. **1.** 9. **2.** 6. **3.** $11\frac{1}{4}$. **4.** $\frac{9}{10}$. **5.** $8\frac{8}{9}$. **6.** $\frac{7}{15}$. **7.** 12. **8.** 8. **9.** $\frac{4}{9}$. **10.** $7\frac{1}{2}$.

Page 78. **1.** $\frac{23}{25}$. **2.** $\frac{7}{8}$. **3.** $\frac{8}{9}$. **4.** $\frac{11}{12}$. **5.** $\frac{15}{16}$. **6.** $\frac{23}{24}$. **7.** $\frac{31}{32}$. **8.** $\frac{41}{42}$. **9.** $\frac{3}{4}$. **10.** $\frac{2}{3}$. **11.** $\frac{5}{6}$. **12.** $\frac{11}{13}$. **13.** $\frac{17}{18}$. **14.** $\frac{2069}{2156}$. **15.** $\frac{35}{36}$. **16.** $\frac{51}{52}$. **17.** $\frac{7}{12}$. **18.** $\frac{8}{15}$. **19.** $\frac{11}{24}$. **20.** $\frac{7}{15}$. **21.** $\frac{5}{12}$. **22.** $\frac{9}{16}$. **23.** $\frac{17}{35}$. **24.** $\frac{11}{24}$. **25.** $\frac{17}{25}$. **26.** $\frac{27}{35}$. **27.** $\frac{21}{25}$. **28.** $\frac{32}{35}$. **29.** $\frac{35}{37}$. **30.** $\frac{15655}{24338}$. **31.** $\frac{11}{24}$. **32.** $\frac{11}{27}$.

Page 80. **1.** $5\frac{5}{28}$. **2.** $3\frac{25}{39}$. **3.** 6. **4.** $13\frac{27}{28}$. **5.** $6\frac{59}{62}$. **6.** $4\frac{23}{24}$. **7.** 12. **8.** $4\frac{1}{3}$. **9.** $13\frac{2}{3}$. **10.** 13. **11.** 44. **12.** 26. **13.** $52\frac{1}{4}$. **14.** $5\frac{1}{36}$. **15.** $10\frac{1}{2}$. **16.** $71\frac{1}{9}$. **17.** $29\frac{53}{63}$. **18.** $27\frac{1}{36}$. **19.** $31\frac{1}{2}$. **20.** $46\frac{2}{9}$. **21.** 30. **22.** $36\frac{19}{96}$. **23.** $120\frac{1}{2}$. **24.** $5\frac{11}{162}$.

Page 81. **1.** $\frac{192}{7}$. **2.** $\frac{346}{9}$. **3.** $\frac{131}{7}$. **4.** $\frac{328}{11}$. **5.** $\frac{547}{12}$. **6.** $\frac{511}{8}$. **7.** $\frac{810}{7}$. **8.** $\frac{1574}{9}$. **9.** $\frac{2227}{16}$. **10.** $\frac{3138}{5}$. **11.** $\frac{3515}{8}$. **12.** $\frac{3624}{5}$. **13.** $\frac{5471}{8}$. **14.** $\frac{8518}{9}$. **15.** $\frac{1893}{8}$. **16.** $\frac{2661}{5}$. **17.** $\frac{5651}{9}$. **18.** $\frac{663}{5}$. **19.** $\frac{197}{3}$. **20.** $\frac{580}{7}$. **21.** $\frac{584}{9}$. **22.** $\frac{279}{5}$. **23.** $\frac{614}{7}$. **24.** $\frac{317}{11}$. **25.** $\frac{887}{12}$. **26.** $\frac{309}{8}$. **27.** $\frac{307}{4}$. **28.** $\frac{794}{17}$. **29.** $\frac{458}{5}$. **30.** $\frac{455}{6}$.

Page 82. **1.** .5, $.071\frac{3}{7}$, .375, .625, $.312\frac{1}{2}$, $.562\frac{1}{2}$, $.687\frac{1}{2}$, $.937\frac{1}{2}$, $.265\frac{5}{8}$, $.421\frac{7}{8}$, $.703\frac{1}{8}$, $.921\frac{7}{8}$. **2.** $.333\frac{1}{3}$, $.166\frac{2}{3}$, $.666\frac{2}{3}$, $.833\frac{1}{3}$, $.583\frac{1}{3}$, $.916\frac{2}{3}$, $.416\frac{2}{3}$, $.458\frac{1}{3}$, $.708\frac{1}{3}$, $.791\frac{2}{3}$, $.208\frac{1}{3}$, $.041\frac{2}{3}$. **3.** .2, .6, .8, $.142\frac{6}{7}$, $.428\frac{4}{7}$, $.571\frac{3}{7}$, $.714\frac{2}{7}$, $.222\frac{2}{9}$, $.444\frac{4}{9}$, $.555\frac{5}{9}$, $.777\frac{7}{9}$, $.888\frac{8}{9}$, $.181\frac{9}{11}$, $.454\frac{6}{11}$.

1. .75, .5, .875, .80, **2.** $.6\frac{2}{3}$, $.37\frac{1}{2}$, $.83\frac{1}{3}$, $.58\frac{1}{3}$. **3.** $.57\frac{1}{7}$, .800, $.33\frac{1}{3}$, $.42\frac{6}{7}$. **4.** $.31\frac{1}{4}$, $.43\frac{3}{4}$, $.55\frac{5}{9}$, $.3\frac{3}{4}$.

Page 83. **1.** .75. **2.** .875. **3.** $.833\frac{1}{3}$. **4.** $.583\frac{1}{3}$. **5.** $.666\frac{2}{3}$. **6.** $.916\frac{2}{3}$. **7.** $.437\frac{1}{2}$. **8.** .72. **9.** .65. **10.** $.566\frac{2}{3}$. **11.** $.416\frac{2}{3}$. **12.** .925. **13.** .94. **14.** $.562\frac{1}{2}$. **15.** .575. **16.** .85.

Page 84. **1.** $\frac{4}{9}$. **2.** $\frac{1}{12}$. **3.** $\frac{1}{80}$. **4.** $\frac{1}{9}$. **5.** $\frac{1}{16}$. **6.** $\frac{1}{6}$. **7.** $\frac{5}{7}$. **8.** $\frac{1}{350}$. **9.** $\frac{3}{16}$. **10.** $\frac{1}{300}$. **11.** $\frac{3}{320}$. **12.** $\frac{1}{800}$. **13.** $\frac{3}{160}$. **14.** $\frac{17}{60}$. **15.** $\frac{4}{625}$. **16.** $\frac{1}{150}$. **17.** $\frac{13}{80}$. **18.** $\frac{2}{45}$. **19.** $\frac{11}{80}$. **20.** $\frac{1}{750}$. **21.** $\frac{2}{9}$. **22.** $\frac{1}{6}$. **23.** $\frac{1}{40}$. **24.** $\frac{20630}{90000}$. **25.** $\frac{13}{400}$. **26.** $\frac{1}{12}$. **27.** $\frac{6}{175}$. **28.** $\frac{3}{32}$. **29.** $\frac{1}{150}$. **30.** $\frac{13}{160}$. **31.** $\frac{8}{625}$. **32.** $\frac{2}{15}$. **33.** $\frac{13}{40}$. **34.** $\frac{1}{3}$. **35.** $\frac{2}{3}$. **36.** $\frac{1}{9}$.

1. $\frac{14}{24}$, $\frac{21}{24}$, $\frac{18}{24}$, $\frac{20}{24}$. **2.** $\frac{21}{48}$, $\frac{20}{48}$, $\frac{32}{48}$, $\frac{22}{48}$. **3.** $\frac{42}{70}$, $\frac{40}{70}$, $\frac{35}{70}$, $\frac{48}{70}$. **4.** $\frac{24}{28}$, $\frac{18}{28}$, $\frac{21}{28}$, $\frac{14}{28}$. **5.** $\frac{16}{24}$, $\frac{18}{24}$, $\frac{15}{24}$, $\frac{14}{24}$. **6.** $\frac{15}{120}$, $\frac{48}{120}$, $\frac{80}{120}$, $\frac{20}{120}$. **7.** $\frac{136}{144}$, $\frac{138}{144}$, $\frac{135}{144}$, $\frac{96}{144}$. **8.** $\frac{18}{28}$, $\frac{12}{28}$, $\frac{21}{28}$, $\frac{14}{28}$.

Page 85. **1.** 1. **2.** $1\frac{4}{5}$. **3.** $1\frac{4}{5}$. **4.** $1\frac{17}{56}$. **5.** $2\frac{13}{280}$. **6.** $2\frac{1}{36}$. **7.** $2\frac{2}{15}$. **8.** $1\frac{7}{8}$. **9.** $2\frac{5}{42}$. **10.** $1\frac{13}{36}$. **11.** $2\frac{1}{4}$. **12.** $1\frac{373}{396}$. **13.** $1\frac{254}{429}$. **14.** $2\frac{3}{10}$. **15.** $2\frac{77}{120}$. **16.** $2\frac{3}{8}$. **17.** $2\frac{17}{180}$. **18.** $1\frac{7}{12}$. **19.** $2\frac{17}{60}$. **20.** $1\frac{19}{40}$. **21.** $1\frac{34}{63}$. **22.** $1\frac{1}{9}$. **23.** $1\frac{29}{45}$. **24.** $3\frac{101}{420}$. **25.** $1\frac{13}{42}$. **26.** $1\frac{2}{3}$. **27.** $2\frac{9}{40}$. **28.** $1\frac{5}{9}$. **29.** $1\frac{19}{24}$. **30.** $2\frac{7}{40}$. **31.** 2. **32.** $1\frac{25}{66}$. **33.** $1\frac{1}{6}$. **34.** $2\frac{1}{3}$. **35.** $2\frac{1}{3}$. **36.** $1\frac{79}{144}$.

Page 87. **1.** 3696.90609. **2.** 6059.3083. **3.** 1507.6047. **4.** 476,645.47765.

Page 88. **1.** $1744\frac{5}{12}$. **2.** $1871\frac{11}{16}$. **3.** $1411\frac{4}{9}$. **4.** $1768\frac{29}{56}$. **5.** $1734\frac{1}{15}$. **6.** $2925\frac{13}{36}$.

1. $671.6581\frac{2}{3}$. **2.** $1407.1206\frac{2}{3}$. **3.** $1944.9452\frac{7}{9}$. **4.** $440.9161\frac{1}{9}$. **5.** $236.9143\frac{107}{126}$. **6.** $611.0841\frac{2}{3}$. **7.** $50.5108\frac{1}{3}$. **8.** $295.6825\frac{5}{7}$. **9.** $1016.2260\frac{20}{33}$. **10.** $885.1352\frac{8}{21}$. **11.** 112.9625. **12.** $327.8341\frac{2}{3}$. **13.** \$1012.0495. **14.** \$75.20.

Page 89. **1.** $16\frac{1}{12}$. **2.** $25\frac{7}{60}$. **3.** $20\frac{19}{24}$. **4.** $20\frac{11}{60}$. **5.** $17\frac{7}{12}$. **6.** $24\frac{27}{40}$. **7.** $45\frac{11}{18}$. **8.** $58\frac{13}{30}$. **9.** $62\frac{13}{24}$. **10.** 136^2. **11.** 138^1. **12.** 98^1. **13.** $95\frac{11}{24}$. **14.** 154.0575+. **15.** 586.6. **16.** $501\frac{7}{36}$. **17.** $353\frac{2}{3}$. **18.** 485.5416+. **19.** $382\frac{5}{18}$. **20.** $302\frac{17}{24}$. **21.** 487.65. **22.** $392\frac{2}{9}$. **23.** $207\frac{1}{5}$. **24.** 147.4177+.

Page 90. **1.** 52 in. **2.** 45 in. **3.** 140 in. **4.** 2.9475 mills. **5.** 10,875.4217. **6.** $5218.2788\frac{1}{3}$. **7.** $1198\frac{23}{189}$. **8.** $3488\frac{17}{36}$. **9.** $2598\frac{19}{54}$. **10.** $2245\frac{15}{28}$. **11.** $2185\frac{97}{108}$.

Page 91. **1.** $2\frac{13}{48}$. **2.** $3\frac{1}{18}$. **3.** $85\frac{29}{56}$. **4.** $7012\frac{29}{72}$. **5.** 22,136.6755. **6.** 11,753.3296. **7.** 5103.8468. **8.** $8565.1767\frac{1}{2}$. **9.** 3403.4835. **10.** $3730\frac{3}{4}$. **11.** $3901\frac{1}{16}$. **12.** $3479\frac{1}{9}$. **13.** $4454\frac{1}{2}$. **14.** $2776\frac{1}{2}$, $2704\frac{3}{4}$, $1754\frac{2}{3}$, $2226\frac{1}{6}$, total $9462\frac{1}{12}$.

Page 92. **1.** $\frac{5}{24}$. **2.** $\frac{17}{48}$. **3.** $\frac{1}{4}$. **4.** $\frac{1}{32}$. **5.** $\frac{23}{100}$. **6.** $\frac{49}{171}$. **7.** $\frac{101}{200}$. **8.** $\frac{11}{91}$. **9.** $\frac{17}{36}$. **10.** $\frac{11}{36}$. **11.** $\frac{13}{56}$. **12.** $\frac{17}{48}$. **13.** $\frac{1}{12}$. **14.** $\frac{5}{32}$. **15.** $\frac{13}{144}$. **16.** $\frac{7}{120}$. **17.** $\frac{13}{96}$. **18.** $\frac{16}{75}$. **19.** $\frac{5}{84}$. **20.** $\frac{1}{54}$.

Page 93. **1.** .1828. **2.** .0129. **3.** .0174. **4.** .9702. **5.** .196. **6.** .004. **7.** .0144. **8.** .0099.

1. $\frac{1}{3}$. **2.** $\frac{1}{8}$. **3.** $\frac{1}{4}$. **4.** $\frac{5}{16}$. **5.** $\frac{1}{2}$. **6.** $\frac{1}{3}$. **7.** $\frac{7}{24}$. **8.** $\frac{13}{32}$. **9.** .267. **10.** $\frac{9}{64}$. **11.** $\frac{5}{12}$. **12.** .012. **13.** .103. **14.** .163. **15.** $.778\frac{2}{3}$. **16.** .668. **17.** .359. **18.** .048. **19.** .173. **20.** $\frac{1}{24}$. **21.** $\frac{3}{16}$. **22.** $\frac{3}{16}$. **23.** .075. **24.** .46.

Page 94. **1.** $4\frac{7}{8}$. **2.** $16\frac{5}{6}$. **3.** $13\frac{5}{6}$. **4.** $24\frac{31}{40}$. **5.** $8\frac{7}{9}$. **6.** $17\frac{7}{9}$. **7.** $8\frac{13}{20}$. **8.** 17.3. **9.** 16.5. **10.** 25.05. **11.** 17.017. **12.** 9.05. **13.** 24.585. **14.** $57.8\frac{5}{9}$. **15.** 42.275. **16.** $137\frac{1}{6}$. **17.** 111.93. **18.** 7.239. **19.** 215.9. **20.** 121.98. **21.** 83.521.

1. $\frac{1}{6}$. **2.** $\frac{1}{6}$. **3.** 0. **4.** $\frac{1}{36}$. **5.** $\frac{5}{28}$. **6.** $\frac{1}{4}$. **7.** $\frac{2}{21}$. **8.** $\frac{4}{45}$. **9.** $\frac{103}{280}$. **10.** $.46\frac{2}{3}$. **11.** .662. **12.** .32895+. **13.** $.94\frac{9}{11}$. **14.** .349. **15.** .1. **16.** $.278\frac{2}{3}$.

17. $13.00\frac{2}{3}$. **18.** 43.8. **19.** $26.038\frac{1}{3}$. **20.** $62.83\frac{2}{3}$. **21.** 38.36. **22.** $110\frac{3}{8}$. **23.** 39.24. **24.** 32.425.

Page 95. 1. $\frac{29}{132}$. **2.** $\frac{19}{48}$. **3.** $\frac{41}{96}$. **4.** $\frac{5}{36}$. **5.** $\frac{5}{84}$. **6.** $\frac{1}{96}$. **7.** 99.99. **8.** $.157\frac{4}{7}$. **9.** 40.2114. **10.** 85.5767. **11.** 3044.25. **12.** $122.333\frac{1}{3}$. **13.** 134.2576. **14.** 128.794. **15.** $41.458\frac{2}{3}$. **16.** $1499.666\frac{2}{3}$. **17.** $2417\frac{5}{18}$. **18.** $2095\frac{1}{14}$.

Page 96. 1. $11\frac{5}{9}$. **2.** $44\frac{3}{8}$. **3.** $36\frac{6}{7}$. **4.** $25\frac{3}{8}$. **5.** $40\frac{2}{3}$. **6.** $21\frac{2}{3}$. **7.** $69\frac{7}{12}$. **8.** $131\frac{5}{17}$. **9.** $226\frac{2}{7}$. **10.** $115\frac{5}{6}$. **11.** $102\frac{6}{7}$. **12.** $160\frac{2}{7}$. **13.** 96. **14.** $3632\frac{3}{4}$. **15.** 4730. **16.** 12,462. **17.** 4935. **18.** 13,515. **19.** 9362. **20.** 9362.

Page 97. 1. 27. **2.** 15. **3.** 4.76. **4.** 4.365. **5.** 17.36. **6.** 33.25. **7.** 64. **8.** 10. **9.** 5.52. **10.** 18.216. **11.** 1.3572. **12.** 8.154. **13.** 21.168. **14.** 111.52. **15.** 47.4. **16.** 3.515. **17.** .009058. **18.** .9723. **19.** 208.8. **20.** .7568.

Page 98. 1. .1875. **2.** $\frac{35}{96}$. **3.** $\frac{7}{9}$. **4.** $\frac{5}{8}$. **5.** .195. **6.** 183.69. **7.** .012852. **8.** 1.575. **9.** .16875. **10.** $36\frac{8}{9}$. **11.** 1.01. **12.** .26195. **13.** 1.125. **14.** 448. **15.** 187.6446. **16.** $\frac{52}{75}$. **17.** .728. **18.** $\frac{3}{4}$. **19.** 82.614. **20.** 45.81. **21.** 5.74. **22.** 6.84. **23.** $\frac{16}{41}$. **24.** 33.124.

Page 99. 1. $600\frac{5}{12}$. **2.** $1105\frac{2}{3}$. **3.** $769\frac{29}{40}$. **4.** $412\frac{1}{20}$. **5.** $635\frac{11}{14}$. **6.** $602\frac{1}{3}$. **7.** 472.44. **8.** $560\frac{5}{16}$. **9.** $931\frac{43}{63}$. **10.** $799\frac{5}{8}$. **11.** 175.85625. **12.** $486\frac{8}{15}$. **13.** 682.55. **14.** $2641\frac{15}{32}$. **15.** $3841\frac{1}{2}$. **16.** $599\frac{13}{30}$. **17.** $449\frac{19}{24}$. **18.** $853\frac{1}{24}$. **19.** $1033\frac{1}{2}$. **20.** $985\frac{1}{18}$. **21.** 422.24. **22.** $22.5194\frac{2}{3}$. **23.** $128.8546\frac{2}{3}$. **24.** 78.375. **25.** $407\frac{3}{64}$. **26.** $3685\frac{1}{9}$. **27.** 350.979237. **28.** 3189.195. **29.** 2470.286. **30.** $38463\frac{13}{24}$.

1. 393. **2.** 450. **3.** 830. **4.** 871. **5.** $1276\frac{1}{2}$. **6.** 1258. **7.** 1266. **8.** 1096. **9.** $801\frac{2}{3}$. **10.** $1751\frac{2}{5}$. **11.** 742. **12.** 791. **13.** $1066\frac{1}{2}$. **14.** $1002\frac{2}{3}$. **15.** 1495. **16.** 884. **17.** $255\frac{3}{4}$. **18.** 118.365. **19.** 156.88. **20.** 271.76. **21.** 680. **22.** $430\frac{1}{2}$ **23.** 849.235. **24.** $1184\frac{1}{6}$.

1. $20\frac{17}{32}$. **2.** 28. **3.** $36\frac{3}{16}$. **4.** $33\frac{13}{15}$. **5.** $8\frac{5}{9}$. **6.** $16\frac{12}{35}$. **7.** $10\frac{4}{7}$. **8.** $32\frac{73}{84}$. **9.** 1.156. **10.** .1827. **11.** 10.125. **12.** $12.38\frac{1}{8}$. **13.** 9.624. **14.** 106.5. **15.** 317.36. **16.** $5\frac{29}{56}$. **17.** 5.882. **18.** $\frac{13}{116}$. **19.** $153.3\frac{1}{7}$. **20.** 128.76 **21.** $202.08\frac{8}{9}$. **22.** $306.8\frac{2}{3}$. **23.** 8.4. **24.** 10.30125.

Page 100. 1. $22.58. **2.** $21.07. **3.** $15.85. **4.** $724.81. **5.** $209.24. **6.** $901.84. **7.** $83.49. **8.** $100.25. **9.** $1004.85.

Page 101. 1. $126.00, $52.50. **2.** $13\frac{1}{2}'' \times 20\frac{1}{4}'' \times 9''$. **3.** $4\frac{7}{12}$ yd. **4.** $4.24. **5.** $2.47. **6.** $1.95. **7.** $.37. **8.** $42\frac{1}{2}''$; $63\frac{3}{4}$ sq. ft. **9.** 22,167.6 lb. **10.** 5011.875 lb. **11.** 11722.5 lb. **12.** Copper, 1726.7 lb.; tin, 27.85 lb.; zinc, 1030.45 lb. **13.** $\frac{144}{400000}$ or .362475. **14.** 1049.76 mi. **15.** 10.10 mills. **16.** $.07837.

Page 103. 1. 12, $17\frac{1}{3}$, $20\frac{5}{8}$, $31\frac{1}{3}$, 35. **2.** 49, $23\frac{8}{9}$, $38\frac{2}{5}$, $18\frac{2}{3}$, $47\frac{1}{2}$. **3.** 20, $8\frac{2}{7}$, $10\frac{4}{9}$, $37\frac{1}{2}$, $13\frac{5}{7}$. **4.** 108, 300, 4.76, 17.654, 1.3104. **5.** 287, 64, $77\frac{1}{7}$, $215\frac{5}{8}$, 40. **6.** 136.937, 28.54584, .735, 1746.4, 2.4. **7.** $630\frac{5}{24}$, $1199\frac{5}{12}$, $1939\frac{8}{9}$, $2911\frac{3}{80}$,

$5170\frac{63}{80}$. **8.** $1461\frac{1}{3}$, $1659\frac{1}{5}$, $4708\frac{1}{2}$, $4433\frac{3}{5}$, $2231\frac{1}{30}$. **9.** $340\frac{3}{5}$, $2893\frac{1}{45}$, $570\frac{10}{27}$, $4274\frac{1}{5}$, 572. **10.** $15.40, $5.79, $9.01, $6.46, $12.45. **11.** $2.29, $3.49, $7.65, $4.22, $7.46. **12.** $90.94, $59.34, $87.20, $103.31, $71.31.

Page 105. **1.** .501. **2.** $1.145\frac{1}{7}$. **3.** .375. **4.** .061. **5.** 2.005. **6.** 29.62. **7.** 15.89. **8.** .4935. **9.** $7.76\frac{23}{24}$. **10.** $28.4\frac{8}{45}$. **11.** .61836. **12.** $72.01\frac{5}{12}$. **13.** $.6005\frac{8}{9}$. **14.** 2.305. **15.** 1.71. **16.** 3.0505. **17.** $2.546\frac{2}{19}$. **18.** $3.079\frac{2}{17}$. **19.** $3.679\frac{8}{23}$. **20.** $48.50\frac{15}{19}$.

1. $1\frac{1}{4}$. **2.** $1\frac{2}{3}$. **3.** $\frac{1}{3}$. **4.** $1\frac{2}{9}$. **5.** $\frac{1}{2}$. **6.** 4. **7.** $\frac{1}{4}$. **8.** 1. **9.** $7\frac{7}{8}$. **10.** 3. **11.** $1\frac{17}{32}$. **12.** $2\frac{2}{3}$. **13.** 2. **14.** $\frac{1}{2}$. **15.** $1\frac{1}{2}$. **16.** $1\frac{1}{3}$. **17.** $1\frac{1}{4}$. **18.** $\frac{8}{9}$. **19.** $\frac{9}{10}$. **20.** $1\frac{1}{6}$.

Page 106. **1.** $\frac{5}{6}$. **2.** $\frac{11}{24}$. **3.** $\frac{3}{4}$. **4.** $2\frac{1}{32}$. **5.** $5\frac{5}{21}$. **6.** $\frac{19}{48}$. **7.** $173\frac{5}{16}$. **8.** $22\frac{3}{8}$. **9.** $32\frac{1}{8}$. **10.** $54\frac{1}{18}$. **11.** $210\frac{11}{16}$. **12.** $331\frac{11}{20}$. **13.** $\frac{2}{5}$. **14.** $1\frac{1}{3}$. **15.** $\frac{28}{45}$. **16.** $355\frac{13}{16}$. **17.** $85\frac{11}{16}$. **18.** $268\frac{22}{49}$. **19.** $1\frac{1}{5}$. **20.** $4\frac{3}{32}$. **21.** $4\frac{1}{16}$. **22.** $1990\frac{23}{32}$. **23.** $34\frac{41}{54}$. **24.** $54\frac{2}{15}$.

Page 107. **1.** $30\frac{22}{75}$. **2.** $23\frac{41}{75}$. **3.** $10\frac{71}{160}$. **4.** $22\frac{6}{119}$. **5.** $22\frac{17}{250}$. **6.** $17\frac{337}{3552}$. **7.** $7\frac{353}{370}$. **8.** $8\frac{66}{109}$. **9.** $10\frac{71}{160}$. **10.** $16\frac{26}{57}$. **11.** $18\frac{422}{429}$. **12.** $11\frac{149}{514}$. **13.** $13\frac{3}{11}$. **14.** $47\frac{127}{131}$. **15.** $111\frac{37}{52}$. **16.** $47\frac{215}{374}$. **17.** $31\frac{19}{47}$. **18.** $132\frac{87}{91}$.

1. $22.9865\frac{5}{23}$. **2.** $31.2612\frac{28}{31}$. **3.** $142.3292\frac{4}{13}$. **4.** $68.7670\frac{590}{923}$. **5.** $2171.3235\frac{5}{17}$. **6.** 6778.88. **7.** $38515.8119\frac{77}{117}$. **8.** $298.5174\frac{34}{109}$. **9.** $.0350\frac{450}{6617}$. **10.** $.0628\frac{636}{92213}$. **11.** $92.0903\frac{1379}{4107}$. **12.** $36.2086\frac{237}{239}$. **13.** $23463.8356\frac{12}{73}$. **14.** $134.2807\frac{47}{179}$. **15.** $16.9115\frac{215}{859}$. **16.** $130.3907\frac{43}{151}$. **17.** $1.8022\frac{118}{231}$. **18.** $9.7181\frac{67}{68}$.

Page 109. **1.** $3746.23. **2.** $1224, $1836, $1224. **3.** 2184 cu. ft. **4.** 162.06 cu. in. **5.** 48.84 sq. ft. **6.** $3116.48. **7.** $9\frac{1}{2}'' \times 11\frac{1}{2}'' \times 17\frac{1}{4}''$. **8.** 24. **9.** Top and bottom, $16\frac{3}{4}'' \times 33\frac{3}{8}''$; sides, $12\frac{1}{2}'' \times 33\frac{3}{8}''$; ends, $12\frac{1}{2}'' \times 14\frac{3}{4}''$; strap, $100\frac{1}{4}''$. **10.** $3410. **11.** $9312.50. **12.** $33.47. **13.** $1\frac{3}{8}¢$; $\frac{7}{8}¢$; $10.94. **14.** $134.46. **15.** $188.98. **16.** $42.67. **17.** $4\frac{293}{318}$. **18.** $5\frac{263}{328}$. **19.** $14\frac{271}{330}$. **20.** $20\frac{245}{944}$. **21.** $12\frac{321}{880}$. **22.** $11\frac{1613}{2240}$. **23.** $13\frac{653}{3600}$. **24.** $8\frac{95}{896}$. **25.** $6\frac{1983}{2968}$.

Page 112. **1.** 5.01. **2.** $1.145\frac{1}{7}$. **3.** .079. **4.** 1140. **5.** 40,103. **6.** $\frac{153}{184}$. **7.** $\frac{164}{297}$. **8.** $\frac{4}{5}$. **9.** $2\frac{53}{80}$. **10.** $6\frac{1}{8}$. **11.** 155.9. **12.** $177\frac{29}{32}$. **13.** $107\frac{1}{4}$. **14.** $145\frac{7}{8}$. **15.** $109\frac{1}{9}$. **16.** $164\frac{4}{15}$. **17.** $12\frac{130}{379}$. **18.** $14\frac{505}{879}$. **19.** $8\frac{464}{5283}$. **20.** $18\frac{554}{1011}$. **21.** $8\frac{67}{109}$. **22.** $20\frac{139}{160}$. **23.** $8\frac{71}{167}$. **24.** $4\frac{320}{429}$. **25.** $23\frac{549}{1285}$. **26.** $11\frac{49}{115}$. **27.** $11\frac{130}{131}$. **28.** $111\frac{41}{52}$. **29.** $11\frac{1337}{14996}$. **30.** $31\frac{19}{47}$.

Page 116. **1.** 2,689,550. **2.** 154,100. **3.** 30,225. **4.** 64,400. **5.** 22,400. **6.** 23,325. **7.** 7800. **8.** 18,825. **9.** 2025. **10.** 7800. **11.** 12,725. **12.** 3,646,750. **13.** 15,600. **14.** 4200. **15.** 142,725. **16.** 155,100. **17.** 295,800. **18.** 43,350. **19.** 33,918.75. **20.** 121,600. **21.** 12,618.75. **22.** 42,837.5. **23.** $89,533\frac{1}{3}$. **24.** $106,162\frac{1}{2}$.

Page 117. **1.** 1210. **2.** 1250. **3.** 15,600. **4.** 5900. **5.** 3000. **6.** 103,000. **7.** 52,000, **8.** 52,000. **9.** 2510. **10.** 7600. **11.** 955. **12.** 11,675. **13.** 9275. **14.** 104,300. **15.** 152,300. **16.** 248,950. **17.** 11,675. **18.** 50,800. **19.** 38,000. **20.** 189,500. **21.** 5950. **22.** 14,250. **23.** $6333\frac{1}{3}$. **24.** 42,625.

Page 118. **1.** \$317.25. **2.** \$210.75. **3.** \$490. **4.** \$532.88. **5.** \$133.13. **6.** \$161.88. **7.** \$354.38. **8.** \$134.44. **9.** \$132. **10.** \$260. **11.** \$364.67. **12.** \$423.33. **13.** \$70.42. **14.** \$155.75. **15.** \$352. **16.** \$95.63. **17.** \$108.94. **18.** \$252.31. **19.** \$300. **20.** \$658.50. **21.** \$327.50. **22.** \$841.75. **23.** \$71.63. **24.** \$160.50. **25.** \$560. **26.** \$182.81. **27.** \$222.63. **28.** \$162. **29.** \$419.17. **30.** \$233.44.

Page 119. **1.** $817\frac{1}{2}$. **2.** $1783\frac{1}{3}$. **3.** 6400. **4.** $3137\frac{1}{2}$. **5.** 59,250. **6.** $213{,}333\frac{1}{3}$. **7.** $16{,}012\frac{1}{2}$. **8.** 441,250. **9.** 76,500. **10.** 310,875. **11.** 3690. **12.** 59,000. **13.** 2560. **14.** 32,375. **15.** 755,125. **16.** $432{,}666\frac{2}{3}$. **17.** 232,500. **18.** 3925. **19.** $3966\frac{2}{3}$. **20.** 110,000. **21.** $4{,}418\frac{3}{4}$. **22.** $84{,}333\frac{1}{3}$. **23.** 686,250. **24.** $2746\frac{2}{3}$.

Page 120. **1.** \$10.88. **2.** \$29.38. **3.** \$55.13. **4.** \$40.50. **5.** \$125.63. **6.** \$43.88. **7.** \$63.25. **8.** \$115.38. **9.** \$118.75. **10.** \$117. **11.** \$57. **12.** \$86.88. **13.** \$187.50. **14.** \$102.38. **15.** \$154.13. **16.** \$248.63. **17.** \$281.75. **18.** \$534.38. **19.** \$432.50. **20.** \$257.13.

Page 121. **1.** \$31.50. **2.** \$19.00. **3.** \$18.13. **4.** \$7.50. **5.** \$27.38. **6.** \$41.13. **7.** \$32.00. **8.** \$35.25. **9.** \$127.75. **10.** \$67.50. **11.** \$126. **12.** \$199.38.

1. \$59. **2.** \$133.83. **3.** \$150.50. **4.** \$131.03. **5.** \$233.58. **6.** \$190.02. **7.** \$116.01. **8.** \$181. **9.** \$247.58. **10.** \$171.13. **11.** \$140.94. **12.** \$180.92.

Page 122. **1.** 27.5. **2.** 114. **3.** 81. **4.** 552. **5.** 73. **6.** 165. **7.** 97. **8.** 110. **9.** 276. **10.** 35.5. **11.** 112. **12.** 206. **13.** 174. **14.** 628. **15.** 330. **16.** 258. **17.** 284. **18.** 131. **19.** 148. **20.** 148.5. **21.** 40.5. **22.** 62. **23.** 214. **24.** 7.425. **25.** 16.5. **26.** 34. **27.** 105.3. **28.** 51.9. **29.** 165. **30.** 130. **31.** 11.85. **32.** 51. **33.** 60. **34.** 1400, 438, 788,501, 72, 333, $118\frac{4}{5}$, 39, 267,120, **35.** 442, rem. 1¢; 430, rem. 6¢; 96, rem.7¢; 70, rem. 2¢; 79, rem. 10¢; 49, rem. 9¢; 681, rem. 3¢; 144, rem. 5¢; 64, rem. 15¢; 459, rem. 4¢.

Page 124. **1.** $84.45\frac{1}{3}$. **2.** $44.17\frac{7}{9}$. **3.** $271.62\frac{2}{3}$. **4.** $60.62\frac{6}{11}$. **5.** $120.46\frac{2}{5}$. **6.** $16.28\frac{4}{13}$. **7.** $71.90\frac{6}{7}$. **8.** $110.09\frac{1}{3}$. **9.** 30.944. **10.** $39.63\frac{3}{7}$. **11.** 125.248. **12.** 85.11. **13.** 74.82. **14.** $35.83\frac{7}{11}$. **15.** 219.048. **16.** 109.664. **17.** $82.34\frac{2}{7}$. **18.** 101.4

Page 125. **1.** \$13.13. **2.** \$39.60. **3.** \$9.56. **4.** \$56.88. **5.** \$8.63. **6.** \$56.50. **7.** \$39.60. **8.** \$8.55. **9.** \$70.50. **10.** \$11.05. **11.** \$48.13. **12.** \$9.90. **13.** \$292.56. **14.** \$83.10. **15.** \$153.13. **16.** \$76.77. **17.** \$23.14. **18.** \$49.42. **19.** \$37.75. **20.** \$58.30.

Page 127. **1.** 6.691 mi. **2.** \$28.85. **3.** \$82.60. **4.** \$29.76. **5.** \$69.48. **6.** *a*, \$8.10; *b*, \$2.70; *c*, \$4.05; *d*, \$6.75; *e*, \$9.45; *f*, \$.54; *g*, \$1.08; *h*, \$1.62; *i*, \$2.16. **7.** *a*, \$.78; *b*, \$1.56; *c*, \$3.12; *d*, \$4.68; *e*, \$1.95; *f*, \$2.73; *g*, \$1.87; *h*, \$2.81; *i*, \$3.90. **8.** *a*, \$7.00; *b*, \$10.50; *c*, \$14.; *d*, \$8.75; *e*, \$11.20; *f*, \$14.88; *g*, \$10.85; *h*, \$11.03; *i*, \$11.08. **9.** \$1.25. **10.** \$4.86. **11.** \$9.90. **12.** \$.37. **13.** \$.72. **14.** \$.47. **15.** \$.75. **16.** \$.92. **17.** \$2.17. **18.** \$1.65. **19.** \$2.19.

20. \$2.25. **21.** \$4.27. **22.** \$4.65. **23.** \$1.25. **24.** \$89.63. **25.** \$53.33. **26.** \$150. **27.** \$32.19. **28.** \$11.38.

Page 128. **1.** \$242.13. **2.** \$445.95. **3.** \$2685.30. **4.** \$3193. **5.** \$266.88. **6.** \$1222.63. **7.** \$39.78. **8.** \$109.83.

Page 130. **1.** \$617.31. **2.** 24–\$20; 6–\$10; 7–\$5; 15–\$2; 5–\$1; 11–50¢; 4–25¢; 5–10¢; 3–5¢; 17–1¢. **3.** 7–\$20; 5–\$10; 6–\$5; 14–\$2; 4–\$1; 9–50¢; 6–25¢; 9–10¢; 4–5¢; 26–1¢; total \$259.36. **4.** 9–\$20; 3–\$10; 4–\$5; 10–\$2; 5–\$1; 4–50¢; 8–25¢; 13–10¢; 6–5¢; 25–1¢; total \$260.85. **5.** 19–\$20; 11–\$10; 5–\$5; 10–\$2; 9–\$1; 7–50¢; 10–25¢; 8–10¢; 6–5¢; 23–1¢; total \$551.33. **6.** 20–\$20; 7–\$10; 8–\$5; 17–\$2; 6–\$1; 10–50¢; 6–25¢; 9–10¢; 9–5¢; 35–1¢; total \$558.20. **7.** \$37.80. **8.** \$38.48. **9.** 55¢; $61\frac{7}{8}$¢. **10.** \$3.18, \$3.24, \$3.36, \$3.69, \$3.58, \$3.80, \$3.47, \$3.30; total \$27.62. **11.** \$426.88. **12.** $\$17.30\frac{10}{13}$; \$75; no; $4\frac{1}{3}$ wk. **13.** $\$21.92\frac{4}{13}$. **14.** 31–\$20; 6–\$10; 10–\$5; 11–\$2; 9–\$1; 6–50¢; 8–25¢; 9–10¢; 5–5¢; 33–1¢; total \$767.48.

Page 135. **1.** $\$7.30\frac{1}{2}$. **2.** \$10.80. **3.** The second, by \$5.65. **4.** \$279.35. **5.** $\$31.75\frac{1}{2}$; \$4731.50. **6.** \$56.75. **7.** Corn per bu. $33\frac{1}{6}$¢; barley per bu. 40¢; barley better 98¢ per A. **8.** 2.12 tons; $1.1\frac{5}{6}$ tons. **9.** \$153.55. **10.** \$81.41; $\$.969\frac{1}{6}$. **11.** \$1600 better to feed. **12.** \$.105+; \$241.53+; \$261.44+. **13.** \$531.16; \$471.16. **14.** \$419.12. **15.** \$931.84; \$512.72. **16.** Corn meal by $\frac{17}{28}$¢ per lb. **17.** 41 bu. **18.** $\$.81\frac{55}{64}$.

Page 137. **1.** \$309. **2.** \$36.80; \$3.07. **3.** $\frac{17}{25}$. **4.** $19\frac{1}{2}''\times34''\times10\frac{1}{2}''$ or $26''\times25\frac{1}{2}''\times10\frac{1}{2}''$ or $17''\times39''\times10\frac{1}{2}''$. **5.** $\frac{1547}{13824}$ (approx. $\frac{1}{9}$ or .112); $4\frac{11}{384}$ cu. ft. or 4 cu. ft. $49\frac{1}{2}$ cu. in.; $20''\times34\frac{1}{2}''\times11''$, $26\frac{1}{2}''$x$26''\times11''$, and others. **6.** $3''\times6''$.

Page 138. **1.** \$8.64. **2.** 5¢. **3.** \$61.18. **4.** \$17,373.74; \$173.74. **5.** \$250; \$8.33; 412,800 sq. ft. **6.** \$6.82. **7.** 2280.96—tons. **8.** \$.0496+; \$.036+. **9.** 436; \$.0041. **10.** Mar., .025; Apr., .0192; diff., .0058. **11.** \$.01325. **12.** \$.2566; \$.2256; \$.1533; \$.05; \$.6855.

Page 140. **1.** \$70.36. **2.** \$3.60. **3.** \$8.40; 56¢. **4.** \$21.84. **5.** $52\frac{1}{18}$ no. bu. **6.** 5.58+bu. **7.** \$1,187,374.90; \$1,223.66. **8.** \$2549.50; \$6.37+. **9.** $\$9.04\frac{1}{11}$. **10.** \$58.546. **11.** \$1810. **12.** \$65.61. **13.** \$1470. **14.** \$652. **15.** \$.85. **16.** \$222.75.

Page 142. **1.** \$9844.26. **2.** \$8781.46; loss, \$3482.08 **3.** \$19,663.87. **4.** \$20,374.65. **5.** \$28,413.23. **6.** \$34,063.53. **7.** \$6456.08. **8.** \$6455.90. **9.** \$545.91. **10.** \$1171.09.

Page 144. **1.** \$50. **2.** $7\frac{7}{12}$ mi. **3.** 27.8, 25.2, 24.3, 24.8, 25.5. **4.** \$470.94. **5.** 67.3°. **6.** \$252.25. **7.** 1.1 in. **8.** \$12,866.785. **9.** No; \$315.; \$100.; \$150. **10.** \$173.42; \$.3942. **11.** 83.08 bu. **12.** 339; 347. **13.** \$337.98. **14.** 138.14 A; \$39.60. **15.** \$364.80. **16.** \$34.37. **17.** 53 mi., 51 mi. **18.** \$39,072,520; \$107,048.

Page 149. **1.** \$3.6461. **2.** 1616 lb., 1016 lb., 104 lb., 1264 lb. **3.** \$43.28; \$20,644.56. **4.** \$8,847,447. **5.** \$1,379,650. **6.** 12 yr. **7.** 9984.87 sq. mi. **8.** \$10,345.41. **9.** \$309,375. **10.** \$108 less. **11.** 4.1877%, 2.673%, 2.4948%, 2.3166%, 2.0493%, 1.6038%, 1.3365%, 1.1583%; 579 pupils.

Page 151. **1.** $33\frac{1}{3}$%. **2.** 98%. **3.** 58.44%. **4.** Incr. 8.2%, incr. 36%, incr. 8.8%, incr. 9.97%, decr. 21.3%, decr. 7.6%, incr. 273.58%, decr. 7.8%; 4.08% total incr. **5.** .22%. **6.** 17.39+%; 5[illegible]+%. **7.** 34.7%, 38.5%, 58.6%, 30.3%, 35.3%, 18.2%, 67.4%, 34.5%. **8.** 1.2%. **9.** Tonnage: Decr. 2%, incr. 2%, incr. 10%, decr. 7%, incr. 13%, incr. 8%, incr. 11%, incr. 11%, decr. 1%. Pass: Incr. 20%, incr. 40%, decr. 31%, decr. 10%, decr. 3%, incr. 10%, incr. 17%, decr. 3%, incr. 6%. **10.** 29%, 33%, 35%. **11.** 21.25%. 13.83%. **12.** 50,742.2%. **13.** 11.26%. **14.** 12.55%, 17.43%, 20.97%. **15.** Sales: 1.6% under, 28.7% over, 11.9% under, 16.4% over, 31.6 under. Exp.: 5.2% under, 8.7% over, 1.2% under, 15.3% under, 13.1% over.

Page 155. **1.** \$235,640.55. **2.** \$1250. **3.** \$10,538.17. **4.** \$1933, \$14,915, \$533,923, \$795, \$1.297,415. **5.** W \$5600, M \$4000. **6.** 300 A; corn, 156 A; past., 54 A; wheat, 63 A; mead., 27 A. **7.** \$2329.70. **8.** 25,570, 67,469, 33,195, 127,628, 34,400. **9.** \$6394.74. **10.** \$13,758.22. **11.** \$4150.40. **12.** \$15,693.27. **13.** \$91,739,436.62, \$78,712,436.62. **14.** Incr. 200%, incr. 250%, incr. 175%, incr. 20%, decr. $16\frac{2}{3}$%, decr. 25%, decr. 55%, incr. 100%, decr. $13\frac{1}{3}$%, incr. 25%, decr. $7\frac{1}{2}$%, incr. 50%, decr. 80%, decr. 50%, decr. $33\frac{1}{3}$%, incr. $66\frac{2}{3}$%, decr. 50%, incr. 58.8%, decr. $33\frac{1}{3}$%. **15.** \$40,000, \$168,000, \$150,000, \$203,000, \$46,000, \$1,400,000, \$180,000, \$20,000, \$10,000, \$52,000, \$1700, \$72,000, \$1300, \$9000, \$1500, \$32,000, \$1,080,000, \$10,700,000 **16.** Lang.,116,982; gram., 44,970. **17. 1923:** .2463, .1837, .1652, .1437, .1079, .0708, .0469, .0355. **1914:** .2306, .1784, .1652, .1525, .1073, .077, .052, .0368. **1925:** .2108, .1719, .1623, .1541, .12, .0818, .0558, .0433. **1926:** .2086, .1529, .1525, .1538, .1244, .0935, .0677, .0466. **1927:** .2039, .1512, .1482, .1489, .1278, .0991, .0693, .0516. Averages: 6446, 4911, 4653, 4425, 3457, 2490, 1722, 1261. Pop. 163,000. **18. 1922:** \$6000, \$40,000, \$3500, \$5000, \$27,000, 80,000, \$72,000, \$400,000, \$800, \$40,000. **1923:** \$2000, \$75,000, \$1750, \$230,000, \$45,000, \$112,000, \$42,000, \$360,000, \$2400, \$32,000. **19.** $86\frac{1}{9}$%; \$41,765.63; \$77,730.47.

Page 165. **1.** \$60, \$2940. **2.** \$180, \$4320. **3.** \$283.50, \$5386.50. **4.** \$404.37, \$6335.13. **5.** \$417.66, \$7176.14. **6.** \$237.27, \$3413.02. **7.** \$125.08, \$3448.72. **8.** \$120.33, \$2553.57. **9.** \$75.86, \$2091.56. **10.** \$48.93, \$1908.27. **11.** \$38.25, \$571.71. **12.** \$102.93; \$1338.06. **13.** \$62.84, \$1002.21. **14.** \$52.01 \$537.85. **15.** \$38.86, \$885.36. **16.** \$107.50, \$1497.83. **17.** \$170.70, \$2181.17. **18.** \$105.51, \$2522.95. **19.** \$24.01, \$787.31. **20.** \$65.74, \$1528.45. **21.** \$184.27. **22.** \$625.73. **23.** $1\frac{1}{4}$¢ per bu.; \$1.56. **24.** \$55.30; \$2.30. **25.** \$87.00. **26.** \$166.57. **27.** 46¢. **28.** \$206.77. **29.** \$3092.46.

Page 166. **1.** \$46,774.38. **2.** \$3910.53, \$1865.53, \$9209.25, \$13,000, \$4177.56, \$3048.01, \$1970.09, \$1380.62. **3.** 150. **4.** \$500, \$25; \$600, \$30;

$300, $12; $900, $18; $800, $56; $700, $14; $650, $32.50; $1238.50, $37.16; $1473.50, $29.47; $1829.40, $18.29; $1179.14, $70.75; $1273.50, $101.88. **5.** $5.43. **6.** 257, $32.75 rem. **7.** 121. **8.** 115, $2.60. **9.** 83, $8.82. **10.** $50,262.50. **11.** $1541.29. **12.** $2593.38. **13.** $2063.52. **14.** $1.72. **15.** $1.145525. **16.** $6494.80. **17.** $6143.17. **18.** $163.91. **19.** $10,000

Page 171. **1.** 15,840 ft. **2.** $2\frac{5}{8}$ mi. **3.** 33,644 in. **4.** 35,840 oz. **5.** $148\frac{1}{2}$ in. **6.** $1075\frac{7}{8}$ lb. **7.** 108,900 sq. ft. **8.** 7,260 sq. yd. **9.** 145,152 cu. in. **10.** 53,576 cu. in. **11.** 120 pt. **12.** $72\frac{1}{2}$ qt. **13.** 20 pt. **14.** $25\frac{1}{2}$ pt. **15.** 29.76 oz. **16.** 67° 30′. **17.** $240\frac{1}{2}$ hr. **18.** 310. **19.** 138 gal., 3 qt., 1 pt. **20.** 82 lb., 5 oz. **21.** 49 rd., 2 ft. **22.** 7 cu. ft., 430 cu. in. **23.** 4 sq. yd., 3 sq. ft., 113 sq. in. **24.** 1 hr., 20 min., 35 sec. **25.** 2° 44′ 9″. **26.** 1 mi., 239 rd., 4 yd., 2 ft., 6 in. **27.** 10 sq. mi., 129 A. **28.** $.0454\frac{6}{11}$ rd. **29.** .29924+ mi. **30.** $\frac{7}{216}$ cu. yd. **31.** $\frac{3}{28}$ or $.107\frac{1}{7}$. **32.** $\frac{1}{6480}$. **33.** $\frac{89}{8640}$. **34.** $\frac{12}{73}$. **35.** 3, 2, $2\frac{1}{2}$, $\frac{1}{4}$, $1\frac{3}{5}$, $2\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{1}{10}$, $1\frac{3}{5}$, $1\frac{1}{4}$, $3\frac{1}{2}$, $5\frac{1}{2}$, $\frac{7}{10}$, $1\frac{1}{3}$, $3\frac{2}{5}$, $2\frac{1}{4}$, $5\frac{3}{10}$, $2\frac{5}{6}$.

Page 173. **1.** 129A., 9 sq. rd. **2.** 80 rd., 16 ft. **3.** 38 sq. ft., 94 sq. in. **4.** 103 ft., 10 in.

1. 5 yr., 6 mo., 26 da. **2.** 1 yr., 4 mo., 18 da. **3.** 1 mi., 318 rd., 2 ft. **4.** 19 gal., 3 qt. **5.** 25 T., 1500 lb. **6.** 3 yrs., 8 mo., 3 da. **7.** 1 yr., 2 mo., 8 da.; 1 yr., 9 mo.; 7 mo., 7 da.; 90 da.

Page 175. **1.** 63 lb. **2.** 25,200; $68.04. **3.** $4459. **4.** 201 rd., 15 ft., 6 in. **5.** 450 bu.; $2925. **6.** $26,514. **7.** $71,890.26. **8.** $670.70. **9.** $1.49.

Page 176. **1.** 980 bu.; $686. **2.** 21. **3.** 320 rails; 140.8 T. **4.** $2062\frac{1}{2}$ bu. **5.** $5.09. **6.** 102 mi. **7.** 432. **8.** 4 wk., 2 da., 5 hr. **9.** 72¢ per yr.; 2¢ per day. **10.** $\frac{9}{50}$. **11.** $143.91.

Page 182. *Exact days*: **1.** 38. **2.** 67. **3.** 119. **4.** 125. **5.** 135. **6.** 116. **7.** 147. **8.** 117. **9.** 77. **10.** 166. *Comp. sub.*: **1.** 1 mo., 8 da. **2.** 2 mo., 6 da. **3.** 3 mo., 26 da. **4.** 4 mo., 2 da. **5.** 4 mo., 12 da. **6.** 3 mo., 24 da. **7.** 4 mo., 24 da. **8.** 3 mo., 25 da. **9.** 2 mo., 15 da. **10.** 5 mo., 13 da. *Months and days*: **1.** 1 mo., 8 da. **2.** 2 mo., 6 da. **3.** 3 mo., 27 da. **4.** 4 mo., 2 da. **5.** 4 mo., 12 da. **6.** 3 mo., 25 da. **7.** 4 mo., 25 da. **8.** 3 mo., 25 da. **9.** 2 mo., 16 da. **10.** 5 mo., 13 da.

Comp. Sub.: **1.** 1 yr., 6 mo., 4 da. **2.** 2 yr., 3 mo., 3 da. **3.** 3 yr., 3 mo., 15 da. **4.** 2 yr., 3 mo., 19 da. **5.** 2 yr., 9 mo., 16 da. **6.** 4 yrs., 6 mo., 5 da. **7.** 5 yr., 2 mo., 21 da. **8.** 8 yr., 3 mo., 19 da. **9.** 4 yr., 3 mo., 23 da. **10.** 7 yr. 3 mo., 23 da. *Exact days:* **1.** 1 yr., 188 da. **2.** 2 yr., 95 da. **3.** 3 yrs., 105 da. **4.** 2 yr., 109 da. **5.** 2 yr., 290 da. **6.** 4 yr., 187 da. **7.** 5 yr., 83 da. **8.** 8 yr., 112 da. **9.** 4 yr., 115 da. **10.** 7 yr., 116 da.

1. 2 yr., 4 mo., 24 da. **2.** 3 yr., 10 mo., 20 da. **3.** 1 yr., 4 mo., 16 da. **4.** 4 yr., 5 mo., 27 da. **5.** 2 yr., 9 mo., 14 da. **6.** 9 yr., 5 mo., 11 da. **7.** 6 yr. 3 mo., 18 da. **8.** 3 yr., 2 mo., 1 da. **9.** 4 mo., 11 da. **10.** 2 yr., 3 mo.

Page 183. **1.** $175.50 **2.** $375.84. **3.** $449.17. **4.** $111.09. **5.** $177.52. **6.** $243.20. **7.** $88.63. **8.** $101.83. **9.** $88.22. **10.** $270.94.

Page 185. **1.** \$5.60. **2.** \$53.19. **3.** \$14.87. **4.** \$94.08. **5.** \$16.14. **6.** \$2.50. **7.** \$36.59. **8.** \$3.72. **9.** \$28.94. **10.** \$1.89.

Page 186. **1.** \$31.20. **2.** \$101.49. **3.** \$128.90. **4.** \$12.71. **5.** \$7.43. **6.** \$63.24. **7.** \$6.24. **8.** \$27.47. **9.** \$38.94. **10.** \$160.23.

1. \$12.40. **2.** \$14.50. **3.** \$9.13. **4.** \$8.10. **5.** \$6.17. **6.** \$19.48. **7.** \$19.10. **8.** \$18.68. **9.** \$9.44. **10.** \$7.44. **11.** \$30.42. **12.** \$32.40. **13.** \$86.33. **14.** \$116.80. **15.** \$73.88. **16.** \$118.50. **17.** \$169.40. **18.** \$26.10. **19.** \$57.50. **20.** \$58.42.

Page 187. **1.** 6%. **2.** 4%. **3.** 3%. **4.** 5%. **5.** 6%. **6.** 6%. **7.** 7%. **8.** 6%. **9.** 8%. **10.** 6%. **11.** $7\frac{1}{2}$%. **12.** 6%.

Page 188. **1.** 4 mo. **2.** 8 mo., 15 da., **3.** 8 mo., 20 da. **4.** 8 mo., 10 da. **5.** 8 mo., 17 da. **6.** 7 mo., 17 da. **7.** 1 yr., 4 mo., 29 da. **8.** 1 yr., 4 mo., 24 da. **9.** 1 yr., 5 mo., 16 da. **10.** 4 yr., 8 mo., 22 da. **11.** 3 yr., 6 mo., 14 da. **12.** 4 yr., 4 mo., 19 da.

1. \$485.71. **2.** \$949.56. **3.** \$1111.76. **4.** \$5066.67. **5.** \$12,828.57. **6.** \$1736.97. **7.** \$1834.06. **8.** \$1616.64. **9.** \$1880.32. **10.** \$1851.84. **11.** \$19,617.95. **12.** \$5872.38.

Page 189. **1.** \$1136.20. **2.** \$2106.63. **3.** \$3106.82. **4.** \$4609.27. **5.** \$3648.69. **6.** \$465.01. **7.** \$570.53. **8.** \$1301.61. **9.** \$2405.13. **10.** \$789.06.

Page 190. **1.** \$7.50, \$507.50. **2.** \$5, \$405. **3.** \$8, \$608. **4.** \$9.60, \$809.60. **5.** 8 mo. **6.** \$10, \$510. **7.** \$600, \$648. **8.** \$2000, \$2150. **9.** 4 mo. **10.** \$300, \$24. **11.** 2 yr., 6 mo. **12.** \$145, \$159.50. **13.** 8%. **14.** 6%. **15.** 3%. **16.** 4%. **17.** 8 mo., \$260. **18.** \$16, \$416. **19.** \$300, \$18. **20.** \$500, \$35. **21.** \$54, \$414. **22.** 1 yr., 4 mo. **23.** \$205, \$223.45. **24.** 1 mo., 15 da.

Page 191. **1.** \$993.44. **2.** \$605.89. **3.** \$2292.23. **4.** \$767.51, \$783.17, \$783.17, \$810.71. **5.** \$10,400. **6.** \$6200. **7.** $16\frac{2}{3}$ yr., 20 yr., $12\frac{1}{2}$ yr., $14\frac{2}{7}$ yr. **8.** \$45.87, \$45.66, \$46.29. **9.** \$39.75. **10.** \$63.16. **11.** \$29.66. **12.** 15.6+%. **13.** \$8,474.58.

Page 192. **1.** \$18.85. **2.** $14\frac{1}{2}$%. **3.** $143\frac{1}{2}$%. **4.** \$16,777.11. **5.** \$7.66. **6.** \$1795. **7.** 35.86%. **8.** \$465.09. **9.** 8.35%. **10.** \$270.25, \$229.75. **11.** 102.02−%. **12.** \$1097.48 at 6 % simple int. **13.** \$2224.10. **14.** 25.64%. **15.** 12.4. **16.** \$1866.82, \$.3044$\frac{1}{3}$, 132 days before. **17.** \$1283.19, \$1248.02.

Page 196. **1.** \$1.00. **2.** \$5.83. **3.** \$2.29. **4.** \$1.94. **5.** \$384.94. **6.** \$23.81. **7.** \$2.25. **8.** \$3.83. **9.** \$42.00. **10.** \$5.69.

Page 199. \$200, \$201, \$202, \$203.03, \$207.

Page 203. **1.** May 18, May 17. **2.** June 27, June 25. **3.** Aug. 14, Aug. 12. **4.** Feb. 28, Feb. 28. **5.** Aug. 24, Aug. 23. **6.** Feb. 10, Feb. 6. **7.** Jan. 21, Jan. 19. **8.** Mar. 22, Mar. 22. **9.** Feb. 28, Mar. 2. **10.** Feb. 28, Mar. 1. **11.** Dec. 14, Dec. 13. **12.** Feb. 28, Feb. 28. **13.** Apr. 28, Apr. 29. **14.** Sept. 12, Sept. 9. **15.** Aug. 27, Aug. 26. **16.** Sept. 30, Sept. 30.

1. 8/12, 8/11. **2.** 1/4/20, 1/2/20. **3.** 2/26/19, 2/24/19. **4.** 9/30, 9/30. **5.** 2/28/18, 2/27/18. **6.** 2/28/18, 2/28/18. **7.** 2/28/18, 3/1/18. **8.** 2/28/18, 3/2/18. **9.** 6/30, 6/30. **10.** 6/30, 6/29. **11.** 6/30, 6/29. **12.** 3/28, 3/30. **13.** 9/30, 9/29. **14.** 12/30, 12/29. **15.** 1/30/18, 1/29/18. **16.** 1/31/18, 1/29/18. **17.** $203. **18.** $508.42.

Page 204. 1. 78 da. **2.** 37 da. **3.** 52 da. **4.** 30 da. **5.** 54 da. **6.** 83 da. **7.** 12 da. **8.** 7 da. **9.** 89 da. **10.** 73 da. **11.** 92 da. **12.** 72 da. **13.** 91 da. **14.** 28 da. **15.** 61 da. **16.** 54 da. **17.** 29 da. **18.** 84 da. **19.** 112 da. **20.** 165 da.

Page 206. 1. $595.50. **2.** $375.82. **3.** $350.05. **4.** $1144.63. **5.** $862.58. **6.** $388.74. **7.** $2911. **8.** $905.32. **9.** $4422.75. **10.** $7134.40. **11.** $357.32 **12.** $1268.76. **13.** $837.60. **14.** $124.32. **15.** $277.09. **16.** $384.57. **17.** $3428.69. **18.** $706.51. **19.** $376.76. **20.** $511.47. **21.** $840.73. **22.** $1567.60. **23.** $4433.25. **24.** $6122.50. **25.** $2459.09.

Page 208. 1. $892.20. **2.** $470.61. **3.** $1618.44. **4.** $365.69. **5.** $790.67. **6.** $747.04. **7.** $974.65. **8.** $427.60. **9.** $210.20. **10.** $1,340.62. **11.** $4205.14. **12.** $3726.02. **13.** $1258.85. **14.** $14.75. **15.** $3141.92. **16.** $16.91.

Page 210. 1. $501.41. **2.** $1254.08. **3.** $1157.61. **4.** $1481.47. **5.** $274.93. **6.** $729.97. **7.** $417.38. **8.** $3519.86. **9.** $1249.15. **10.** $1636.39. **11.** $761.85. **12.** $472.92. **13.** $267.24. **14.** $1438.61. **15.** $1139.29. **16.** $4494.23. **17.** $1276.67. **18.** $2167.11. **19.** $1243.97. **20.** $745.64.

Page 211. 1. $8338.23. **2.** $1218.28. **3.** $1182.80. **4.** $578.67. **5.** $570.11. **6.** $1781.40. **7.** $3461.58. **8.** $2985.50.

Page 212. 1. $356.49, $147.46, $96.36, $238.34, $75.73, $439.52, total $1353.90. **2.** $1630.21. **3.** $19,762.50. **4.** $1599.57. **5.** $1997.35. **6.** $160.53.

Page 215. 1. $28.86. **2.** $.89. **3.** $89.06. **4.** $9.56. **5.** Either. **6.** (*a*) 40%, (*b*) $34\frac{3}{8}$%, (*c*) 32%, (*d*) 22%, (*e*) 46%, (*f*) $44\frac{1}{2}$%, (*g*) 50%, (*h*) 50%, (*i*) $41\frac{2}{3}$%, (*j*) $66\frac{2}{3}$%, (*k*) $47\frac{1}{2}$%, (*l*) $46\frac{2}{3}$%, (*m*) $35\frac{1}{5}$%, (*n*) $23\frac{1}{20}$%, (*o*) $31\frac{3}{5}$%, (*p*) $35\frac{7}{8}$% (*q*) 75%, (*r*) 82%, (*s*) 84.4%, (*t*) $43\frac{3}{4}$%, (*u*) $48\frac{4}{5}$%, (*v*) $77\frac{1}{2}$%, (*w*) 78.175%, (*x*) 27.1%. **7.** $26\frac{2}{13}$%.

Page 217. 1. $3.64 Nov. 12. **2.** $44.20 Aug. 29. **3.** $33.04 Mar. 17. **4.** $6.64 net. **5.** $18.71 Jan. 18. **6.** $19.89 July 26. **7.** $536.53 Oct. 24. **8.** $178.71 Oct. 24.

Page 222. 1. Second, by $1.59. **2.** Second, by $95.47. **3.** $432; 125%. **4.** Second, by $2.70. **5.** 2.4¢ per lb.

Page 223. 1. 32.2%. **2.** $7\frac{9}{13}$%. **3.** $29,400. **4.** $600; less by $\frac{3}{5}$%. **5.** 9.4—%; 8.6—%. **6.** 4.6%. **7.** 2.3%. **8.** 1.96%. **9.** $26.32.

Page 229. **1.** 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400. **2.** 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728, 2197, 2744, 3375, 4096, 4913, 5832, 6859, 8000. **3.** 1, 16, 81, 256, 625, 1296, 2401, 4096, 6561, 10,000, 14,641, 20,736, 28,261, 38,416, 50,625, 65,536, 83,521, 104,976, 130,321, 160,000. **4.** $\frac{1}{16}$, $\frac{9}{64}$, $\frac{49}{144}$, $\frac{25}{36}$, $\frac{9}{49}$, $\frac{9}{16}$, $\frac{25}{64}$, $\frac{1}{4}$. **5.** $\frac{27}{125}$, $\frac{1}{64}$, $\frac{27}{512}$, $\frac{125}{512}$, $\frac{343}{1728}$, $\frac{125}{216}$, $\frac{1}{27}$, $\frac{1}{8}$. **6.** $\frac{81}{256}$, $\frac{1}{16}$, $\frac{81}{4096}$, $\frac{256}{625}$, $\frac{1}{256}$, $\frac{625}{1296}$, $\frac{625}{20736}$, $\frac{81}{625}$. **7.** .0625, .5625, .2116, .3025, .7225, .0256, .0225, .0324. **8.** .001728, .00020736; .003375, .00050625; .005832, .00104976; .008, .0016; .015625, 00390625; .091125, .04100625; .019683, .00531441; .032768, .01048576. **9.** See prob. 1. **10.** 40 A. **11.** $336.

Page 231. **1.** 18. **2.** 21. **3.** 23. **4.** 27. **5.** 36. **6.** 42. **7.** 53. **8.** 68. **9.** 75. **10.** 85. **11.** 93. **12.** 98. **13.** 2.17. **14.** 3.518+. **15.** 10.724. **16.** 4.141+. **17.** 5.568+. **18.** 24.32. **19.** .759. **20.** .086+. **21.** $\frac{5}{7}$. **22.** $\frac{1.414+}{1.782+}$. **23.** .316+. **24.** $\frac{1}{2}$.

Page 234. **1.** 180 sq. ft.; 20 sq. yd. **3.** 10 sq. yd., 10 sq. yd., $12\frac{1}{2}$ sq. yd., $12\frac{1}{2}$ sq. yd.; 59 sq. yd. **4.** $272\frac{1}{4}$ ft. **6.** 40 rd. **7.** Areas: 108 sq. ft., 315 sq. yd. 1400 sq. yd., $1462\frac{1}{2}$ sq. rd., 3947.14 sq. rd., 117.12 sq. ch. Perimeters: 51.633+ ft., 87.619+ yd., 190.622+ yd., 189.056+ rd., 335.081+ rd., 53.432+ ch. **8.** $3\frac{1}{16}$ A. **9.** 134.666+ sq. ft., 384 sq. rd., 191.245+ sq. ft., 100.118+ sq. ft., 509.901+ sq. yd., 854.925+ sq. rd. **10.** $1467.38. **11.** 91.787+ rd. **12.** 104.985+ ft. **13.** 195.959+ ft. **14.** 127.279 ft. **15.** 16 ft., 8 in. **16.** 10.9956 ft. or 11 ft. **17.** 530.7+. **18.** 20.16+ in. **19.** 2.5 ft.; .5 ft. **20.** 50.2656 sq. yd. **21.** 12 A., 117 sq. rd., 5 sq. yd., 3 sq. ft., 86.2+ sq. in.

Page 237. **1.** 136 sq. in., 169 sq. in.; 108 sq. in., $148\frac{1}{2}$ sq. in.; 120 sq. in., 132 sq. in.; 264 sq. in., 347.136 sq. in. **2.** 325 sq. in. **3.** $71\frac{1}{9}$ sq. yd. **4.** 164.934 sq. ft. **5.** 163.3632 sq. yd. **6.** $4.1888\frac{5}{9}$ sq. yd. **7.** 128 sq. ft. **8.** 144 sq. ft. **9.** 36 cu. ft. **10.** 3392.928 gal. **11.** 10.971+ in. **12.** 9.506+ in. **13.** 138.6 bbl.

Page 238. **1.** 2880 bd. ft. **2.** $99.00. **3.** $31.36. **4.** $94.60. **5.** $106.33.

Page 239. **1.** 2. **2.** $3\frac{3}{4}$. **3.** 2.226+. **4.** $6\frac{3}{4}$. **5.** $1\frac{7}{8}$.

Page 240. **1.** 19.439+ ft. **2.** 12-14 pitch. **3.** 6-10 pitch; 11.661 ft.; 12.646 ft. **4.** 479.352 bd. ft. **6.** $146\frac{2}{3}$ ft.; $2.87.

Page 241. **1.** 65, 29, 67, 10, 25, 18. **2.** 61. **3.** 62. **4.** 38. **5.** 25. **6.** $91.44. **7.** $60.13. **8.** $49. **9.** $76.50. **10.** $107.10.

Page 243. **1.** 3024. **2.** $31.50. **3.** 575. **4.** 3988. **5.** 3287. **6.** $264.25. **7.** $160.44. **8.** $82.65. **9.** $87.28.

Page 246. **1.** 66 sq. yd. **2.** 217 sq. yd. **3.** $186.30. **4.** $5.19. **5.** $4.55. **6.** $63.40. **8.** 5. **9.** $15\frac{1}{2}$. **10.** $11.34. **11.** $149.28. **12.** $18.75. **13.** $286.01. **14.** $70.95. **15.** $405.41. **16.** $390. **17.** $.076625, $.09575.

Page 248. **1.** 4400. **2.** $138.75. **3.** $178.50. **4.** $237.24. **5.** $231. **6.** $336. **7.** 2904 pressed, 10186 common. **8.** $150.85. **9.** 327,967.

Page 249. **1.** $76\frac{4}{11}$. **2.** $121\frac{7}{33}$. **3.** $266\frac{2}{3}$. **4.** $7446.74.

Page 250. **1.** 1.243 or 1.24. **2.** 1.1745 or 1.17. **3.** 1.05705 or 1.06. **4.** 10.79+ or 44 bags = 11 bbl.

Page 251. **1.** Sand: .71, .86, .95, 1.01, .51, .46, .42, .52, .45, .46, .52, .47, Stone: 0, 0, 0, 0, .68, .77, .85, .77, .89, .92, .86, .94. **2.** $17\frac{1}{2}$ bbl. cement, 6 cu. yd. sand, 8 cu. yd. gravel. **3.** $31\frac{3}{4}$ bbl. cement, 10 cu. yd. sand, 19 cu. yd. gravel. **4.** $53\frac{3}{4}$ bbl. cement, 24 cu. yd. sand, 48 cu. yd. stone. **5.** $22\frac{1}{2}$ bbl. cement, 7 cu. yd. sand, 14 cu. yd. stone. **6.** $418.54. **7.** $417.04. **8.** $18.78.

Page 252. **1.** $843\frac{1}{3}$. **2.** $506.25. **3.** $1579.10. **4.** $2407.93. **5.** $169.30.

Page 253. **1.** $65,397,555; 68%. **2.** $178,500; $212\frac{1}{2}$%. **3.** 25 mills; $250; $36.40; 6.868+ yr., or 6 yr. 10 mo. 13 da. **4.** $112,500; $1\frac{7}{9}$ yr. or 1 yr. 9 mo. 10 da. **5.** $1,487,500. **6.** $.2494\frac{11}{31}$; $\frac{7}{31}$%.

Page 256. **1.** 40 sq. ft. **2.** 8. **3.** 274 sq. ft.. **4.** 152 ft. $9\frac{1}{2}$ in. **5.** 1251.3 **6.** Neither. **7.** 3128.3. **8.** 129. **9.** 45. **10.** 78 sq. ft. **11.** 12. **12.** 7 sections. **13.** 13, 11, 14, 13, 4, 5. **14.** 11.

Page 259. **1.** 8 A.M.; 7 A.M.; 6 A.M. **2.** Three times; Pittsburgh, North Platte, Salt Lake City; and others. **3.** 6:42 P.M. **4.** 52 min. **5.** 2 hr. behind. **6.** Set watch back 2 hours. **7.** No. **8.** 2 P.M. **9.** 2 P.M. **10.** 34.6+ mi. per hr. **11.** 12 hr. 40 min.; 17 hr., 40 min.; 2 da., 2 hr. 25 min.; 3 da. 10 hr., 40 min.; 1 da., 8 hr., 27 min. Page 260, bottom: 1984 mi.; Laramie, Wyo.; $6640\frac{3}{7}$ mi.; 41 mi. per hr.

Page 263. **1.** 1 sq. mi. **2.** 320 A. **3.** 160 A. **5.** $1300. **6.** 400 A. **7.** $10; $5. **8.** $242.40. **9.** $31.68.

Page 264. **1.** 12.8 bu. **2.** 38.4 bu. **3.** 576 bu. **4.** 1632 bu. **5.** 720 bu. **6.** 58.8 bu. **7.** 63.7 bu. **8.** 84 bu. **9.** 1911 bu. **10.** 30.8 bu. **11.** 10.08 bu. **12.** 30.24 bu. **13.** 453.6 bu. **14.** 1285.2 bu. **15.** 567 bu. **16.** 46.305. **17.** 50.16375 bu. **18.** 66.15 bu. **19.** 1504.9125 bu. **20.** 24.255 bu. **21.** 578.566+ bu.; 452.798+ bu. **22.** Add 3.56+ bu.; deduct 1.11− bu. **23.** 277.2 bu. **24.** 2304 bu. **25.** 1814.4 bu. **26.** $2646\frac{7}{8}$ bu. **27.** $31\frac{1}{4}$ ft. **28.** $4\frac{2}{3}$ ft. **29.** 22,528 lb. in all but 4 states. **30.** $252.18. **31.** $1.40; $644. **32.** $64.13.

Page 266. **Corn in the ear:** **1.** 6.4 bu. **2.** 19.2 bu. **3.** 288 bu. **4.** 816 bu. **5.** 360 bu. **6.** 29.4 bu. **7.** 31.85 bu. **8.** 42 bu. **9.** $955\frac{1}{2}$ bu. **10.** 15.4 bu.

Corn in the husk: **1.** $4\frac{12}{13}$ bu. **2.** $14\frac{10}{13}$ bu. **3.** $221\frac{7}{13}$ bu. **4.** $627\frac{9}{13}$ bu. **5.** $276\frac{12}{13}$ bu. **6.** $22\frac{8}{13}$ bu. **7.** 24.5 bu. **8.** $32\frac{4}{13}$ bu. **9.** 735 bu. **10.** $11\frac{11}{13}$ bu.

1. $59.55. **2.** $12.93.

Page 267. **1.** $3.47\frac{2}{9}$ tons. **2.** $4.40\frac{11}{245}$ tons. **3.** 6.75 tons. **4.** $4\frac{2}{49}$ tons.

Page 268. **1.** $633\frac{3}{4}$. **2.** 1152. **3.** 588. **4.** 676. **5.** 882. **6.** 1088. **7.** 980. **8.** 1280. **9.** 507. **10.** 686. **11.** $8472\frac{3}{4}$.

Page 269. **1.** 8.83575. **2.** $19.089\frac{7}{12}$. **3.** 31.416. **4.** 28,125. **5.** 30.925+. **6.** 17,812.872 gal. **7.** .59+. **8.** 1767.15. **9.** 176,715. **10.** 1507.9+. **11.** 8.812, 19.04, .595. **12.** 19.48+. **13.** $4\frac{1}{5}$. **14.** 56.1 **15.** 60.54+. **16.** 29.4525. **17.** 21 ft. **18.** $33\frac{1}{3}$ bbl.

Page 271. **1.** 22.644. **2.** 9.732. **3.** 8.764. **4.** 7.769. **5.** 1.075. **6.** 1.33. **7.** .5. **8.** .608. **9.** 4.009. **10.** 2.798. **11.** 2.657. **12.** 1.358.

Page 272. **1.** 166.25. **2.** 58.875. **3.** 59.1875. **4.** 118.75. **5.** 64.375. **6.** 1203.625. **7.** 100.3125. **8.** 168.75. **9.** 35.0625. **10.** 450.0625. **11.** 531.25. **12.** 102.5. **13.** 494.9375. **14.** 59.375. **15.** 654.625.

1. 86.224 lb. **2.** 187.5 lb. **3.** $4182\frac{3}{4}$ lb. **4.** 29.92 exact; 30 approx. **5.** .875. **6.** 10,171.875 tons. **7.** 28,479.3894 lb. **8.** 15,336. **9.** 2775 lb. **10.** 1 lb. 5.9 oz. **11.** 13.76 inches.

Page 273. **1.** 19.013+ in. **2.** 17.44+ in. **3.** 2538. **4.** 949.42+ gal. **5.** Exact, 62.34+, 48.79+; approx., 62.07, 48.8775. **6.** 4.2 in. **7.** $44.28. **8.** $.592. **9.** $.295666. **10.** 113.4375 gal.; 3.6+ bbl. **11.** 139.218+. **12.** Exact, 514.28; approx., 512. **13.** 5.39 bu. more by exact. **14.** 57.63 approx.; 57.89 exact. **15.** 1402.5. **16.** 1496 approx.; 1502.66 exact. **17.** Rye. **18.** 2.205—. **20.** $17\frac{1}{2}$; 19. **21.** 231.8+. **22.** 34+ strokes for 100 gal. **23.** 34.176+ exact, 34.236+ approx. **24.** 85.714+, 67.924+, and 54.545+ mi. per hr. **25.** 352 rails; $5068.80. **26.** 39.75+ exact, 39.68+ approx. **27.** $312.50. **28.** 2347. **29.** 100.2+ lb. **30.** $6\frac{2}{9}$ yd.; $1'' \times 32''$ waste. **31.** 22 in.; waste $5\frac{1}{2}'' \times 16''$. **32.** 40 in. **33.** 2880; 103,680; 22 min. **34.** $93.33. **35.** $47.25.

Page 276. **1.** $48.02; 8.44+%. **2.** $3.023; $.05038. **3.** $24.82; 10.32+%. **4.** $.0169\frac{1}{3}$; $.0225\frac{7}{9}$. **5.** 21.9¢. **6.** 78¢; 80.66+¢. **7.** 131.88+%; 23.26+%; 18.875%. **8.** 18.64+%. **9.** (*a*) 55%; (*b*) 52.38+%; (*c*) 34.375%; (*d*) 54%; (*e*) 87+%; (*f*) 79.09+%; (*g*) 44.164%; (*h*) 82.53+%; (*i*) 28.74—%; (*j*) 21.095%. **10.** $12,710; 101.68%. **11.** (*a*) $107.81; (*b*) $108.28; (*c*) $109.38; (*d*) $109.88; (*e*) $7.82; (*f*) $8.60; (*g*) $.1014; (*h*) $1.62; (*i*) $4.80; (*j*) $107.78. **12.** (*a*) $1.29; (*b*) $1.6125; (*c*) $.14\frac{1}{3}$; (*d*) $.16125; (*e*) $.9675; (*f*) $.0645; (*g*) $2.068; (*h*) $.5442+; (*i*) $3.5846—; (*j*) $1.548. **13.** 271.5 lb.; $.0864—; 65.56 lb.; $5.94.

Page 280. **1.** (*a*) $3.1555, $22.627; (*b*) $2.853, $24.443. **2.** (*a*) 64.35—%, 37.55+%; (*b*) 48.58%, 48.58%. **3.** 42.8¢; 1.04%, 3.07%. **4.** 1.64¢, 3.07¢. **5.** 1.4—¢. 3.22+¢. **6.** 5—¢. **7.** $4.145, $3.317, $.3317, $.4146.

Page 281. **1.** $280.06. **2.** $496.11. **3.** 10¢. **4.** $.10285. **5.** $4.. **6.** $374.53. **7.** $12.07\frac{1}{2}$. **8.** 33¢. **9.** $7.938; $8.7318. **10.** 10.081¢.

Page 283. **1.** dnp/ogk. **2.** dtg/oao. **3.** div/xdk. **4.** dei/xyk. **5.** duo/xen. **6.** opt/xcn. **7.** olv/yxg. **8.** oeg/yik. **9.** nde/gda. **10.** nvo/gdg. **11.** tip/nxo. **12.** tup/aok. **13.** dogp/oeny. **14.** dnvg/xdny. **15.** dten/onie. **16.** dgtp/xgyk.

1. $16,317.05. **2.** $21,564.90. **3.** $33,084.70.

Page 284. **1.** (*a*) 48.42%; (*b*) 68.34%; (*c*) 56.51%; (*d*) 46.87%; (*e*) 43.43%; (*f*) 43.19%; (*g*) 38.64%; (*h*) 28.31%; (*i*) 33.04%; (*j*) 38.19%. Second part, prob. **1.** (*a*) 31.87%; (*b*) 39.05%; (*c*) 32.71%; (*d*) 30.12%; (*e*) 29.13%; (*f*) 28.2%; (*g*) 27.36%; (*h*) 18.8%; (*i*) 24.41%; (*j*) 25.46%. **2.** 63.54%. **3.** $4209.73; 48.05+%.

Page 285. **1.** $24,808.50; $3768.33. **2.** 1.25%; 15.38+%; 31.63+%. **3.** $5.00. **4.** $7.07+. **5.** $1986.50; $6.3466. **6.** $29,681.82. **7.** $55. **8.** $47,573.21; $5328.20. **9.** $1.305; $1.305. **10.** $6025; $1150, $2725, $650, $200, $100, $75, $350, $600, $125, $50. **11.** $180,000.

Page 287. **1.** 2.5%. **2.** $768.75. **3.** $294.12. **4.** $13.64. **5.** $7; 8.75%. **6.** 7.8−%; $8453.48. **7.** $2657.14. **8.** 14.48¢. **9.** $3.95−. **10.** $.0675+; $.4988+. **11.** Lost $8.17. **12.** $5.75. **13.** 2.85+%. **14.** $2922.13. **15.** $26.67; $11\frac{1}{3}$%. **16.** $2.79. **17.** $26.04. **18.** $17,704.63.

Page 289. **1.** 75.16−%. **2.** 1.63+%. **3.** 72.92%. **4.** 35.25%; 14.06%. **5.** 23.51+%. **6.** 14.46%. **7.** 10.39+%. **8.** 62.9−%; 11.22+%; 6.39+%. **9.** 93.02+%. **10.** 41.48−%.

Page 291. **1.** $33.33 loss; $6\frac{2}{3}$%. **2.** $16730.64. **3.** $1437.50. **4.** $3.125. **5.** 73.61%. **6.** $30. **7.** $14,612.50. **8.** $1587; $2380.50. **9.** $4020; $4582.80.

Page 293. 1st Exercise: **1.** $20, $22. **2.** $24, $27. **3.** $30, $36. **4.** $15, $21. **5.** $13.33, $18.33. **6.** $75, $82.50. **7.** $20, $25. **8.** $45, $52.50. **9.** $25, $27. **10.** $75, $87. **11.** $166.67, $171.67. **12.** $83.33, $88.33. **13.** $312.50, $337.50. **14.** $321.43, $366.43. **15.** $666.67, $676.67. **16.** $32, $52. **17.** $80, $81. **18.** $625, $675. **19.** $333.33, $358.33. **20.** $450, $459. **21.** $1800, $1950. **22.** $588.24, $688.24. **23.** $131.58, $181.58. **24.** $450, $486.

2nd Exercise: **1.** $20, $18. **2.** $32, $28. **3.** $20, $15. **4.** $30, $21. **5.** $40, $32. **6.** $13.50, $9. **7.** $15.63, $13.13. **8.** $25, $15. **9.** $250, $245. **10.** $200, $176. **11.** $666.67, $656.67. **12.** $625, $600. **13.** $583.33, $548.33 **14.** $150, $123. **15.** $500, $495. **16.** $71.43, $46.43. **17.** $200, $190. **18.** $1125, $1116. **19.** $1333.33, $1323.33. **20.** $1200, $1182. **21.** $2250, $2235. **22.** $714.29, $564.29. **23.** $18.87, $8.87. **24.** $2160, $2016.

3d Exercise: **1.** $11.82, $1.18. **2.** $14.22, $1.78. **3.** $16.67, $3.33. **4.** $21.43, $8.57. **5.** $21.82, $8.18. **6.** $34.09, $3.41. **7.** $28, $7. **8.** $34.29, $5.71. **9.** $55.56, $4.44. **10.** $77.59, $12.41. **11.** $87.38, $2.62. **12.** $132.08, $7.92. **13.** $185.19, $14.81. **14.** $176.32, $23.68. **15.** $270.94, $4.06. **16.** $193.85, $121.15. **17.** $419.75, $5.25. **18.** $462.96, $37.04. **19.** $813.95, $61.05. **20.** $686.27, $13.73. **21.** $923.08, $76.92. **22.** $1025.64, $174.36. **23.** $1050.72, $399.28. **24.** $1203.70, $96.30.

4th Exercise: **1.** $6.66,\frac{2}{3}$ $6.66\frac{2}{3}$. **2.** $4.57, $.57. **3.** $16, $4. **4.** $24.29, $7.29. **5.** $6.25, $1.25. **6.** $27.75, $9.25. **7.** $28.57, $4.57. **8.** $45, $18. **9.** $38.27, $.77. **10.** $79.55, $9.55. **11.** $76.14, $1.14. **12.** $93.75, $3.75. **13.** $93.09, $5.59. **14.** $231.71. $41.71. **15.** $171.72, $1.72. **16.** $184.62,

$64.62. 17. $381.91, $1.91. **18.** $378.02, $3.02. **19.** $453.40, $3.40. **20.** $598.98, $8.98. **21.** $503.36, $3.36. **22.** $1012.66, $212.66. **23.** $1993.62, $1056.62. **24.** $1232.14, $82.14.

5th Exercise: **1.** 18.18%. **2.** 23.08%. **3.** 42.85%. **4.** 25%. **5.** 20%. **6.** 25%. **7.** 16.67%. **8.** 23.08%. **9.** 3.45%. **10.** 15.38%. **11.** 5.88%. **12.** 3.7%. **13.** 14.29%. **14.** 28.85%. **15.** 3.77%. **16.** 6.78−%. **17.** .23%. **18.** $11\frac{1}{9}$%. **19.** 2.94%. **20.** 1.3%. **21.** 17.65%. **22.** 9.09%. **23.** 3.57%. **24.** 2.85%.

6th Exercise: **1.** 25%. **2.** 50%. **3.** 29.41%. **4.** 34.61%. **5.** 61.54−%. **6.** 19.57%. **7.** 9.43%. **8.** 27.03%. **9.** 11.76%. **10.** 25.53%. **11.** 11.76%. **12.** 21.74%. **13.** 28.57%. **14.** 12.44%. **15.** 2.86%. **16.** 17.24%. **17.** 2.56% **18.** 2.34%. **19.** 2.17%. **20.** 2.96%. **21.** 2.91%. **22.** 15.79%. **23.** 1.06%. **24.** 11.13%.

Page 294. 1. 30¢. **2.** 34¢. **3.** $12.20; 42.43%. **4.** He gained $1.10.

Page 295. 1. 92%. **2.** 44.73%; 110.53%. **3.** 56.52%. **4.** 36%. **5.** 185.71%. **6.** 46.34%. **7.** 77¢. **8.** $7.15. **9.** 1.37\frac{1}{2}$ profit. 12.88%.

Page 296. 1. 31.46+%; 40.19%.

1. $1.90. **2.** 28$\frac{4}{7}$%; 54$\frac{2}{7}$%. **3.** 8.36%; 41.43%. **4.** 39.22−%. **5.** $.79. **6.** 33.36%; $.78 loss. **7.** 38¢.

Page 297. 1. $1.08. **2.** 20%; 60.26%. **3.** $14,000. **4.** 60%. **5.** 27.08¢. **6.** $4168.80. **7.** $1.238. **8.** $2.30.

Page 298. 1. 3.92+%. **2.** 18.24%. **3.** Feed it; $42.05. **4.** $257.09; 124.98−%. **5.** Walking, $2.66; gang, $1.89; **$92.16. 6.** The second plan, by 15¢. **7.** 64.2¢. **8.** 45¢; 3.88¢; 37.5¢.

Page 300. 1. 8%. **2. $105,615. 3.** $2,355.94. **4.** 58.99%. **5.** 76.64% **6.** 13.03%. **7.** incr. of 3.35 in rate. **8.** 6.636%; 8.7%. **9.** $80,061.27. **10.** 3.15% **11.** 4.79%. **12.** 57.24%. **13.** 75.1%. **14.** 19.63%. **15.** 38.59%; 27.84%.

Page 302. 1. $126.20. **2.** $239.94. **3.** $129.73. **4.** $142.42. **5.** $155.93. **6.** $143.25. **7.** $72.56. **8.** $49.66. **9.** $101.41. **10.** $382.58.

Page 304. 1. 10.65 mills. **2.** .326+ mills, .029+ mills, .819+ mills, .154+ mills, .070+ mills. **3.** 11.42 mills.

Page 305. 1. $523,681.74. **2.** 15.79 mills, $1.579, $15.79, 1.579%. **3.** $82,146.23. **4.** $9.59. **5.** .667 mill. **6.** 1.5 mills. **7.** .04742 mill. **8.** .2581 mill. **9.** .2847 mill. **10.** 5.84%. **11.** $2,359,669.86, $471,933.97, $353,950.47, $401,143.88, $94,386.79. **12.** $233,845.41.

Page 308. 1. $115.31. **2.** $118.98. **3.** $130.25.

Page 311. 1. $84. **2.** $437.50. **3.** $82.50. **4.** $411.60. **5.** $58.25. **6.** $562.50. **7.** $58. **8.** $961.50. **9.** $241.11. **10.** $480. **11.** $504. **12.** $18.625. **13.** $544. **14.** $379.33. **15.** $2475. **16.** $117. **17.** $155. **18.** $1155 **19.** $1276.80. **20.** $403.20. **21.** $186.90. **22.** $46.20. **23.** $443.10. **24.** $582.80. **25.** $2478.35.

Page 314. **1.** $11.98. **2.** $44.76. **3.** $119. **4.** 30¢.

1. $3000. **2.** $3000. **3.** $3000. **4.** $500. **5.** $3000. **6.** $2625. **7.** $5750.

Page 316. **1.** $5000. **2.** $3000. **3.** $1953.13. **4.** $300. **5.** $375. **6.** $703.13. **7.** 545.45. **8.** $711.11.

Page 317. **1.** $1600, $1000, $600, $800. **2.** $300, $180, $180, $240. **3.** $2777.78, $1851.85, $4629.63, $4629.63. **4.** $180, $180, $240. **5.** $1350, $900.

Page 318. **1.** $12.38. **2.** $24.75. **3.** $7.96. **4.** $15.39, **5.** $8. **6.** $11.30.

Page 319. **1.** $292.59.

Page 320. **1.** $7000. **2.** B, $5162.50; S, $1837.50. **3.** W, $1500; B $750. **4.** $6.11. **5.** 32.76%. **6.** $84.97. **7.** $6.74. **8.** $1690.

Page 324. **1.** $36.80. **2.** $100.70. **3.** $167.06. **4.** $155.80. **5.** $206.30. **6.** $258.13. **7.** $193.68. **8.** $385.36. **9.** $1171.60. **10.** $298.08. **11.** $2464. **12.** $554.40. **13.** $740.75. **14.** $1556.85. **15.** $184.37. **16.** $2079. **17.** $472.08.

Page 325. **1.** 13 yr. 195 da.; $620. **2.** $137.77. **3.** (1st prob.) 5 yr. 355 da., $297.50; 15 yr., $700. (2d prob.) $137.35, $142.27. **4.** $1251.01. **5.** $161.29. **6.** $438 term, $805.60 ord.; term costs more, $16. **7.** $184.60. **8.** $108. more. **9.** $2058.54 more. **10.** $2,964, 906.16. **11.** $33.13. **12.** $6.75. **13.** $4,805,934.34. **14.** $4.28. **15.** 87.94%.

Page 330. **1.** $45.555. **2.** $54.87. **3.** $1165.60. **4.** $18,070; $19,235.60; $19,985.60. **5.** 6.45+%; 3.89+%. **6.** $28.175. **7.** $56.44.

Page 331. **1.** 25%. **2.** $14.25, $19.95. **3.** $140.65. **4.** $603.24. **5.** $859.87.

Page 332. **1.** 9¢; $30.57. **2.** $46.31. **3.** $1077.25. **4.** $122.11. **5.** $122.61, 6.13¢, $37.39, 30.49%.

Page 334. **1.** $138.69. **2.** 7¢ gain. **3.** $262.14. **4.** $265.96, $425.54

Page 336. **1.** 18.66¢, 13.54¢. **2.** $51.635, $12.64. **3.** $4669.09. **4.** 8.14¢ **5.** 5.55¢.

Page 338. **1.** $1.0625, $2.145, $1.5737, $19.284+. **2.** $3518.04, $4975.30 $6281.30, $2890.36. **3.** $1742.59, $2464.41, $3111.32, $1431.68. **4.** $2581.03, $3650.16, $4608.29, $2120.52. **5.** $17.85. **6.** $7382.81, $11,071.30, $12,304.69, $8616.20; 37.5%. **7.** $19.28. **8.** $19.48. **9.** $11.025, $64.218, $65.391, $703.13. **10.** I. L. $2876.71, $3452.06, $2071.23; B. E., $1307.69, $1634.62, $1307.69; P., $2983.17, $4219.07, $1917.76; I., $181.71, $242.29, $106; T., $171.43, $228.57, $100; D., $937.71, $1250.29, $547; R., $1562.50, $2812.50, $1250; Fact., $1716.67, $2184.85, $1248.48; Misc. S., $276.45, $414.68, $218.87; Misc. Exp., $772.28, $965.35, $212.37.

Page 340. **1.** $70, $105, $140, $105. **2.** $26.03, $52.05, $69.40, $26.02. **3.** $35, $60, $70. **4.** $366.75, $607.51, $782.02. **5.** Dept. O. H.: $.81, $5.37.

$6.51, $8.05, $79.08; Genl., $1.23, $6.53, $7.01, $9.80, $85.18. **6.** $2.70, $3.51, $4.74, $.76; $17.50, $22.87, $29.40, $5.60; $16.25, $22.76, $29.77, $4.73; $26.25, $34.30, $44.10, $8.15; $197.50, $276.58, $361.76, $3.24. **7.** 40.04%; 43.13%; 1.64%; 1.17%; .89%; .89%.

Page 341. **1.** $.887; $2.10. **2.** 1918: $224.73, $315, $258.50. 1919: $116.73, $157.50, none. 1920: $73.53, $94.50, none. **3.** $14.84\frac{3}{8}¢$; $8.91, $6.23, $5.64, $13.66.

Page 342. **1.** $68.23. **2.** 19.8%; 9.59%. **3.** $6.41. **4.** $6.28. **5.** $802.12. **6.** $2.30. **7.** $2.10. **8.** 42.73¢, 77.79¢, 68.81¢. **9.** 10.06%, 8%, 6.65%, 12.36%. **10.** 40.44%, 28.28%, 31.28%. **11.** $2.13. **12.** 8.17%, 8.38%, 12.56%, 11.64%; $7.894, $.9356, $1.2233, $1.0756; 20.81%, same in all depts.; $.9537, $1.129, $1.4779, $1.2995. **13.** 26.31¢. **14.** 60¢. **15.** 27.8¢.

Page 347. **1.** Pp. $.20, pp. $.2575, frt. $1.44, frt. $1.72, Expr. $.715, Expr. $.795, pp. $.315, pp. $.0965, Expr. $.435, pp. $.158, total $6.132. **2.** $.17473. **3.** $1.8\frac{1}{3}¢$; $3.1\frac{1}{3}¢$. **4.** $1.6148, $3.3648. **5.** C. L. by $159. **6.** A, $11.20, B, $38.40, C, $36.80, D, $21.60, E, $48. **7.** $14.37.

Page 349. A, 696,109 lb.; $10,056.21. B, $2245.36, $17,255.65; C, Cent. $6386.37, Read. $10,869.28. D, 1494 mi.; 30.32%, 54.82%, 14.86%.

Page 350. **1.** $1\frac{1}{2}$. **2.** 17. **3.** 15.6¢. **4.** $66.08. **5.** 12. **6.** Royal. **7.** 4400 lb. **8.** 162−lb. **9.** 12 lb. per rm; $2396.16. **10.** $31.50. **11.** $58.80. **12.** $24\frac{1}{2}$, 18, $25\frac{1}{2}$; $29\frac{1}{2}$, $21\frac{1}{2}$, $30\frac{1}{2}$.

Page 351. **1.** 17 days; $12,551; 2201%. **2.** Smoke dept., $1841.85. **3.** $4882.25. **4.** 227 days. 11.1 days; $343.35.

Page 353. **1.** $4.26. **2.** 7.2. **3.** U. S. $\frac{20}{33}$ lb., Fr. $1\frac{2}{3}$ lb. or U. S. $\frac{1}{3}$% reduction to give Fr. 8% incr. **4.** $23\frac{1}{3}$%; 25.3559%. **5.** $2.459. **6.** 9.80375¢. **7.** 3817−%. **8.** 43.8%.

Page 355. **1.** $247\frac{1}{2}$ mi. **2.** 6 hr. 11 min. **3.** 45 mi. **4.** 15 hr. **5.** $6\frac{7}{8}$ da., $2\frac{10}{11}$ A. **6.** $18\frac{2}{11}$. **7.** $62.04. **8.** 180 rows; 378 hills. **9.** $5\frac{5}{16}¢$. **10.** 50¢; 8 pt. **11.** 43.3 ft.; 822 trees.

Page 356. **1.** $1.08. **2.** $2.47. **3.** $2.35. **4.** $41.90. **5.** 1658.8. **6.** $505.40. **7.** $11.57; $8.45. **8.** $323.71.

Page 358. **1.** $162.43. **2.** $323.07. **3.** $593.28. **4.** $4017.10. **5.** $212.24 **6.** $1689.74.

1. $346.81. **2.** $3277.23. **3.** $1796.07. **4.** $18.42. **5.** $52,433.70. **6.** $3377.82. **7.** $5173.78. **8.** 106,792. **9.** 25,398.

Page 361. **1.** $567.44. **2.** $58.23. **3.** $180.34. **4.** $568.88.

Page 362. **1.** $1160. and $776.24. **2.** $461.88 and $815.02. **3.** 59.5+%; 70.26+%. **4.** $770.61. **5.** $6674.20. **6.** $1697.68. **7.** $2750.58. **8.** $196.56. **9.** $233.89. **10.** $478.71. **11.** $156,936.19. **12.** $73,221.33.

Page 367. **1.** $281.14. **2.** $245.44. **3.** $1921.61. **4.** $337.65. **5.** $475.19. **6.** $410.04. **7.** $598.12. **8.** $1152.84. **9.** $892.

Page 370. **1.** $172.70. **2.** $247.10. **3.** $128.63. **4.** $2772.07. **5.** $1542.01; he saved $4.47 in 1st 2 yr. **6.** By second plan pd. 7.132% of $3,000; 5.164% by 1st plan. **7.** $400.06; $305.25.

Page 372. **1.** $297,916.67, $189,583.33. **2.** $121,680; $81,120 **3.** $2,164,861.11, $1,377,638.89. **4.** $72,333.33, $51,666.67. **5.** $348,450; $232,300. **6.** $2650. **7.** $13,500, $5750, $75.

Page 373. **1.** $138.21. **2.** Bond better, $93.75. **3.** $637.50. **4.** 5.025%. **5.** (*a*) 1 to 20.62; (*b*) 79.4%; (*c*) 7.02 to 1. **6.** 99−%.

Page 375. **1.** $366.09. **2.** $328.98. **3.** $220.60. **4.** $129.88. **5.** $324.53. **6.** $4.66. **7.** 6¢ less. **8.** $7.42. **9.** $7.49.

Page 377. **1.** (*a*) 1st N., $13,514.13; 2d N., $21,145.17; 3d N., $18,815.02; 4th N., $9140. (*b*) 1st N., $19,235.80; 2d N., $14,806.50; 3d N., $14,916.19; 4th N., $13,655.83. (*c*) 1st N., $5721.67; 4th N., $4515.83. (*d*) 2d N., $6338.67; 3d N., $3898.83. **2.** Natl. City, Dr. $3695.17; Metr., Cr. $1104.77; Union, Dr. $8992.94; Am. State, Cr $5145.39; City State, Dr. 524.54; Linc., Cr. $6962.49.

Page 381. **1.** $2500.63. **2.** $2996.25. **3.** $184.20. **4.** $3.90 less. **5.** $82.50.

Page 384. **1.** $731.10. **2.** $2913.84. **3.** $244.86. **4.** $1937.66. **5.** $1965.14. **6.** $2307.69. **7.** $1223.93. **8.** $1154.92. **9.** $1774.22. **10.** $1973.72. **11.** 1216.84. **12.** $5816.07. **13.** 502.50. **14.** $1229.68. **15.** $2135.25. **16.** 70.8+%.

Page 385. **1.** £205, 8s, 10.56d. **2.** £512, 16s, 4.9d. **3.** £1026, 13s, 10.56d. **4.** £619, 4s, 8.52d. **5.** £3377, 19s, 2.64d. **6.** £246, 14s, 5.52d. **7.** 2582.5 fr. **8.** 6220.5 fr. **9.** 54,461.51 fr. **10.** 19,439.06 fr. **11.** 16,714.775 fr. **12.** 8465.61 marks. **13.** 51,989.5 marks. **14.** 27,521.83 marks. **15.** 7118.30 marks. **16.** 1575.29 marks. **17.** £163, 6s, 11d, 3 far. **18.** 7779.93. fr.

Page 388. **1.** $175. **2.** $2\frac{1}{2}$%. **3.** $385. **4.** 4.21%. **5.** (*a*) 5.05−%; (*b*) 4.62−%; (*c*) 4.49+%; (*d*) $6\frac{1}{4}$%. **6.** $100, $80, $120, $60, $110. **7.** Stock C, 444 sh., $50. rem. **8.** $96.42. **9.** $150.33. **10.** $1545.

Page 389. **1.** $72,220.625. **2.** $87,745.25. **3.** $125.875. **4.** 59 sh., $21.625 rem. **5.** 46 sh., $140.25. **6.** 403 sh., $26 rem. **7.** 4.94−%. **8.** 4.79+%. **9.** 5.08−%. **10.** $172.79 less.

Page 391. **1.** $39,164.13. **2.** $172,500. **3.** $45,787. **4.** $44,880.

1. 7.94−%. **2.** 149.875. **3.** 4.18%. **4.** $120. **5.** 4.29%. **6.** 3.79%. **7.** $133.33. **8.** The latter is better by 11.98%. **9.** $16,000. **10.** Latter pays .14% more upon inv. **11.** Decr. $50.58.

Page 393. **1.** $1,285.45 loss. **2.** $1,056.90 profit. **3.** $326.40 profit.

Page 396. **1.** $9508.33. **2.** $10,629.58. **3.** $31,015.20. **4.** $5109.58. **5.** $85,582.22. **6.** $21,430. **7.** $3067.93. **8.** $55,542.36. **9.** $90,815.50.

Page 398. **1.** $555.79. **2.** 260.81. **4.** 1344.10. **5.** $1166.10. **6.** $5\frac{1}{2}\%$ **7.** 6%. **8.** $29,378; $1445. **9.** $3000, $3784.80. **10.** $1528.21. **11.** $1178.85.

Page 401. **1.** $1324.47. **2.** $1377.45. **3.** $925.03. **4.** $971.28. **5.** $215.99, $118.55, $70.75. **6.** $1824.43. **7.** $2056.15. **8.** $2234.16.

Page 402. **1.** $72,000. **2.** $19,800. **3.** $88. **4.** $533\frac{1}{3}$ sh. **5.** 2134.56 sh, 1423.04 sh. **6.** $110,000, $92,000, $69,000. **7.** $100,000; $225,000. **8.** $7500. **9.** $95.00; $11.07. **10.** $117.33, $13.90. **11.** $6080.

Page 405. **1.** No. by $11,174.; no. **2.** yes, and $66,528.40 more; yes. **3.** $100.

Page 406. Example. $168.56.

Page 407. **1.** $135.18. **2.** 110.65. **3.** $145.92. **4.** $144.52. **5.** $539.94; $385.67. **6.** $385.67.

Page 408. **1.** 8.65 to 1. **2.** 61.46 to 1. **3.** 1.92 to 1. **4.** 2.24 to 1.

Page 409. **1.** $29,450. **2.** $555,000. **3.** $1,082,500. **4.** 7.9%. **5.** 5.72%. **6.** $12,500, $500. **7.** $7,000,000. **8.** $3,424,830. **9.** $200,000. **10.** $70,000. **11.** 3.33.

Page 413. **1.** $3358.18. **2.** $48,113.81. **3.** $11,753.61. **4.** $51,604.28 less. **5.** $8179.09. **6.** .0285+ mills.

Page 415. **1.** $221.74. **2.** $11,869.82. **3.** $1353.37, $1646.63. **4.** $201.65.

Page 417. **1.** $52,188.02, $104.50. **2.** $395.48, $415.51, $436.53, $458.64, $481.86. **3.** $2761.98. **4.** $102.875. **5.** $105.60.

Page 418. **1.** 98.14145. **2.** $444.24, $457.57, $471.30, $485.44; **1919,** $673.03; **1920,** $942.80; **1921,** $242.72. **3.** 99.125. **4.** $9997.22. **5.** $3400.19. **6.** $4197.72.

Page 419. **1.** $33,000. **2.** $6987.58; $86; $68.80. **3.** $153,875. **4.** $18,000; $120,000; $2503.07. **5.** 9.02%; 14.93%. **6.** $2.89. **7.** $40.99. **8.** $1.60; $36.

Page 421. **1.** $25,200,000. **2.** $20,810,000. **3.** $20,000. **4.** $907,500; $1,815,000. **5.** $1,089,000. **6.** $61\frac{1}{9}$ mi. **7.** $504,166\frac{2}{3}$; $504,166\frac{2}{3}$; $1,008,333\frac{1}{3}$. **8.** $605,000; $74,000. **9.** $201\frac{1}{9}$ mi.; $1,694,000. **10.** $3,077,508.46; $1,391,845.79.

Page 422. **1.** $21,000. **2.** Stock $14\frac{6}{11}\%$ better. **3.** $691.88. **4.** $360; 6.31%. **5.** Incr. $217.50.

1. Cash, by $55.16. **2.** $4425.59. **3.** $22.59. **4.** $1041.01. **5.** $494.55. **6.** $5500. **7.** $32,000. **8.** $1085.85. **9.** $77,217.35. **10.** $100, $105, $104, $103, $102, $101. **11.** $5250. **12.** $168,000.

Page **425.** **1.** Jan. 27. **2.** May 20. **3.** July 19. **4.** June 10. **5.** Nov. 2. **6.** July 2. **7.** June 5. **8.** Aug. 12. **9.** Nov. 17. **10.** June 4. **11.** May 8. **12.** Apr. 5. **13.** Dec. 9. **14.** Aug. 13.

Page **427.** **1.** Sept. 20. **2.** May 12. **3.** Sept. 3. **4.** Nov. 30. **5.** June 3.

Page **429, bottom.** $697.98.

Page **430.** **1.** $193.30. **2.** $1168.68. **3.** $823.12. **4.** $791.08. **5.** June 26, Nov. 11, Feb. 3, Dec. 27. **6.** $207.04. **7.** $484.52.

Page **434.** **1.** M., $6150; S., $1650. **2.** S., $1675; B., $675. **3.** B., $2113.06; C., $1584.79. **4.** S., $4150, T., $5700; S., $3460, T., $5240. **5.** $6232.14, $7478.57, $3739.29. **6.** S., $12,626.72, B., $8713.36. **7.** W. H. C $11,392.62; F. G. C., $11,172.62; $24,485.24. **8.** L. $2868.33; O., $3381.67. **9.** A., $2731.79, B., $2268.21. **10.** D., $6680.59; E., $5163.48; F., $9071.93. **11.** C. takes $5198.70; D. owes $150.65. **12.** Debts, $2000; M., $6000; N., $3500. **13.** G., $1857.14; H., $1142.86.

Page **438.** **1.** $412.50. **2.** $1,075.00. **3.** $6\frac{2}{3}\%$. **4.** 4.7%; 80.5. **8.** 69+%. **6.** 200. **7.** 4%. **8.** 25% more. **9.** $2\frac{2}{3}\%$. **10.** $2\frac{1}{2}\%$. **11.** $691.87; $360.00; 7.2%. **12.** $127,625; 5.48+%; keep it, by $175.00, with $67.37 unexpended. **13.** $10,000.00; $600.00. **14.** $196,000.00. **15.** $4,375.00. **16.** 10%. **17.** 25%. **18.** $12.50 gain. **19.** sales, $8,032.58; cost of G. S. $5,921.65; profit A, $383.86; B, $158.00; C, $32.26; D, $952.96; E, $408.49; F, $175.36; total, $2,110.93. A, 44−%; B, 30−%; C, 18+%; D, 36+%; E, 54−%; F, 19−%. Average, 35.6+%. **20.** $1,646.96; 10.52+%. **21.** 22.8%. **22.** Oldest son, $6,400.00; second son, $5,333.33; third son, $4,571.43; fourth son, $4,000.00; each daughter, $5,847.62; estate, $32,000.00. **23.** $70.00 gain. **24.** Frt., $63\frac{3}{4}\%$; pass., $29\frac{3}{4}\%$; mail, $2\frac{1}{4}\%$; expr., 3+%; misc., $1\frac{1}{5}\%$. **25.** $17\frac{1}{2}$; $\frac{1}{5}$+%.

Page **443.** **1.** $1.33\frac{1}{3}$. **2.** $7.50. **3.** $1.39. **4.** 60%. **5.** $30.00. **6.** $5.04. **7.** $2.40.

Page **444.** **1.** $4.50; $2.25. **2.** 80%. **3.** 5%. **4.** $62,500.00. **5.** $5.00. **6.** $4.00. **7.** $3.28. **8.** $300.98. **9.** $1578.95. **10.** $.104358\frac{1}{3}$. **11.** $8.5755. **12.** Cost $.7535, gross cost $.866525, profit 73.1%. **13.** Gained $15.40; $14.60; 6.53+%. **14.** $.0332; $.0068; $33.50. **15.** 30%. **16.** Order (dozens) $\frac{1}{4}$, $1\frac{1}{6}$, $3\frac{1}{3}$, $2\frac{5}{6}$, $2\frac{5}{6}$, $1\frac{1}{4}$, $\frac{1}{3}$. **17.** $4\frac{1}{6}$, $9\frac{1}{3}$, $9\frac{1}{3}$, $9\frac{1}{4}$, $9\frac{1}{3}$, $4\frac{2}{3}$, $3\frac{11}{12}$. **18.** $1\frac{7}{12}$, $3\frac{5}{12}$, $4\frac{2}{3}$, $3\frac{5}{6}$, $3\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{2}$.

Page **448.** **1.** $128.32. **2.** $329.96. **3.** $4,315.72. **4.** $86.45. **5.** $4,900.

Page **449.** **5.** Width 8 in., length 16 in. **6.** Hat $8.00, gloves $2.00, shoes $4.00. **7.** 1st. 135, 2nd. 142. **8.** Calif. 11, Iowa 11, Ohio 22.

Page **451.** **1.** $2.26. **2.** $2.58. **3.** $.69. **4.** $.45.

Page **452.** **1.** $269.06. **2.** $79.26. **3.** $231.34. **4.** $2.22. **5.** 4.5%. **6.** The former by 14¢. **7.** No. 1 $26.41, No. 2 $12.72, No. 3 $27.13, No. 4 $25.63; No. 2 worth $47.39. **8.** Better to sell at $63\frac{1}{2}$ by 14¢. **9.** Gain in 6 mo. $19.80.

Page 454.

1. Would have gained $18,600. **2.** $3,850. **3.** Lose $60,400.

Page 455.

1. $204.23. **2.** Saving $237. **3.** Saves $105. **4.** Gain $4.50. **5.** 31¢.

Page 456.

1. Net saving $714. **2.** B's farm worth $1,000 more than A's. **3.** $162. **4.** 4840. **5.** 8.6%. **6.** Saved $3. **7.** 87+ acres. **8.** 600 days. **9.** Corn $514\frac{2}{7}$ bu., silage 61 T., oats $312\frac{1}{2}$ bu., hay $19\frac{1}{2}$ T. **10.** $95\frac{5}{11}$ bu. **11.** $127.72. **12.** $5\frac{1}{3}$¢. **13.** Gain 18¢. **14.** $730. **15.** Gains $311.66\frac{2}{3}$. **16.** Gain more by feeding, $930. **17.** Shelter saves $650. **18.** Makes $1,330. **19.** More by $2,650. **20.** $1,000; $24; gains $4. **21.** Gains $11.90. **22.** Gains $113.37. **23.** $2.2\frac{6}{7}$%. **24.** $212.50. **25.** $1,044. **26.** $36. **27.** $2,481.40. **28.** $9. **29.** $6.86. **30.** $1,400. **31.** Cash by $5. **32.** Daisy $33.36; Bossy $38.88; Jersey $30.69. **33.** $88.61.

22 Bricks to Cut ft.

May 26, 27